1. D
2. B
3. C
4. B
5. A
6. B
7. B
8. B
9. E
10. C
11. D
12. A
13. C
14. B
15. D
16. B
17. B
18. D
19. C
20. E
21. C
22. D
23. A
24. B
25. C
26. C
27. B
28. E
29. C
30. A

1. D The number 1 appears 14 times in the ones place of the numbers, from 1, 11, 21, etc to 131. It appears 20 times in the tens place, from numbers 10 to 19 and 110 to 119. Finally, it appears 38 times in the hundreds place from 100 to 137. Also 10 extra spots in the 10's place. Adding these up gives us 72.
2. B The sum of the first n positive integers is n^2. So, the sum of the first 500 positive integers is 250,000.
3. C If we let $a_1 = m$, then $a_1 + a_2 + \cdots + a_n = m + (m+1) + \cdots + (m+n-1) = \frac{n(2m+n-1)}{2} = 2022$. Then we have $n(2m+n-1) = 4044 = 2^2 \cdot 3 \cdot 337$. The largest possible value for $n = 12$.
4. B The y coordinates of the bee follow a geometric sequence with first term 6 and common ratio $-\frac{1}{4}$, and the y coordinates of the bee follow a geometric sequence with first term 3 and common ratio $-\frac{1}{4}$. Finding the sum of these sequences, the bee will end at the coordinates $(\frac{12}{5}, \frac{24}{5})$. This is a distance of $\frac{12\sqrt{5}}{5}$ away from the origin.
5. A $a_{19} = (a_1 + a_2 + \cdots + a_{19}) - (a_1 + a_2 + \cdots + a_{18}) = (19! + 20!) - (18! + 19!)$ $= 20! - 18! \equiv 0 + 1 \equiv 1\ (mod 19)$.
6. B Note that $S = (40-38)(40+38) + (39-37)(39+37) + \cdots + (4-2)(4+2) + (3-1)(3+1) = 2(40+39+38+\cdots+2+1) = 40 \cdot 41 = 1640$. Thus, the hundreds digit of S is 6.
7. B The first circle has a radius of 8. The inside square has a side length of $8\sqrt{2}$, so the next circle has a radius of $4\sqrt{2}$. The ratio of the lengths is $1:\sqrt{2}$, so the ratio of the areas is 1:2. This means the areas of the circles form a geometric sequence with first term 64π and common ratio .5, which has a sum of 128π.
8. B If we rationalize the fraction, we get $\frac{\sqrt{n+1}-\sqrt{n}}{1}$. This series will telescope, because every positive square root will cancel out with the negative square root of the next term. The only terms that are left over are $-\sqrt{1}$ and $\sqrt{100}$, which add up to make 9.
9. E Each light will get jumped on a number of times equal to how many factors it has. Therefore, only perfect squares that have an odd number of factors will be off at the end. There are 44 perfect squares less than 2022, so that means 1978 lights will stay on.
10. C The formula for sum of squares of integers is $\frac{(n)(n+1)(2n+1)}{6}$. Plugging in 35 gives us 14910.
11. D A quadratic sequence has the form $ax^2 + bx + c$. We can plug 1, 2, and 3 for x into this to get the equations $a + b + c = 11, 4a + 2b + c = 22$, and $9a + 3b + c = 39$. Solving, we get $a = 3, b = 2$, and $c = 6$. So, the 10^{th} term is 326.
12. A Note that $a_1 + a_2 + \cdots + a_{20} = 6(a_1 + a_2 + \cdots + a_{10})$. If we let $a_n = a + (n-1)d$, then we get $\frac{20(2a+19d)}{2} = 6\frac{10(2a+9d)}{2}$. Simplifying the expression we get $a = -2d$. Thus, $\frac{a_1}{a_2} = \frac{a}{a+d} = 2$.
13. C It first falls for 60 feet. Then every bounce afterwards it travels twice the distance (bouncing up and falling). This sequence starts with 40 and has a common ratio of $\frac{2}{3}$,

which sums to 120. Multiplying this by 2 and adding the original 60 feet gives us 300 feet.

14. B Except for the last day, the frog climbs 3 feet every day. For the first 22 days, he climbs up 3 feet, getting him to 66 feet, and then the last day he climbs the last 9 feet and makes it out of the well. Day 23 is a Tuesday.

15. D $7^1 = 7, 7^2 = 49, 7^3 = 343$ and $7^4 = 2401$. So, the last two digits of 7^n repeat in cycles of 4. 7^{48} would end in 01, so 7^{50} ends in 49. Then, $(49)(2022)$ ends in 78.

