1. C
2. E
3. A
4. E
5. B
6. D
7. B
8. D
9. A
10. D
11. B
12. D
13. C
14. B
15. C
16. C
17. E
18. A
19. B
20. C
21. E
22. B
23. C
24. E
25. B
26. C
27. C
28. B
29. C
30. E

1. C By power rule we know: $\int(2x^3+2x+2)dx=\frac{x^4}{2}+x^2+2x+C$. Now plugging in both bounds yields $696-0=696$

2. E Using angle sum identities, we know:

$$\sin((20+22)x)=\sin(20x)\cos(22x)+\cos(22x)\sin(20x)$$
$$\sin\big((20-22x)\big)=\sin(20x)\cos(22x)-\cos(22x)\sin(20x)$$

So $2\sin(20x)\cos(22x)=\sin(42x)-\sin(2x)$.

$$I=\frac{1}{2}\int_0^{2022\pi}(\sin(42x)-\sin(2x))dx=1/2\left(-\frac{\cos(42x)}{42}+\frac{\cos(2x)}{2}\right)$$

Now evaluating both bounds yields:

$$I=\frac{1}{2}\left(-\frac{1}{42}+\frac{1}{2}\right)-\frac{1}{2}\left(-\frac{1}{42}+\frac{1}{2}\right)=0$$

3. A Observe $-x^2+8x-15=-(x-5)(x-3)$ which is a downward sloping parabola and is only nonnegative when $3\le x\le 5$. Thus:

$$M=-\int_3^5(x-3)(x-5)dx$$

Making the substitution $u=x-4$ yields:

$$M=-\int_{-1}^{1}(u+1)(u-1)du=-2\int_0^1(u^2-1)\,du=\frac{4}{3}$$

4. E Notice that $\frac{1}{x-2022}$ is undefined and unbounded only at x = 2022. Since $\int\frac{dx}{x-2022}=\ln|x-2022|+C$, the integral of the function is also unbounded since $2021\le 2022\le 2022+e$.

5. B Notice that:

$$I=\int_0^{2022}\sqrt{2022x-x^2}dx=\int_0^{2022}\sqrt{1011^2-(x-1011)^2}dx$$

Making the substitution $u=x-1011$ yields:

$$I=\int_{-1011}^{1011}\sqrt{1011^2-u^2}du=2\int_0^{1011}\sqrt{1011^2-u^2}du$$

Making the substitution $u=1011\sin(t)$ yields:

$$I=2\cdot 1011^2\int_0^{\frac{\pi}{2}}\cos^2(t)\,dt$$

Now observe:

$$\int\cos^2(t)\,dt=\int\frac{1+\cos(2t)}{2}dt=\frac{t}{2}+\frac{\sin(2t)}{4}+C$$

Evaluating the desired bounds gives the answer.

6. D Observe:

$$I=\int_1^2 \frac{dx}{x(x^4+1)}=\int_1^2 \frac{x^3dx}{x^4(x^4+1)}$$

Making the substitution $u=x^4$ yields:

$$I=\frac{1}{4}\int_1^{16}\frac{du}{u(u+1)}$$

Now observe:

$$\int \frac{du}{u(u+1)}=\int\left(\frac{1}{u}-\frac{1}{u+1}\right)du=\ln\left|\frac{u}{u+1}\right|+C$$

Plugging in the desired bounds gives:

$$I=\frac{\ln\left(\frac{32}{17}\right)}{4}\to 1+32+17+4=54$$

7. B With $u=x^2+1$ the integral becomes:

$$\frac{1}{2}\int_1^2\frac{du}{\sqrt{u}}=\sqrt{2}-1\approx 0.4$$

8. D Observe:

$$\int(x^4+4x^3+6x^2+4x+2)dx=\int((x+1)^4+1)dx=\frac{(x+1)^5}{5}+x+C$$

Now evaluating the desired bounds gives $I=6/5$

9. A Substituting $\cos^2(x)=1-\sin^2(x)$ we find:

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos(x)\,dx}{3+\sin^2(x)}$$

Making the substitution $u=\sin(x)$ yields:

$$I=\int_0^1\frac{du}{u^2+3}$$

Making the substitution $u=v\sqrt{3}$ yields:

$$I=\frac{\sqrt{3}}{3}\int_0^{\frac{1}{\sqrt{3}}}\frac{dv}{v^2+1}=\frac{\pi\sqrt{3}}{18}$$

10. D Using power rule for integrals this evaluates to:

$$\left(\frac{2}{3}x^{\frac{3}{2}}+\frac{3}{2}x^{\frac{4}{3}}+\frac{6}{7}x^{\frac{6}{7}}\right)\Big|_0^1=\frac{2}{3}+\frac{3}{2}+\frac{6}{7}=\frac{127}{42}$$

11. B First note that $f(x)=(x+2)^2+1$. Thus, when $0\le a\le 1, f(-a)<f(a)$. This means that the solid formed by revolving the region in [0,3] absorbs the solid formed by revolving the region in [-1,0]. Now using shell method:

$$V_1=2\pi\int_0^3(x^3+4x^2+5x)\,dx=2\pi\left(\frac{81}{4}+36+\frac{45}{2}\right)=\frac{315\pi}{2}$$

