1. E
2. C
3. D
4. D
5. A
6. B
7. D
8. C
9. E
10. A
11. B
12. C
13. B
14. D
15. C
16. A
17. A
18. C
19. D
20. A
21. D
22. E
23. A
24. D
25. C
26. A
27. B
28. B
29. A
30. D

1. E $(x-6)(x+1)=0 \rightarrow x=6,-1$
2. C Let $x=\log_2 a$, then the equation becomes
$\frac{x}{4}+\frac{x}{2}+x=\frac{8}{x}+\frac{9}{x}+\frac{11}{x} \rightarrow \frac{7x}{4}=\frac{28}{x} \rightarrow x^2=16 \rightarrow x=\pm 4$
3. D There are two ways to get x^3 term, using either $(2x^2)^2\left(\frac{1}{x}\right)^1$ or x^3. There are 3 ways to get the first, giving $12x^3$ and 1 way to get the second, give x^3. The coefficient is 13.
4. D The $p\%$ discount makes the price $100-p$, adding $p\%$ back gives
$$(100-p)\left(1+\frac{p}{100}\right)=100+p-p-\frac{p^2}{100}=100-\frac{p^2}{100}$$
5. A Cubing the given equation gives $x^3+3x+\frac{3}{x}+\frac{1}{x^3}=3\sqrt{3}$, thus $x^3+\frac{1}{x^3}=0$
6. B When $x=0$, $f(0)+f(1)=0$, so $f(1)=1$, repeat the process for $x=1,2,\ldots,9$ to eventually get $f(10)=5$
7. D $(1+i)^2=2i$, so $(1+i)^4=-4$, so for the given equation to be true, n must be a multiple of 4, since $-4\neq 4$, $n=8, m=2$
8. C $x\equiv 1(mod\ 2), x\equiv 2(mod\ 3), x\equiv 3(mod\ 5)$. The first two congruences yield $x\equiv 5(mod\ 6)$, combining with the last to get $x\equiv 23(mod\ 30)$
9. E $S_k=\left(\frac{k(k+1)}{2}\right)^2$, so it is always a perfect square.
10. A $r_1+r_2+r_3=10, r_1+r_2+r_4=-2, r_1r_2r_3=30, r_1r_2r_4=-42$. From these, we get $r_3-r_4=12, and\ \frac{r_3}{r_4}=-\frac{5}{7}$. Solving gives $r_3=5, r_4=-7$.
11. B Setting the fraction equal to x, we have $x=\frac{1}{1+\frac{1}{2+x}}$. Simplifying gives $x^2+2x-2=0$, or $x=-1\pm\sqrt{3}$. Since x is clearly positive, it just be $-1+\sqrt{3}$.
12. C $\log m=11\log 3+14\log 5$, plugging in the approximations, it is 15.033.
13. B Compare the logs of each value, $\log 3^{29}\approx 8.729, \log 5^{20}\approx 13.980, \log 7^{15}\approx 12.675, \log 10^{13}=13$.
14. D We consider 3 cases: $x^2-x-1=1, x^2-x-1=-1, x^2-2x-8=0$. Note that x^2-2x-8 must be even in the second case. Case 1 has two solutions: $2,-1$. Case two has one solution, 1. Case 3 has 2 solutions, $4,-2$, for 5 solutions total.
15. C $0.\overline{10}_2=\frac{10_2}{11_2}=\frac{2}{3}, 0.\overline{20}=\frac{20_3}{22_3}=\frac{6}{8}=\frac{3}{4}, 0.\overline{30}_4=\frac{30_4}{33_4}=\frac{12}{15}=\frac{4}{5}$
16. A If the fourth side is of length 6, then the quadrilateral is degenerate, as $4+7+6=17$.
17. A From the question, we get 3 equations:
$$\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}=2$$
$$\sqrt[3]{abc}=6$$
$$\frac{a+b+c}{3}=10$$
Simplifying the third equation, we get $a+b+c=30$. Squaring both sides, we get $a^2+b^2+c^2+2ab+2bc+2ac=900$. From this, we get

$$a^2 + b^2 + c^2 = 900 - 2(ab + bc + ac)$$

Simplifying the first equation, we get $\frac{3}{\frac{bc+ac+ab}{abc}} = \frac{3abc}{bc+ac+ab} = 2$. Multiplying both sides by $bc + ac + ab$, we get $3abc = 2(bc + ac + ab)$. From the second equation, by cubing both sides, we get $abc = 216$. So, substituting this into the second equation, we get $3(216) = 648 = 2(ab + ac + ab)$. Substituting this back into our first equation, we get $a^2 + b^2 + c^2 = 900 - 684 = 252$

18. C Multiply the two given equations yields $\frac{a+b+c}{a} + \frac{a+b+c}{b} + \frac{a+b+c}{c} = 80$. The left side is the desired expression with an additional 1 $\left(\frac{a}{a}, \frac{b}{b}, \frac{c}{c}\right)$ on each fraction. So the final answer is 77.

