1. B
2. A
3. D
4. B
5. C
6. E
7. B
8. A
9. B
10. D
11. B
12. B
13. C
14. C
15. C
16. D
17. D
18. A
19. C
20. C
21. A
22. C
23. E
24. A
25 A
26. D
27. A
28. B
29. B
30. B

1. B Intersection of lines: $\begin{cases} 3x-4y=8 \\ -2x+9y=1 \end{cases} \to \begin{cases} 6x-8y=16 \\ -6x+27y=3 \end{cases} \to 19y=19 \to (4,\ 1).$

Midpoint between intersection and (6, 17): $\left(\dfrac{6+4}{2},\ \dfrac{17+1}{2}\right) = (5,\ 9) \to 5+9=14.$

2. A $9x^2+16y^2=144 \to \dfrac{x^2}{16}+\dfrac{y^2}{9}=1 \to a^2=16,\ b^2=9,\ c^2=7,$ so the foci are at $\left(\pm\sqrt{7},\ 0\right).$

$r^2=\left(\sqrt{7}-0\right)^2+\left(0-3\right)^2=16.$ The circle is $x^2+(y-3)^2=16 \to$

$x^2+y^2-6y-7=0.$

3. D Find the distance between the vertex and the line. Then double that value to find the length of the diagonal. $\dfrac{|3(-1)-4(1)+8|}{\sqrt{3^2+4^2}}=\dfrac{1}{5} \to d=\dfrac{2}{5}.$ The area of a square given its diagonal is $\dfrac{1}{2}d^2 \to \dfrac{1}{2}\left(\dfrac{4}{25}\right)=\dfrac{2}{25}.$

4. B The slope of the line passing through (2, –1) and (5, –3) is $\dfrac{-3+1}{5-2}=-\dfrac{2}{3}.$ The equation of this line is $y+1=-\dfrac{2}{3}(x-2) \to 2x+3y-1=0.$ Solving for *a* and *b*, we get $2a+12-1=0 \to a=-\dfrac{11}{2}$ and $-4+3b-1=0 \to b=\dfrac{5}{3}.$ The line containing $\left(-\dfrac{11}{2},\ \dfrac{5}{3}\right),$ among the given choices, is $2x+6y+1=0.$

5. C Vertex $R(x,\ y)$ must be equidistant from the other vertices:

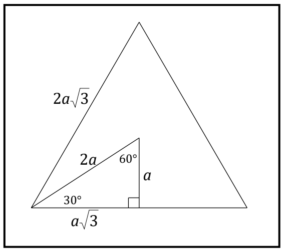
$\sqrt{(x-1)^2+(y-3)^2}=\sqrt{(x+2)^2+(y-7)^2} \to -2x+1-6y+9=4x+4-14y+49 \to$

$6x-8y+43=0.$ The vertex must lie on this line. Among the choices, B and C satisfy the equation, but choice B lies on the base of the triangle.

6. E An equilateral triangle cannot have vertices with coordinates that are all rational.

7. B The *x*-coordinates have a range of 5, so the *x*-coordinates increase by 5/3. The *y*-coordinates have a range of 4, so the *y*-coordinates decrease by 4/3. The places *W* at $\left(-\dfrac{1}{3},\ \dfrac{11}{3}\right)$ and *F* at $\left(\dfrac{4}{3},\ \dfrac{7}{3}\right).$ The midpoint of $\overline{WF}$ is therefore $\left(\dfrac{1}{2},\ 3\right).$ The distance from the origin is $\sqrt{\dfrac{1}{4}+9}=\sqrt{\dfrac{37}{4}}=\dfrac{\sqrt{37}}{2}.$

8. A

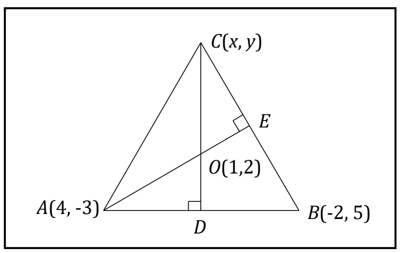



The side length is distance from the vertex to the centroid, multiplied by $\sqrt{3}$. $s=\sqrt{3}\sqrt{\left(2+\frac{2\sqrt{3}}{3}\right)^2+\left(2+\frac{2\sqrt{3}}{3}\right)^2}=$ $\frac{\sqrt{3}}{3}\sqrt{36+12+36+12}=\frac{\sqrt{3}}{3}\left(4\sqrt{6}\right)=4\sqrt{2}.$

9. B $A=\frac{s^2\sqrt{3}}{4}=\frac{\left(4\sqrt{2}\right)^2\sqrt{3}}{4}=8\sqrt{3}.$

10. D The area of the rhombus will be twice the area of the triangle with vertices at the given points. $2\left(\frac{1}{2}\begin{vmatrix}2 & -3 & 1\\ 6 & 5 & 1\\ -2 & 1 & 1\end{vmatrix}\right)=48.$