12. D Observe that:

$$A = \int_{-1}^{3} (x^2 + 4x + 5)\, dx = 9 + 18 + 15 - \left(-\frac{1}{3} + 2 - 5\right) = \frac{136}{3}$$

13. C Since $(x^2 + 4x + 5)' = 2x + 4$ we have that:

$$2c + 4 = \frac{1}{4}\int_{-1}^{3} (2x + 4)\, dx$$
$$8c + 16 = (x^2 + 4x)|_{-1}^{3} = 24 \to c = 1$$

Remark: It is not an accident that the correct answer is the average of the endpoints. The result holds for all linear functions when applying the MVT for integrals and all quadratic polynomials when applying the MVT for derivatives. The proof is left as an exercise to the reader.

14. B

$$V_2 = \pi \int_{-1}^{3} ((x + 2)^2 + 1)^2\, dx = \pi \int_{-1}^{3} ((x + 2)^4 + 2(x + 2)^2 + 1)\, dx$$
$$= \pi(625 + \frac{250}{3} + 3 - \left(\frac{1}{5} + \frac{2}{3} - 1\right) = \pi\left(628 + \frac{250}{3} + \frac{2}{15}\right)$$
$$= \pi\left(\frac{2134}{3} + \frac{2}{15}\right) = \pi\left(\frac{10672}{15}\right)$$

15. C Notice that:

$$I = \int_{-\infty}^{\infty} \frac{2022^{-x^2}}{1 + 2022^{-x}}\, dx$$

Making the substitution $x = -x$ yields:

$$I = \int_{-\infty}^{\infty} \frac{2022^{-x^2}}{1 + 2022^{x}}\, dx.$$

Adding both integrals gives:

$$2I = \int_{-\infty}^{\infty} 2022^{-x^2}\left(\frac{1}{1 + 2022^{-x}} + \frac{1}{1 + 2022^{x}}\right) dx$$
$$= \int_{-\infty}^{\infty} 2022^{-x^2}\left(\frac{2022^{-x} + 2022^{x} + 2}{2022^{-x} + 2022^{x} + 2}\right) dx \to 2I = \int_{-\infty}^{\infty} 2022^{-x^2} dx.$$

Making the substitution $x = u\sqrt{\log_{2022} e}$ yields:

$$2I = \left(\sqrt{\log_{2022} e}\right) \int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}\left(\sqrt{\frac{\ln(e)}{\ln(2022)}}\right) \to I = \frac{\sqrt{\pi}}{2\sqrt{\ln(2022)}}$$

16. C Notice that $x^3 - x^2 - 2x = x(x - 2)(x + 1)$.

Define $G(x) = \int_0^x (t^3 - t^2 - 2t) dt = \frac{x^4}{4} - \frac{x^3}{3} - x^2$

Now we compute G() at the roots of the integrand and the given bounds

$$G(-3) = \frac{81}{4} + 9 - 9 = \frac{81}{4}$$
$$G(-1) = \frac{1}{4} + \frac{1}{3} - 1 = -\frac{5}{12}$$

$G(0) = 0$
$G(2) = 4 - \frac{8}{3} + 4 = -\frac{8}{3}$
$G(3) = \frac{81}{4} - 9 - 9 = \frac{9}{4}$
Now we know:

$$I = |G(-3) - G(-1)| + |G(-1) - G(0)| + |G(0) - G(2)| + |G(2) - G(3)|$$
$$= \frac{81}{4} + \frac{5}{12} + \frac{5}{12} + \frac{8}{3} + \frac{8}{3} + \frac{9}{4} = \frac{86}{3}$$

17. E Making the substitution $x = \frac{1}{u}$ yields:

$$I = \int_0^\infty \frac{u^2 - u^4}{1 + u^8} du.$$

Adding this to the original integral gives $2I = 0$.

18. A Observe that:

$$A = \int_0^{\frac{\pi}{2}} \sin(x) (\sin^2(x))\, dx = \int_0^{\frac{\pi}{2}} \sin(x) (1 - \cos^2(x)) dx$$

Now applying the u substitution $u = \cos(x)$ yields:

$$A = \int_0^1 (1 - u^2) du = \frac{2}{3}$$

19. B First notice that:

$$\left(uf(u)\right)' = uf'(u) + f(u)$$

Via product rule for derivatives.
Thus:

$$A = \int_0^1 uf'(u) du = \int_0^1 (5u^5 + 3u^3 + u) du = \frac{5}{6} + \frac{3}{4} + \frac{1}{2} = \frac{25}{12}$$

20. C Since the bounds are always positive, we have $|x|^3 = x^3$.
Thus, the integral evaluates to:

$$\int_1^2 x^6 dx = \frac{127}{7}$$

21. E Taking the derivative of both sides yields:

$$1 + f(x) = -\frac{f'(x)}{2022}.$$

Separation of variables yields:

$$dx(-2022) = \frac{d\left(f(x)\right)}{1 + f(x)}.$$

Integrating both sides yields:

$$-2022x + C = \ln(|1 + f(x)|).$$

Now taking x = 0 in the original equation gives f(0) = 0, so C = 0.

