Alpha

Individual

Test #521

Directions:

1. Fill out the top left section of the scantron. Do not abbreviate your school name.

2. In the Student ID Number grid, write your 9-digit ID# and bubble.

3. In the Test Code grid, write the 3-digit test# on this test cover and bubble.

4. Scoring for this test is 5 times the number correct plus the number omitted.

5. TURN OFF ALL CELL PHONES.

6. No calculators may be used on this test.

7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

8. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

9. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

10. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

11. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

12. Unless a question asks for an approximation or a rounded answer, give the exact answer.

For all problems, answer choice E, NOTA, denotes "none of the above."

1. What is the smallest positive coterminal angle, in degrees for -1126°?

   A. 46 B. 214 C. 274 D. 314 E. NOTA

2. Billy, Philip, Hunter, and Andrew all want to go to prom with a Buchholz math team girl. The probability that Billy gets a date is $\frac{7}{8}$; the probability Philip gets a date is 0.6; and the probability that Hunter gets a date is $\frac{1}{6}$. If the probability that at least one of the four guys gets a date is $\frac{31}{32}$, what is the probability that Andrew gets a date? Each person's chances are independent of the others.

   A. 0.125 B. 0.25 C. 0.5 D. 0.75 E. NOTA

3. What is the domain of $y = \frac{1}{\sqrt{\ln\sqrt{x^2-1}}}$?

   A. $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ B. $(-\sqrt{2}, \sqrt{2})$

   C. $[-1,1]$ D. $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$ E. NOTA

4. In triangle XYZ, angle Y has a tangent of $\frac{1}{5}$. $XY = 2\sqrt{26}$ and $YZ = 8$. Find the length of XZ.

   A. 2 B. $2\sqrt{2}$ C. $8\sqrt{2}$ D. $2\sqrt{34}$ E. NOTA

5. Compute $\sum_{n=0}^{\infty}\left(\frac{-\pi}{3}\right)^n$.

   A. $\frac{3}{3-\pi}$ B. $\frac{\pi}{3-\pi}$ C. $\frac{3}{3+\pi}$ D. $\frac{\pi}{3+\pi}$ E. NOTA

6. Compute $\tan\frac{11\pi}{12}$.

A. $-\sqrt{3}$ B. $-2-\sqrt{3}$ C. $-2+\sqrt{3}$ D. $2+\sqrt{3}$ E. NOTA

7. Find the equation of the set of points equal distance from $(-2,8)$ and the line $x=2$.

A. $(y-8)^2=8x$ B. $(y-8)^2=-8x$

C. $(y-8)^2=\frac{1}{8}x$ D. $(y-8)^2=\frac{-1}{8}x$ E. NOTA

8. The probability that event L occurs is 3/4; the probability that event U occurs is 2/3. Let Z be the probability that both L and U occur. What is the smallest interval that necessarily contains Z?

A. $\left[\frac{1}{12},\frac{1}{2}\right]$ B. $\left[\frac{5}{12},\frac{1}{2}\right]$ C. $\left[\frac{1}{2},\frac{2}{3}\right]$ D. $\left[\frac{5}{12},\frac{2}{3}\right]$ E. NOTA

9. If m and n are positive real numbers and each of the two equations $x^2+mx+2n=0$ and $x^2+2nx+m=0$ has real solutions for x. Then find the smallest possible value of $m+n$.

A. 3 B. 4 C. 5 D. 6 E. NOTA

10. Diagonal RU of rectangle MRLU is divided into three segments of length 1 by parallel lines that pass-through M and L and are perpendicular to RU. The area of rectangle MRLU is?

A. $2\sqrt{3}$ B. $3\sqrt{2}$ C. $1+2\sqrt{3}$ D. 4.5 E. NOTA

11. Find the sum of the values of n for which the sum of the coordinates of the x and y intercept(s) of $y=|x-3n|-n$ is 24

A. -3 B. 3 C. 6 D. 15 E. NOTA

12. Two Superheroes (Captain Wiggie and MuLu) fly back and forth across a land mass with constant but different speeds, turning at the opposite end without loss of time. They leave opposite ends at the same instant, meeting for the first time 700 miles from one end and meet for the second time 300 miles from the opposite end. What is the width of the land mass, in miles?

A. 1000 B. 1200 C. 1400 D. 1800 E. NOTA

13. A line is given by the equation $2x-3y=6$. A point W at coordinates $(4,8)$ is reflected over this line. What is the abscissa of the reflected point?

