1. D
2. B
3. D
4. C
5. B
6. E
7. A
8. A
9. B
10. C
11. C
12. A
13. A
14. D
15. B
16. A
17. C
18. C
19. D
20. B
21. C
22. A
23. D
24. C
25. D
26. C
27. C
28. D
29. B
30. A

1. D From 11 students, 4 can be selected in ${}_{11}C_4 = 330$ ways. For there to be more girls than boys, there must be either 3 girls and 1 boy (${}_6C_3 * {}_5C_1 = 20 * 5 = 100$) or 4 girls and 0 boys (${}_6C_4 = 15$). Thus, the probability is $\frac{10+15}{330} = \frac{23}{66}$.
2. B The constant term is ${}_5C_2(4x^3)^2\left(\frac{-2}{x^2}\right)^3 = -1280.$
3. D A set with n elements has 2^n subsets. $2^6 = 64$.
4. C Andrew has (4 dresses)*(5 hats)*(${}_6C_2 = 15$ accessories)=300 outfits.
5. B $P(X \cap Y) = P(X) + P(Y) - P(X \cup Y) = 0$, so E and F must be mutually exclusive
6. E The 4 distinct numbers on the slips can be arranged in 4!=24 ways. Of these 24 ways only one arrangement is in descending order, so the probability is 1/24.
7. A Using Bayes Rule: $P(Bottom\ Banana \mid Boba\ Without\ Ice) = P(Bottom\ Banana\ and\ Boba\ Without\ Ice)/P(Boba\ Without\ Ice) = (.3 * .16)/.2 = .24$
8. A The prime factorizations of 420 and 2860 are $2^2 * 3 * 5 * 7$ and $2^2 * 5 * 11 * 13$, respectively. Any positive integral divisors of 420 that are also divisors of 2860 must come from $2^2 * 5$ since that is the part of the factorization shared by both. Therefore, there are a total of $(2 + 1)(1 + 1) = 6$ such divisors.
9. B There is a probability of 4/5 for a die to not show a 2 since the roll cannot show a 1. For 3 dice, $\left(\frac{4}{5}\right)^3 = 64/125$.
10. C Within each square, there is a smaller square where the center of the disk can land without the disk touching the square's side. The smaller square's length is $length\ of\ square - 2(radius) = 16 - 2 \cdot 2 = 12$. Thus, $P(not\ touching) = \frac{12^2}{16^2} = \frac{9}{16}$.
11. C The initial probability of drawing a certain toy is the same as the probability of drawing that certain toy at any future point, so the probability of getting a dog toy on the fourth draw is the same as getting it on the first draw, or 5/8.
12. A In order for Bryan to win on the fifth match, both Bryan and Andy must win two times in the first four matches in any order, and Bryan must win the fifth game. Thus, the probability is ${}_4C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)$.
13. A We can think of this as a dividing marbles into bags problem. We have three containers (x, y, and z) that we wish to fill with 10 marbles. Since each x, y, z are positive integers, each container has at least one marble, so we only need to sort 7 items into 3 containers/choose where to place two dividers between 7 items: $\binom{9}{2}$.
14. D Let us calculate the probability of not getting a matched pair. This occurs if we pick three same hand black gloves, two same hand white gloves and any black glove, or two same hand black gloves and any white glove.
BBB: $\frac{6}{10} * \frac{2}{9} * \frac{1}{8} = \frac{1}{60}$
BBW: $\left(\frac{6}{10} * \frac{2}{9} * \frac{4}{8}\right) * 3 = \frac{12}{60}$ (multiplying by 3 for the 3 permutations of BBW, BWB, WBB)

WWB: $(\frac{4}{10} * \frac{1}{9} * \frac{6}{8}) * 3 = \frac{6}{60}$
$P = 1 - (\frac{1}{60} + \frac{12}{60} + \frac{6}{60}) = \frac{41}{60}$.

15. B We want to find the numbers of integers up to 732 containing a digit 1.
In each set of 100 integers, the number of integers with units digit 1 is 10 and with tens digit 1 is 10. We subtract one for the overlap (11) for a total of 19.
0-99, 200-299, 300-399, 400-499, 500-599 and 600-699 will contain 19 integers containing 1: $19 * 6 = 114$
100-199 will have 1 in each integer: 100
For 700-732, we have 3 numbers with 1 in the units digit and 10 in the tens digit: 13
Total: $114 + 100 + 13 = 227$

16. A If all 5 elements are fixed, the identity map works.
If 3 of the 5 elements are fixed, there are ${}_5C_3$ ways to choose these 3 elements and exactly 1 way to assign the last two if they satisfy the property.
If 1 of the 5 elements are fixed, there are ${}_5C_1$ ways to choose this element. WLOG assume 1 is not fixed. Then there are exactly 3 ways to choose which element 1 maps to, which makes the remaining two elements fixed.
Totally there are 26 cases that work out of the possible 120.