Thus:

$$|1 + f(x)| = e^{-2022x} \to f(x) = \pm e^{-2022x} - 1$$

Thus, the requested limit is -1.

22. B First observe $-3x^2 - 4x + 11 = -3(x^2 + 2x + 5) + (2x + 26)$, and $x^2 + 2x + 5 = (x+1)^2 + 4$. Thus:

$$I = \int_0^1 \frac{dx}{(x+1)^2 + 4}\left(-3 + \frac{2x + 26}{(x+1)^2 + 4}\right) dx\ .$$

Now applying the substitution $x + 1 = 2\tan(a)$ yields (where $r = \arctan\left(\frac{1}{2}\right)$):

$$I = \int_r^{\frac{\pi}{4}} \frac{2\sec^2 a}{4\sec^2 a}\left(-3 + \frac{4\tan(a) + 24}{4\sec^2 a}\right) da$$

$$I = \frac{1}{4}\int_r^{\frac{\pi}{4}} (-6 + 2\sin(a)\cos(a) + 12\cos^2(a)) da$$

$$I = \frac{1}{4}\int_r^{\frac{\pi}{4}} (-6 + \sin(2a) + 6 + 6\cos(2a)) da$$

$$I = \frac{1}{4}\int_r^{\frac{\pi}{4}} (\sin(2a) + 6\cos(2a)) da$$

Now making the substitution $b = 2a$ yields:

$$I = \frac{1}{8}\int_{2r}^{\frac{\pi}{2}} (\sin(b) + 6\cos(b)) db$$

Now we can easily calculate the indefinite to be:

$$G(b) = \frac{6\sin(b) - \cos(b)}{8}.$$

The final answer is then $G\left(\frac{\pi}{2}\right) - G(2r) = \frac{3}{4} - G(2r)$

Now since $r = \arctan\left(\frac{1}{2}\right)$, $\sin(r) = \frac{\sqrt{5}}{5}$, $\cos(r) = \frac{2\sqrt{5}}{5}$, so:

$$\sin(2r) = 2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{2\sqrt{5}}{5}\right) = \frac{4}{5}$$

$$\cos(2r) = 2\left(\frac{2\sqrt{5}}{5}\right)^2 - 1 = \frac{3}{5}$$

Thus $G(2r) = 21/40$ and thus the integral evaluates to 9/40 .

23. C Consider the function $f(a) = \int_0^\infty \frac{\sin(ax)}{x} dx$, $a \neq 0$. Making the substitution $x = \frac{u}{a}, dx = \frac{da}{a}$, yields:

$$f(a) = \int_0^\infty \frac{\sin(u)\, du}{u} = f(1) = \frac{\pi}{2}.$$

24. E Because $\sin^2(x) > 0, \forall x \neq n\pi$, it immediately follows that:

$$\int_0^{2\pi} \sin^2(x)\,dx = K$$

Where K is some positive constant

Thus the integral across infinitely many periods of sin^2(x) is necessarily infinity.

25. B Recall the triple angle formula $\sin(3x) = 3\sin(x) - 4\sin^3(x)$.

Rearranging yields:

$$\sin^3(x) = \frac{3\sin(x) - \sin(3x)}{4}$$

Thus, $I = \frac{3f(1)-f(3)}{4} = \frac{f(1)}{2} = \frac{\pi}{4}$ (using notation and results from #23).

26. C Observe that:

$$(e^{-x})' = -e^{-x}.$$

Thus, the requested answer is:

$$-e^{-x}|_0^\infty = 1.$$

27. C Applying the substitution $x \to \frac{\pi}{2} - x$ yields:

$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\cot(x))^{2022}} = \int_0^{\frac{\pi}{2}} \frac{\tan^{2022}(x)}{\tan^{2022}(x) + 1}\,dx.$$

Adding this to the original integral gives:

$$2I = \int_0^{\frac{\pi}{2}} \frac{1 + \tan^{2022}(x)}{1 + \tan^{2022}(x)}\,dx \to I = \frac{\pi}{4}$$

28. B We have:

$$I = \int_0^1 \frac{dx}{1 + \frac{1}{x}} = \int_0^1 \frac{x\,dx}{x+1} = \int_0^1 \left(1 - \frac{1}{x+1}\right)dx = x - \ln(x+1)\,|_0^1 = 1 - \ln(2).$$

29. C We wish to compute:

$$I = \int_0^\infty \frac{dx}{1 + x^2} = \arctan(x)\,|_0^\infty = \frac{\pi}{2}$$

30. E Observe that on it’s domain the derivative of $\ln(x)$ is always positive. Thus the function is 1-1.

Now observe that:

$$\ln\left(x^{ln(y)}\right) = \ln(y) \cdot \ln(x) = \ln\left(y^{ln(x)}\right).$$

Thus:

$$x^{\ln(y)} = y^{\ln(x)} \to x^{\ln(y)} - y^{\ln(x)} = 0$$

And so the integral evaluates to 0.