Alpha

Combinations and Probability

Test #423

Directions:

1. Fill out the top section of the Round 2 Google Form answer sheet and select **Alpha-Combinations and Probability** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10.   Unless a question asks for an approximation or a rounded answer, give the exact answer.

For all questions, answer E) NOTA means none of the above answers is correct. Any question mentioning "die/dice", "cards", or "coins" assumes that those objects are fair and standard, unless otherwise stated. Good luck and have fun!

1. 4 students are to be selected at random out of 5 boys and 6 girls. What is the probability that selection has more girls than boys?
    A. 1/33   B. 12/33   C. 15/33   D. 23/66   E. NOTA

2. What is the constant term in the expansion of $(4x^3 - \frac{2}{x^2})^5$?
    A. 1280   B. -1280   C. 2560   D. -2560   E. NOTA

3. If set $A = \{1,2,3,5,8,13\}$, how many subsets does set A have?
    A. 6   B. 16   C. 32   D. 64   E. NOTA

4. Andrew is dressing up his virtual character and can make one outfit consisting of exactly one dress, one hat, and two accessories. He has four distinct dresses, five distinct hats, and six distinct accessories. Given his choices, how many distinguishable outfits can Andrew make?
    A. 30   B. 150   C. 300   D. 600   E. NOTA

5. If $P(X) = 0.30, P(Y) = 0.50$, and $P(X \cup Y) = 0.80$, then events X and Y must be:
    A. Independent   B. Mutually Exclusive   C. Both A and B   D. Neither A nor B   E. NOTA

6. Ace is drawing raffle winners from a box of paper slips, numbered 1 through 50. He picks four slips at random without replacement, one after the other. What is the probability that the numbers on the slips, in the order he picks them up, are in descending order?
    A. 1/4   B. 1/5   C. 1/12   D. 1/50   E. NOTA

7. In Murica, 30% of people peel their bananas from the bottom and 16% of people who peel their bananas from the bottom drink their boba without ice. Given that 20% of people in Murica drink their boba without ice, what is the probability that a randomly selected person peels their bananas from the bottom if they drink their boba without ice?

A. 0.24   B. 0.16   C. 0.048   D. 0.024   E. NOTA

8. How many positive integral factors of 420 are also factors of 2860?

A. 6   B. 8   C. 10   D. 12   E. NOTA

9. Three fair dice are rolled. If none show a 1, what is the probability that none show a 2?

A. 8/27   B. 64/125   C. 125/216   D. 2/3   E. NOTA

10. A square grid is composed of 100 small squares, each with side length of 16. A circular disk of diameter 4 is tossed at the grid, and the disk's center lands on the grid. What is the probability that the disk does not cover any part of any side of any of the 100 small square within the grid?

A. 5/16   B. 7/16   C. 9/16   D. 11/16   E. NOTA

11. A box contains 3 cat toys and 5 dog toys. One by one, every toy is selected at random without replacement. What is the probability that the fourth toy selected is a dog toy?

A. 1/4   B. 1/2   C. 5/8   D. 2/3   E. NOTA

12. Andy has a $\frac{2}{3}$ chance of winning any given chess match against Bryan. In a best of five series, what is the probability that Bryan wins the series on the fifth match?

A. 8/81   B. 16/81   C. 4/243   D. 8/243   E. NOTA

13. If x, y, z are positive integers, how many solutions does the following equation have?
$$x + y + z = 10$$

A. $\binom{9}{2}$ B. $\binom{13}{3}$ C. 10^3 D. $10 \cdot 9 \cdot 8$ E. NOTA

14. A box contains three pairs of black gloves and two pairs of white gloves. Each pair consists of a left-hand glove and a right-hand glove. The gloves are separated and mixed in the box. If three gloves are randomly selected from the box, what is the probability that a matched set (a left- and right-hand glove of the same color) will be among the three gloves selected?

    A. 3/10   B. 23/60   C. 7/12   D. 41/60   E. NOTA

15. A probability textbook contains 732 pages numbered from 1 to 732. If a page is selected at random from the book, what is the probability that the page number contains the digit 1?

    A. 184/732   B. 227/732   C. 233/732   D. 254/732   E. NOTA

16. Let $S = \{1,2,3,4,5\}$ and P be the set of all permutation functions of S. Let f be a randomly selected function from P. What is the probability that, for all pairs $(e1, e2) \in (S \times S)$, that f has the property that if $f(e1) = e2$, then $f(e2) = e1$?
    Note: a permutation function $g \in P$ is a bijection of S onto itself. For example, $g(1) = 2$, $g(2) = 3, g(3) = 5, g(4) = 4, g(5) = 1$, corresponds to the permutation 2, 3, 5, 4, 1.