17. C
$$P(Rd\ 1) + P(Rd\ 2) + P(Rd\ 3) = \frac{1}{7} + \frac{2}{7}\left(\frac{3}{4}\right)\left(\frac{3}{5}\right) + \frac{4}{7}\left(\frac{3}{4}\right)^2\left(\frac{3}{5}\right)^2 = \frac{271}{700}$$

18. C You could get this easily going from the 4th row to the 5th row of Pascals Triangle or you could take the annoying route of expanding it. You get 161051. Answer 14

19. D Base can be 2, 4, 8, or 64. Case work
Base=2 then AW=6 4 ways Base=4 then AW=3 2 ways
Base=8 then AW=2 2 ways Base =64 then AW=1 1 way 4+2+2+1=9

20. B We can always arrange the points so the x coordinates in order. The probability that their y and z coordinates happen to fall in order in this configuration is $\frac{1}{24}^2 = \frac{1}{576}$, since there exists exactly 1 way to arrange each set of coordinates in ascending alphabetical order.

21. C This is the same thing as computing the expected value of one cup and subtracting it from the total amount of money under the hood, which gives \$120-\$20 = \$100.

22. A By drawing out the possibilities (for example, in a table), it becomes clear that the distributions of the sum of each of the two players' dice are then same, so by symmetry, the probability Buff's is bigger is $\frac{1-P(tie)}{2}$. We have
$$P(tie) = \frac{1^2+2^2+3^2+4^2+5^2+6^2+5^2+4^2+3^2+2^2+1^2}{36^2} = \frac{146}{1296}$$
Using the form above gives A.

23. D First note that the expression in question is also $sin(\theta_1 + \theta_2)$. No matter what Caroline gets, Lion can pick t so that Caroline's number plus all possible results in the distribution will be nonnegative by making it always fall between $[\pi, 2\pi]$ where sine is negative. For example, if Caroline rolls 1, then Lion would pick his interval to be $[\pi - 1, 2\pi - 1]$. Therefore, the probability of winning is 1.

24. C Must be a multiple of 420 that is 4 digits. That is 1260 so 1+2+6+0=9

25. D Using Bayes' Rule: $\frac{2(4)}{2(4)+2(2)+1(1)} = \frac{8}{13}$.

26. C We can use a state-recurrence using the parity of the current running sum to solve this. Note that 0 is even, so we will be solving for Se the even state (So is the odd state):

$$Se = \frac{1}{3}Se + \frac{1}{2}So + \frac{1}{6} \qquad So + Se = 1$$

This system solves to give $Se = \frac{4}{7}$.

27. C Drawing the perpendicular bisectors between the center and each of the vertices and taking the intersections of the regions where points are closer to the center will give a region in the center that is also a regular hexagon. The ratio of the side lengths is $\sqrt{3}: 1$ for the bigger hexagon to smaller hexagon. This makes the probability $\left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

28. D If Olivia goes first and misses, then the game becomes the same as if Jeff would have started, so the ratio of the two probabilities is $1 - p2$.

29. B To find the range of points in which a cevian will be shorter than the median, we can simply reflect the median over the altitude of the triangle (minimum length cevian). This gives us an isosceles triangle, in which we need to compute the length of the base, with legs of length 5. Note that the altitude has length 24/5 while the hypotenuse of half the isosceles triangle has length 5, so $sin(\theta) = \frac{24}{25}$ if θ is a base angle which makes the base of the triangle $\frac{2*7}{5} = \frac{14}{5}$.This makes the probability if chosen uniformly and at random $\frac{14}{5}/10 = \frac{7}{25}$.

30. A We can use tangent subtraction to get that $tan(arctan(1) - arctan(1/2)) = 1/3$. However, we are looking for the value of $\frac{arctan\left(\frac{1}{3}\right)}{2}$ to get the probability. We can use the half angle formula to get that the tangent of this is $\sqrt{10} - 3$.