Theta

Equations and Inequalities

Test #412

Directions:

1. Fill out the top section of the Round 2 Google Form answer sheet and select **Theta-Equations and Inequalities** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

- NOTA = None of the given answer choices are correct
- $i = \sqrt{-1}$
- Unless otherwise noted, $\log x = \log_{10} x$
- The symbol $[x]$ denotes the greatest integer less than or equal to x.

1. Solve for x: $x^2 - 5x - 6 = 0$

   A. $2, 3$ B. $-2, 3$ C. $2, -3$ D. $-2, -3$ E. NOTA

2. Solve for a over the integers:

   $$\log_{16} a + \log_4 a + \log_2 a = \log_a 256 + \log_a 512 + \log_a 2048$$

   A. 4 B. 8 C. 16 D. 32 E. NOTA

3. Find the coefficient of the x^3 term in the expansion of $\left(2x^2 + x + \frac{1}{x}\right)^3$

   A. 1 B. 5 C. 12 D. 13 E. NOTA

4. Kevin is selling pots for 100 rupees, and as a special promotion, he's giving a p% discount to the first 50 customers. However, after serving the first 50 customers, he erroneously tries to fix the price by adding back p% of the discounted price. What price, in rupees, do the pots actually cost now?

   A. $100 + \frac{p}{100}$ B. $100 - \frac{p}{100}$ C. $100 + \frac{p^2}{100}$ D. $100 - \frac{p^2}{100}$ E. NOTA

5. Given that $x + \frac{1}{x} = \sqrt{3}$ for some $x \in \mathbb{C}$, what is the value of $x^3 + \frac{1}{x^3}$?

   A. 0 B. $2\sqrt{3}$ C. $2\sqrt{3} - 2$ D. $3\sqrt{3}$ E. NOTA

6. Let f be a function such that $f(x) + f(x+1) = x$. Given that $f(0) = 0$, what is the value of $f(10)$?

   A. 4 B. 5 C. 6 D. 7 E. NOTA

7. Let m, n be positive integers such that $(1+i)^m = 4^n$. What is the smallest possible value of $m+n$?

A. 3 B. 5 C. 8 D. 10 E. NOTA

8. When x is divided by 2, it leaves a remainder of 1. When x is divided by 3, it leaves a remainder of 2. Finally, when x is divided by 5, it leaves a remainder of 3. What is the remainder when x is divided by 30?

A. 11 B. 17 C. 23 D. 29 E. NOTA

9. Consider the sequence $S_k = \sum_{n=1}^{k} n^3$. What is the smallest positive integer value of k such that S_k is not a perfect square.

A. 2 B. 3 C. 5 D. 7 E. NOTA

10. It is known that $f(x) = x^3 - 10x^2 + ax - 30$ has roots r_1, r_2, r_3 and that $g(x) = x^3 + 2x^2 + bx + 42$ has roots r_1, r_2, r_4. What is $r_3 + r_4$?

A. -2 B. 4 C. 6 D. 12 E. NOTA

11. Evaluate the following:

$$\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\ddots}}}}}$$

A. $\frac{-1+\sqrt{2}}{2}$ B. $-1+\sqrt{3}$ C. $\frac{-1+\sqrt{5}}{2}$ D. $-2+\sqrt{5}$ E. NOTA

12. Let $m = 3^{11}5^{14}$ what is the value of $\lfloor \log m \rfloor$? (Hint: $\log 3 \approx 0.477, \log 5 \approx 0.699$)

A. 9 B. 12 C. 15 D. 25 E. NOTA

13. Which of the following is the largest: $3^{29}, 5^{20}, 7^{15}, 10^{13}$? (Hint: $\log 7 \approx 0.845$)

A. 3^{29} B. 5^{20} C. 7^{15} D. 10^{13} E. NOTA

14. How many real solutions does the following equation have?

$$(x^2 - x - 1)^{x^2-2x-8} = 1$$

A. 2 B. 3 C. 4 D. 5 E. NOTA

15. Which of the following is the largest: $0.10\overline{10}_2, 0.20\overline{20}_3, 0.40\overline{40}_5$?

A. $0.10\overline{10}_2$ B. $0.20\overline{20}_3$ C. $0.40\overline{40}_5$ D. All equal E. NOTA

16. A non-degenerate quadrilateral has 3 of its sides with lengths 4, 7, and 17. Which of the following cannot be the length of the fourth side?