    A. 13/60   B. 31/120   C. 3/10   D. 41/120   E. NOTA

17. Hiro and Haru are playing in a Super Smash Bros. tournament, which consists of a single-elimination bracket with 8 participants - each participant is randomly paired against another, and the winner advances to the next round and plays against one of the other winners (and so on until only one is left). Haru has a 60% chance of beating Hiro and an 75% chance of beating anyone else in the tournament, while Hiro only has a 60% chance of beating anyone other than Haru. If there are no chances for a tie, what is the probability that Hiro and Haru play against each other in the tournament?

    A. 81/700   B. 619/700   C. 271/700   D. 429/700   E. NOTA

18. What is the sum of the digits in 11^5 ?

    A. 11   B. 12   C. 14   D. 17   E. NOTA

19. How many ordered triples (J, A, W) of positive integers satisfy $(J^A)^W = 64$?
    A. 4   B. 6   C. 8   D. 9   E. NOTA

20. Four points are selected at random within the unit cube with $(0, 0, 0)$ and $(1, 1, 1)$ as opposite vertices. The four points are then listed in three ascending orders for each of the three coordinates. What is the probability that all three orderings are the same?
    A. 1/13824   B. 1/576   C. 1/288   D. 1/24   E. NOTA

21. Trick2g puts out 6 cups, and he tells you that there are 3 cups that each contain a $10 bill, 2 cups that each contain a $20 bill, and 1 cup that contains a $50 bill. Trick2g is generous, so he will shuffle the cups while upside down, let you choose 5 cups, and give you their contents. What is the expected amount of money, in dollars, that you will get from this deal?
    A. 90   B. 95   C. 100   D. 105   E. NOTA

22. Ryan and Buff each have two dice. Ryan's dice are both standard 6-sided die, while Buff's dice are non-standard (though still fair). One of Buff's dice has faces {1,2,2,3,3,4} and the other has faces {1,3,4,5,6,8}. If Buff and Ryan each roll both their dice once, what is the probability that the sum of the top faces of Buff's dice will be greater than that of Ryan's?
    A. 575/1296   B. 625/1296   C. 325/648   D. 175/324   E. NOTA

23. Caroline and Lion are playing a game. To start the game off, Caroline will generate a real number θ_1 uniformly at random between $[0, \pi]$. After seeing Caroline's result, Lion will select a real number t, then generate a real number θ_2 uniformly at random on $[t, t + \pi]$. The result of the game is determined by the sign of $\sin(\theta_1) \cos(\theta_2) + \sin(\theta_2) \cos(\theta_1)$. If it is nonnegative, Caroline wins, otherwise, Lion wins. Assuming Lion plays optimally, p is the probability that Lion will win this game. Compute $p^2 + (1 - p)^2$.
    A. 1/2   B. 5/9   C. 5/8   D. 1   E. NOTA

24. The least 4-digit number divisible by 2, 3, 4, 5, 6, and 7 is M. What is the sum of the digits of M?

    A. 6   B. 8   C. 9   D. 15   E. NOTA

25. Caroline has a bag with 5 unfair coins. Two of the coins are identical and golden, two are identical and silver, and the last one is bronze. A golden coin is twice as likely to come up heads as a silver coin. A silver coin in turn is twice as likely to come up heads as the bronze coin. She selects a coin and random and flips it, resulting in heads. Given the outcome, what is the probability she selected a golden coin?

    A. 2/5   B. 5/9   C. 4/7   D. 8/13   E. NOTA

26. Rick rolls a die repeatedly until he gets a 6. He keeps track of the running sum of his rolls until he finishes. What is the probability that this running sum is even when he finishes rolling?

    A. 2/5   B. 3/7   C. 4/7   D. 3/5   E. NOTA

27. A regular hexagon and its center are drawn. Pix picks a point uniformly and at random in the interior of this hexagon. What is the probability that this point is closer to the center than it is to any of the hexagon's vertices?

    A. 1/4   B. 2/7   C. 1/3   D. 1/2   E. NOTA

28. Jeff and Olivia each have biased coins with probability of landing on heads of p_1 and p_2, respectively. They play a game in which they take turns flipping their coins until one of them gets heads, in which case the game stops and the person who flipped heads wins. Let q_1 be the probability that Jeff wins if he goes first and q_2 be the probability Jeff wins if Olivia goes first. What is $\frac{q_2}{q_1}$?

    A. p_1 B. $1 - p_1$ C. p_2 D. $1 - p_2$ E. NOTA

29. Let ABC be a triangle with $AB = 6$, $BC = 8$, and $AC = 10$. A cevian from B to AC is chosen uniformly and at random and drawn. What is the probability that this cevian is shorter in length than the median from B to AC?

    A. 1/5   B. 7/25   C. 2/5   D. 12/25   E. NOTA

30. A real number x is generated uniformly and at random from $(0,2)$. The probability that $\frac{1}{2} \leq \tan x \leq 1$ is p. Compute $\tan p$.

    A. $\sqrt{10} - 3$ B. 1/6   C. 1/3   D. $3\sqrt{10} - 9$ E. NOTA