A. $\frac{38}{13}$ B. $\frac{124}{13}$ C. $\frac{140}{13}$ D. $\frac{244}{13}$ E. NOTA

14. Compute $\lim_{n\to\infty}\frac{1+4+9+25+...+n^2}{(n-2)^3}$.

A. 0 B. $\frac{1}{6}$ C. $\frac{1}{3}$ D. 1 E. NOTA

15. Find $\cos(x+y)$, if $\tan x=\frac{3}{4}, 0\le x<\frac{\pi}{2}$ and $\sin y=\frac{-5}{13}, \pi\le y<\frac{3\pi}{2}$

A. $-\frac{63}{65}$ B. $-\frac{33}{65}$ C. $-\frac{16}{65}$ D. $\frac{63}{65}$ E. NOTA

16. A circle, tangent to 5x + 3y = 9 at $(3,-2)$ and passing through $(5,-4)$ has an area of $K\pi$. What is the value of K?

A. 8 B. 34 C. 56 D. 72 E. NOTA

17. The sequence $-2, 6, 26, 64, 126, ...$ can be represented by $a_n = An^3 + Bn^2 + Cn + D$, with $a_1 = -2$. What is the value of A – B – C – D?

A. 4 B. 5 C. 8 D. 13 E. NOTA

18. What is the length of time (in years) to double the amount of money invested at a rate of 6% compounded annually?

A. $\frac{\log(1.06)}{2}$ B. $2\log(1.06)$ C. $\log_2 1.06$ D. $\log_{1.06} 2$ E. NOTA

19. If M, R, L, and U are consecutive terms of a positive arithmetic sequence that satisfy: $-M^2+R^2-L^2+U^2=3(M+R+L+U)$, then find the value of $L-U$.

A. -3 B. -2 C. 2 D. 3 E. NOTA

20. Circle L is centered at $(10,-1)$ and is externally tangent to circle U. If circle U has an equation of $x^2+y^2+4x-8y-5=0$, then what is the diameter of circle L?

A. 8 B. 12 C. 13 D. 16 E. NOTA

21. $g(x)=\frac{2x^3-x^2-15x+18}{x^2-4}$. Find the sum of the y-coordinates of the point of discontinuity and the y-intercept of the slant asymptote

A. 0.25 B. 1 C. 2.25 D. 3 E. NOTA

22. How many real solutions does $6x^{\frac{4}{5}}-11x^{\frac{2}{5}}+4=0$ have?

A. 0 B. 1 C. 2 D. 4 E. NOTA

23. If $2^{10}+2^5=K^5+K^{10}$, what is the sum of all possible values of K^5?

A. -1 B. 1 C. 32 D. 33 E. NOTA

24. The locus of points in a plane equidistant from $(-1,-4)$ and $(5,3)$ can be expressed in the form $Ax + By = C$, where A, B, and C are relatively prime integers with $A > 0$. Find the value of $A + B + C$.

A. 24 B. 29 C. 31 D. 43 E. NOTA

25. What is the largest possible distance between two points, one on the sphere of radius 21 with center $(0,-12,5)$ and the other on the sphere of radius 9 with center $(14,6,-16)$?

A. 58 B. 59 C. 60 D. 61 E. NOTA

26. Find $\begin{vmatrix} 4 & 3 & 2 & 1 \\ -1 & 2 & 3 & 4 \\ -4 & -3 & 2 & 1 \\ -1 & -2 & -3 & 4 \end{vmatrix}$.

A. 292 B. 300 C. 308 D. 316 E. NOTA

27. Solve for k: $\ln(k-2)-1=\ln(k+2)$

A. $\frac{2+2e}{1-e}$ B. $\frac{2+2e}{1+e}$ C. $\frac{2-2e}{1+e}$ D. $\frac{2-2e}{1-e}$ E. NOTA

28. The graph of $x+4y=y^2$ NEVER passes through which quadrant?

A. I B. II C. III D. IV E. NOTA

29. Describe the steps of $h(x)=\frac{1}{3}[4x]$:, where [ ] represents the greatest integer function.

A. $\frac{1}{3}$ unit apart, 4 units long B. 4 units apart, 3 units long

C. $\frac{1}{3}$ unit apart, $\frac{1}{4}$ units long D. 3 units apart, 4 units long E. NOTA

30. Mr. Lu's yacht starts at the origin, travels at 20 mph with a bearing (clockwise from the north) of 60 degrees. After 30 minutes, his yacht turns due south for another 30 minutes at a speed of 10 mph. what is his yacht's final bearing, in degrees?

A. 60 B. 90 C. 120 D. 180 E. NOTA