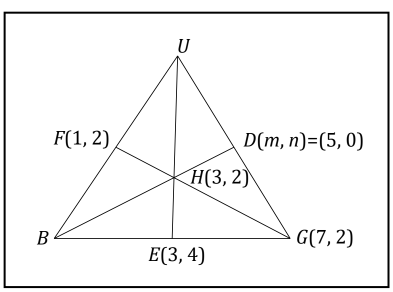
11. B




The orthocenter is the intersection of the altitudes of the triangle. Let the vertices be $A(4, -3)$, $B(-2, 5)$, and $C(x, y)$; $E$ be the foot of the altitude from $A$; and $D$ be the foot of the altitude from $C$. $\overline{CD}\perp\overline{AB}$, $m_{\overline{AB}}=-\frac{4}{3}\to m_{\overline{CD}}=\frac{3}{4}=\frac{y-2}{x-1}\Rightarrow 3x-4y=-5.$ $\overline{CB}\perp\overline{AE}$, $m_{\overline{AE}}=-\frac{5}{3}\to m_{\overline{CB}}=\frac{3}{5}=\frac{y-5}{x+2}\Rightarrow$ $3x-5y=-31.$ $\begin{cases}3x-4y=-5\\ 3x-5y=-31\end{cases}\to(33,\ 26).$

12. B The centroid is located at the "average" of the coordinates of the three vertices but also at the average of the coordinates of the three midpoints of the sides of the triangle. $\left(\frac{m+1+3}{3},\ \frac{n+2+4}{3}\right)=\left(3,\ 2\right)\to(m,\ n)=(5,\ 0)\to 5+0=5.$

13. C




The area of the triangle formed by the midpoints of the three sides is one-four the area of the triangle. The area of $\triangle BUG$ is $4\left(\frac{1}{2}\begin{vmatrix}1 & 2 & 1\\ 5 & 0 & 1\\ 3 & 4 & 1\end{vmatrix}\right)=24.$ Since we want the length of the altitude from $B$, we need $\overline{UG}$ to be the base of the triangle. Since $D$ is the midpoint of $\overline{UG}$, $U$ is (3, –2). $UG=\sqrt{(3-7)^2+(-2-2)^2}=4\sqrt{2}.$ Using the triangle area formula $A=\frac{1}{2}bh,\ 48=4\sqrt{2}h\to h=6\sqrt{2}.$

14. C The image of (2, 3) across the $y$-axis is (–2, 3). In order for (5, 10) and (– 2, 3) to be collinear, the slope from $K(0, k)$ to these two points must be the same.
$\dfrac{k-3}{0+2}=\dfrac{k-10}{0-5}\rightarrow -5k+15=2k-20\rightarrow 7k=35\rightarrow k=5.$ $K$ is located at (0, 5).

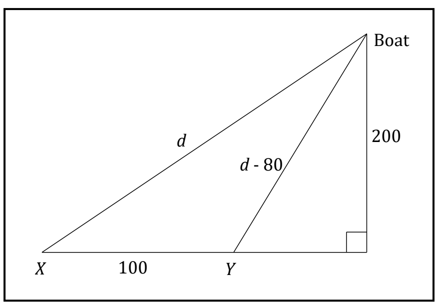
15. C A sketch of these points will show that this gives an isosceles trapezoid, which is a cyclic quadrilateral.

16. D $x^2y^2-9y^2-6x^2y+54y=0\rightarrow y^2\left(x^2-9\right)-6y\left(x^2-9\right)=0\rightarrow$
$(x+3)(x-3)y(y-6)=0.$ Set these four factors equal to 0 and graph them to see a square of side length 6.

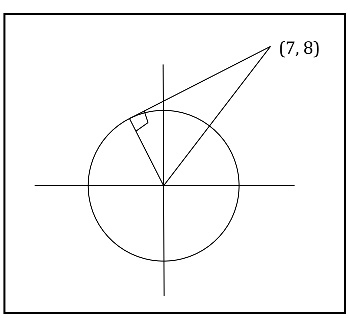
17. D

18. A The point we are looking for is the focus. Let the center of the paraboloid be located at the origin, and let the equation of the parabola be $y=\dfrac{1}{4a}x^2$, where $a$ is the distance from the vertex to the focus. The parabola will pass through (4.5, 2).
$2=\dfrac{1}{4a}\left(\dfrac{9}{2}\right)^2\rightarrow a=\dfrac{81}{32}.$

19. C This is the definition of an ellipse, so the eccentricity must be between 0 and 1.

20. C 


This is based on the definition of a hyperbola, where the boat is the "traveling point," giving $2a=d-(d-80)\rightarrow a=40$ and $2c=100$ $\rightarrow c=50.$ Using $a^2+b^2=c^2,\ b^2=30^2.$ The equation of the hyperbola is therefore $\dfrac{x^2}{40^2}-\dfrac{y^2}{30^2}=1.$ Substituting $y=200$ (the offshore distance of the boat) we can find the $x$-coordinate of the ship: $\dfrac{x^2}{40^2}=1+\dfrac{200^2}{30^2}=1+\dfrac{400}{9}=\dfrac{409}{9}\rightarrow x^2=\left(\dfrac{409}{9}\right)40^2\rightarrow x=\dfrac{40\sqrt{409}}{3}.$