19. D Set the expression to x, we have $x = \sqrt{2 + \sqrt{2 - x}}$ squaring both sides twice (and rearrange after each squaring), it simplifies to $x^4 - 4x^2 + x + 2 = 0$. This solves to $1, -2, \frac{1 \pm \sqrt{5}}{2}$. As $x > 1$, it must be $\frac{1+\sqrt{5}}{2}$

20. A This is the equality case of AMGM, which requires $5 = x = y$, so $x + y = 10$.

21. D Sum of the squares of the roots is $19^2 - 2(0) = 361$.

22. E The smallest integer that satisfies this is 3000, and the largest is 3333. All multiples of 3 between also satisfies, for a total of 112 such integers.

23. A To create this figure, we cut out four 4×4 squares from the corners and fold up the flaps. The volume of the cube is 64, and the total area removed is also 64.

24. D Factor $g(x) = (x - 29)(x + 15)$, thus $x - 25 = 29, -15$ or $x = 54, 10$

25. C These 3 equation are

$$x(x + y + z) = 50$$
$$y(x + y + z) = 40$$
$$z(x + y + z) = 10$$

Since $x, y, z, x + y + z \neq 0$, we can safely divide our equations. Dividing the second equation by the first and the third equation by the first, we get

$$\frac{y}{x} = \frac{4}{5}$$
$$\frac{z}{x} = \frac{1}{5}$$

So, we get $y = \frac{4}{5}x$ and $z = \frac{1}{5}x$. Substituting this back into the first equation, we get $x\left(x + \frac{1}{5}x + \frac{4}{5}x\right) = x(2x) = 2x^2 = 50$. So, we get $x = \pm 5$. However, since the question states $x > 0$, we know that $x = 5$. So, we get $y = 4, z = 1$ and $x + y + z = 10$.

26. A It is fairly obvious that our two numbers must be of the form aba and bab. Without loss of generality, let $a > b$, so $100a + 10b + a = 100b + 10a + b + 728$ or $91a = 91(b + 8)$. Since a, b are single digit numbers, $a = 9, b = 1$. $919 + 191 = 1110$.

27. B Completing the square on the first inequality gives $\frac{(x+2)^2}{36} + \frac{(y-1)^2}{4} < 1$. This is the interior of an ellipse centered at $(-2, 1)$. Note the second inequality is a line that passes through the center, so the desired area is half of the ellipse: $\frac{1}{2}\pi(6)(2) = 6\pi$

28. B Looking at the equations, we notice they look eerily like sum of roots, sum of roots two at a time, and product of roots. Say $z = -c, x = a, and\ z = b$ where a, b, c are the roots of some polynomial. Now, the equations are

$$-a - b - c = -3$$
$$ab + bc + ac = -24$$
$$-abc = 80$$

So, from this, we gather that the polynomial is $x^3 - 3x^2 - 24x + 80$. This factors into $(x-4)^2(x+5)$. So, the ordered triplets are (4,4,-5),(4,5,-4), and (5,4,-4) so the answer is 3.

29. A You could just solve for x and plug it in, however we want to avoid this because there is a lot of multiplying with imaginary numbers which may get confusing. So, we think to use some algebraic manipulation to avoid arithmetic.
We can rewrite what we are finding as

$$(x+1)(x+7)(x+4)^2 = (x^2+8x+7)(x^2+8x+16)$$

From the first equation, we get $x^2 = -3x - 4$. Substituting this into what we are solving for, we get $(5x+3)(5x+12) = 25x^2 + 75x + 36$. Substituting once again, we get $25(-3x-4) + 75x + 36 = -64$

30. D Say $a = x + 3$ and $b = y - 4$. Then, the first equation turns into

$$\sqrt{a-1} + \sqrt{a} + \sqrt{a+1} = \sqrt{b-1} + \sqrt{b} + \sqrt{b+1}$$

From this, we can see this is true when $a = b$ or in other words when $x + 3 = y - 4$. This gives us $y = x + 7$. Plugging this into the second equation, we get

$$(x+7)^2 - 2(x+7) - x^2 + x = 126$$
$$x^2 + 14x + 49 - 2x - 14 - x^2 + x = 126$$
$$13x + 35 = 126$$
$$13x = 91$$
$$x = 7$$
$$y = x + 7 = 7 + 7 = 14$$
$$x + y = 7 + 14 = 21$$