16. B In base 10, these numbers are 1, 3, 5, 7, 9. The 14th term of this sequence is 1 + 13(2), which is 27. In binary, that is 11011.

17. B We can evaluate the sum of the x's first. The x's are from 1 to 15, meaning x+2 is from 3 to 17, which has a sum of 150. Then, the y's are from 1 to 10, meaning y+3 is from 4 to 13, which has a sum of 85. (150)(85) is equal to 12750.

18. D There are $\frac{998}{2} = 499$ multiples of 2, $\frac{999}{3} = 333$ multiples of 3, and $\frac{995}{5} = 199$ multiples of 5. However, in each one of those cases we have counted two of the things we don't want. In counting the 2s we counted 6s and 10s, in counting 3s we counted 6s and 15s, and in counting 5s we counted 10s and 15s. So, we need to subtract twice $\frac{996}{6} + \frac{990}{10} + \frac{990}{15}$, which equals 662. Then, we have counted 30s three times in the first counting and then removed it six times in the second counting. So, we need to add back in 30s $\left(\frac{990}{30}\right)$ three times to get the right amount. Adding all this up, we get 468.

19. C The sum of the first n Fibonacci numbers is equal to $F_{n-2} - 1$. We are looking for the sum of the first 25 numbers, so we need $F_{27} - 1$, which is equal to 75025 + 121393 - 1, or 196417.

20. E Between 1900 and 2000, there are 25 leap years and 75 non leap years, for a total of 75(365)+(25(366) or 36525 days. This gives a remainder of 6 when divided by 7, meaning that January 1, 2000 is a Saturday.

21. C i^n repeats in cycles of 4, and the cycles $(i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1)$ add up to 0. So, all of the terms up to i^{1336} will cancel out and just $i^{1337} = i$ will be left.

22. D The terms of the arithmetic sequence can be written as a, a+d, a+2d and the terms of the geometric sequence can be written as 18, 18r and $18r^2$. Then, a + 18 = 20, so a = 2. 2 + d + 18r = 29 and 2 + 2d + $18r^2 = 40$. Solving this for d, we get 3 or 15.

23. A On Jim's first flip, he has a $\frac{1}{2}$ chance of winning. Then, there is a $\frac{1}{2}$ chance he doesn't flip tails and a $\frac{1}{2}$ chance Bob doesn't flip tails, and then a $\frac{1}{2}$ chance he wins. So, this is a geometric series with first term $\frac{1}{2}$ and common ratio $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$. This sums to $\frac{2}{3}$ for Jim, so Bob has a $\frac{1}{3}$ chance of winning.

24. B Using the information from last question, Jim has a p chance of winning on the first try. Then, there is a (1-p) chance he doesn't flip tails and a (1-p) chance Bob doesn't flip tails, and then a p chance he wins. So, this is a common ratio with first term p and common ratio $(1-p)^2$, which sums to $\frac{p}{1-(1-p)^2}$ and is equal to $\frac{3}{5}$. Solving this, we get that p = $\frac{1}{3}$.

25. C After the 4th sort, the outer groups will both have 8, the middle group will have 48, and the other two groups will have 8+24 = 32. Then after the 5th sort, the groups second from the outside (the ones we want) have 4 + 16 = 20.
26. C A harmonic sequence is the reciprocal of an arithmetic sequence.
27. B After the last number n has been written, the number of terms in the sequence is 1 + 2 + 3 + … + n, which is $\frac{(n)(n+1)}{2}$. So, we want $\frac{(n)(n+1)}{2}$ to be close to 2022. If n = 63, the sum equals 2016, meaning that the 2022nd term will be a 64.
28. E If we call the 5th angle a, then 9a = 7(180) and a = 140. The second angle (a – 3d) equals 125, so the common difference is 5. The largest angle (a+4d) is then equal to 160.
29. C We are looking for the sum of reciprocals of odd perfect squares, which is also equal to the sum of reciprocals of all perfect squares mins the sum of reciprocals of even perfect squares. The sum of reciprocals of even perfect squares is $\sum_{n=1}^{\infty}\frac{1}{(2n)^2}$, which is $\frac{1}{4}$ of the value we are given, or $\frac{\pi^2}{24}$. So, what we want is $\frac{\pi^2}{6}-\frac{\pi^2}{24}=\frac{\pi^2}{8}$.
30. A Similarly to how we did question 12, we can call this x and replace where it repeats. Just looking at the fraction, we have $x=\frac{1}{1+\frac{2}{1+x}}$. Simplifying this, we get $x^2+3x=1+x$, and solving this with quadratic formula, we get $-1+\sqrt{2}$, and then adding the 2 at the beginning, we get $1+\sqrt{2}$.