21. A 


We know that the radius is perpendicular to the tangent line, so we have a right triangle. The distance from the origin to (7, 8) is $\sqrt{49+64}=\sqrt{113}$ and that the radius of the circle is 3. $3^2+b^2=113\rightarrow b^2=104\rightarrow b=2\sqrt{26}.$

22. C The first equation generates the graph of the top half of a circle with center (–2, 0) and radius 2. The second equation generates the bottom half of an ellipse with center (–2, 0)

and radii 2 and 8. The semicircle has area $\frac{1}{2}\pi(2)^2 = 2\pi$ and the semiellipse has area $\frac{1}{2}(2)(8)\pi = 8\pi,$ for a total of $10\pi.$

23. E Using substitution, $x^2+4x=5 \to (x+5)(x-1)=0 \Rightarrow x=1,\ x=-5.$ However, $y^2$ cannot be negative, so we can only use $x=1.$ The points of intersection are (1, 2) and (1, –2), giving a product of –4.

24. A The point (*a*, *b*) is $\left(a, \sqrt{a}\right)$, using the substitution $y=\sqrt{x}.$ The distance from (1, 0) to $\left(a, \sqrt{a}\right)$ is $\sqrt{(1-a)^2+\left(0-\sqrt{a}\right)^2} = D = \sqrt{a^2-a+1}.$ After completing the square, we get $D=\sqrt{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}.$ The *x*-value that minimizes the radicand is the same *x*-value that minimizes the distance, so the point we need is $\left(\frac{1}{2}, \sqrt{\frac{1}{2}}\right)$ or $\left(\frac{1}{2}, \frac{\sqrt{2}}{2}\right).$ $ab=\frac{\sqrt{2}}{4}.$

25 A $4x^2-3xy+9y^2+17x-12y+19=0 \to 4x^2+(-3y+17)x+(9y^2-12y+19)=0$

$$x=\frac{3y-17\pm\sqrt{9y^2-102y+289-16(9y^2-12y+19)}}{8}=x=\frac{3y-17\pm\sqrt{-135y^2+90y-15}}{8}$$

$$x=\frac{3y-17\pm\sqrt{-15(9y^2-6y+1)}}{8}=\frac{3y-17\pm\sqrt{-15(3y-1)^2}}{8}.$$ This gives a real number for *x* only when $y=\frac{1}{3}.$ At this value of *y*, $x=-2,$ so all we have is the $\left(-2, \frac{1}{3}\right)$, which is fully contained in Quadrant II.

26. D We are told that this is non-degenerate, so we can use $B^2-4AC: (-1)^2-4(2)(1)<0,$ which is an ellipse.

27. A $y=-\frac{4}{x^2-x-2} \to yx^2-yx-2y+4=0.$ $x=\frac{y\pm\sqrt{y^2-4y(-2y+4)}}{2y}.$ Now find the domain of the radicand, which will (almost) give us the range:
$y^2+8y^2-16y\ge 0 \to y(9y-16)\ge 0 \Rightarrow (-\infty, 0]\cup\left[\frac{16}{9},\infty\right).$ However, we can see from the original equation that *y* will never be 0, so the range is $(-\infty, 0)\cup\left[\frac{16}{9},\infty\right).$ $A+B-C=0+16-9=7.$

28. B Let $(x, y)$ represent the moving point. $\left(\frac{y-1}{x+2}\right)\left(\frac{y-2}{x-3}\right)=4 \to \frac{y^2-3y+2}{x^2-x-6}=4 \to$

$$4x^2-4x-24=y^2-3y+2 \to 4\left(x^2-x+\frac{1}{4}\right)-\left(y^2-3y+\frac{9}{4}\right)=26+1-\frac{9}{4}=\frac{99}{4} \to$$

$$4\left(x-\frac{1}{2}\right)^2-\left(y-\frac{3}{2}\right)^2=\frac{99}{4}\rightarrow\frac{\left(x-\frac{1}{2}\right)^2}{\frac{99}{16}}-\frac{\left(y-\frac{3}{2}\right)^2}{\frac{99}{4}}=1.\ \ 2a=2\sqrt{\frac{99}{16}}=\frac{3\sqrt{11}}{2}.$$

29. B We are moving 12 left and 10 down. $\left(\frac{5}{8}\right)(12)=\frac{15}{2}$ and $\left(\frac{5}{8}\right)(10)=\frac{25}{4}$. The point we are looking for is $\left(7-\frac{15}{2},1-\frac{25}{4}\right)=\left(-\frac{1}{2},-\frac{21}{4}\right)$. The sum is $-\frac{2}{4}-\frac{21}{4}=-\frac{23}{4}$.

30. B Using long division, we get choice B.