A. 6 B. 10 C. 18 D. 27 E. NOTA

17. There are 3 numbers a, b, c. The harmonic means of the numbers is 2, the geometric mean is 6, and the arithmetic mean is 10. What is the value of $a^2 + b^2 + c^2$?

A. 252 B. 364 C. 648 D. 684 E. NOTA

18. If $a + b + c = 4$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 20$, find the value of $\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}$.

A. 5 B. 24 C. 77 D. 80 E. NOTA

19. Evaluate $\sqrt{2 + \sqrt{2 - \sqrt{2 + \sqrt{2 - \cdots}}}}$

A. 0 B. 1 C. $\frac{-1+\sqrt{5}}{2}$ D. $\frac{1+\sqrt{5}}{2}$ E. NOTA

20. Find the value of $x + y$ that satisfies the following equation: $\frac{5+x+y}{3} = \sqrt[3]{5xy}$.

A. 10 B. 24 C. 30 D. Need more Information E. NOTA

21. Find the sum of the squares of the roots of the following polynomial equation:

$$x^5 - 19x^4 + 18x^2 - 69x + 14 = 0$$

A. 37 B. 120 C. 325 D. 361 E. NOTA

22. How many positive integers n have the property that $\frac{n}{3}$ and $3n$ are both 4-digit integers?

A. 55 B. 111 C. 222 D. 333 E. NOTA

23. Sophia loves hexahedrons, but hates wasting paper. She decides make an open top cube out of a piece of square paper with side length 12 by cutting congruent squares out of each corner and folding up the flaps. What is the numeric difference between the volume of the cube and the area of all four squares of paper she cut out? (Ignore all units.)

A. 0 B. 16 C. 48 D. 180 E. NOTA

24. Let $f(x) = x - 25$ and $g(x) = x^2 - 14x - 435$. What values of x gives $g(f(x)) = 0$?

A. $-10, -54$ B. $-4, 40$ C. $-29, 15$ D. $10, 54$ E. NOTA

25. $x^2 + xy + xz = 50$
$y^2 + yx + yz = 40$
$z^2 + zx + zy = 10$
What is $x + y + z$, where x, y, z are real numbers and $x > 0$?

A. 0 B. 5 C. 10 D. 15 E. NOTA

26. Two 3-digit palindromes differ by 728 and share the same digits (for instance, 131 and 313 share the same digits). What is the sum of the two palindromes?

A. 1110 B. 1156 C. 1221 D. 1331 E. NOTA

27. Find the area of the region that satisfies $x^2 + 4x + 9y^2 - 18y \leq 23$ and $3x + 2y \geq -4$.

A. 4π B. 6π C. 9π D. 12π E. NOTA

28. How many ordered triplets (x, y, z) of real numbers satisfy the following equations

$z - x - y = -3$

$xy - yz - xz = -24$

$xyz = 80$

A. 0 B. 3 C. 6 D. 27 E. NOTA

29. $x^2 + 3x + 4 = 0$. What is $(x + 1)(x + 4)^2(x + 7)$?

A. -64 B. -48 C. -32 D. -16 E. NOTA

30. $$\sqrt{x+2} + \sqrt{x+3} + \sqrt{x+4} = \sqrt{y-3} + \sqrt{y-4} + \sqrt{y-5}$$
$$y^2 - 2y - x^2 + x = 126$$

What is $x + y$?

A. B. 14 C. 18 D. 21 E. NOTA