Theta

Sequences and Series

Test #612

Directions:

1. Fill out the top section of the Round 3 Google Form answer sheet and select **Theta-Sequences and Series** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

1. If a magazine has page numbers written starting at page 1 and ending at page 137, how many times is the number 1 written?

A. 52 B. 58 C. 62 D. 72 E. NOTA

2. What is the sum of the first 500 odd positive integers?

A. 125250 B. 250000 C. 250500 D. 500000 E. NOTA

3. The sequence $a_1, a_2, \cdots, a_n$ consists of n consecutive positive integers whose sum is equal to 2022. What is the largest possible value of n ?

A. 3 B. 4 C. 12 D. 337 E. NOTA

4. A bee starts at the point (0,0) on a coordinate grid. It flies 6 units up, then 3 units to the right, then 1.5 units down, then .75 units to the left, and continues in this pattern. If it does this infinitely, how far from the origin does it end up?

A. $\frac{6\sqrt{5}}{5}$ B. $\frac{12\sqrt{5}}{5}$ C. $\frac{24\sqrt{5}}{5}$ D. 12 E. NOTA

5. Suppose that a sequence a_n satisfies $a_1 + a_2 + \cdots + a_n = n! + (n+1)!$. Find the remainder when a_{19} is divided by 19.

A. 1 B. 2 C. 18 D. 0 E. NOTA

6. When the following sum S is evaluated, find the hundreds digit of S.

$$S = 40^2 + 39^2 - 38^2 - 37^2 + 36^2 + 35^2 - 34^2 - 33^2 + \cdots + 4^2 + 3^2 - 2^2 - 1^2$$

A. 8 B. 6 C. 5 D. 3 E. NOTA

7. A circle with a radius of 8 is drawn. A square is inscribed in this circle, and a circle is then inscribed in that square, and so on. What is the total area of all the circles?

A. 64π B. 128π C. 192π D. 256π E. NOTA

8. What is $\sum_{n=1}^{99} \frac{1}{\sqrt{n+1}+\sqrt{n}}$?

A. 1 B. 9 C. 10 D. 99 E. NOTA

9. There are 2022 lights labeled with the numbers 1 through 2022, and each one has a switch in front of them to turn them on or off. All of the lights start on. There are also 2022 rabbits labeled with the numbers 1 through 2022. Rabbit 1 jumps on all of the lights, turning them all off. Rabbit 2 jumps on lights 2, 4, 6, and so on, turning them all on. Rabbit 3 jumps on lights 3, 6, 9, and so on. After all the rabbits have jumped on the switches, how many lights are still on?

A. 0 B. 45 C. 1977 D. 2022 E. NOTA

10. What is the sum of the squares of the first 35 positive integers?

A. 630 B. 1260 C. 14910 D. 29820 E. NOTA

11. A quadratic sequence has a first term of 11, a second term of 22, and a third term of 39. What is the tenth term?

A. 236 B. 272 C. 300 D. 326 E. NOTA

12. If an arithmetic sequence a_n with $a_1 \neq 0$ satisfies

$$a_{11} + a_{12} + \cdots + a_{20} = 5(a_1 + a_2 + \cdots + a_{10}),$$ what is $\frac{a_1}{a_2}$?

A. 2 B. -2 C. -0.5 D. 5 E. NOTA

13. A super ball is dropped from a height of 60 feet. Each time it hits the ground, it bounces $\frac{2}{3}$ of its previous height. If it bounces forever, what is the total vertical distance it travels, in feet?

A. 120 B. 180 C. 300 D. 360 E. NOTA

14. On a Monday, A frog starts climbing up the side of a well that is 75 feet deep. Each day, it climbs up 10 feet, and each night, it slips back down 7 feet. On which day of the week will the frog to climb out of the well?

A. Monday B. Tuesday C. Wednesday D. Thursday E. NOTA

15. What are the last two digits of $7^{50}(2022)$?

A. 22 B. 46 C. 54 D. 78 E. NOTA

16. The binary numbers 1, 11, 101, 111, 1001, … form an arithmetic sequence. What is the 14th term of this sequence (also written in binary)?

A. 11001 B. 11011 C. 10111 D. 11111 E. NOTA

17. Evaluate $\sum_{y=1}^{10}\sum_{x=1}^{15}(x+2)(y+3)$

A. 12495 B. 12750 C. 13200 D. 13923 E. NOTA

18. How many positive integers are there less than 1000 that are multiples of 2, 3, or 5 but not multiples of 6, 10, or 15?

A. 369 B. 402 C. 432 D. 468 E. NOTA

19. If the 25th term of the Fibonacci sequence is 75025 and the 26th term of the Fibonacci sequence is 121393, what is the sum of the first 25 terms of the Fibonacci sequence?

A. 121393 B. 121394 C. 196417 D. 196419 E. NOTA

20. If January 1, 1900 was a Monday, what day of the week was January 1, 2000?

A. Sunday B. Friday C. Wednesday D. Tuesday E. NOTA

21. Find the sum $\sum_{n=1}^{1337} i^n$, where i = $\sqrt{-1}$.

A. -1 B. 0 C. i D. -i E. NOTA

22. An arithmetic sequence and a geometric sequence are added together to make a new sequence. The first, second, and third term of this sequence are 20, 29, and 40. If the first term of the geometric sequence is 18, and the common ratio and common difference are both positive, what is the largest possible value for the common difference of the arithmetic sequence?

A. 1 B. $\frac{4}{3}$ C. 3 D. 15 E. NOTA

23. Jim and Bob are playing a game. They take turns flipping a coin and whoever flips tails first wins. If Jim starts, what is the probability Bob wins?

A. $\frac{1}{3}$ B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

24. Because Bob doesn't like losing, he brings a counterfeit coin that has a probability of p of flipping tails. They play the game from the previous question, and Jim starts again. If Jim has a probability of $\frac{3}{5}$ of winning the game, what is p?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{3}{5}$ D. $\frac{3}{4}$ E. NOTA

25. 128 balls are dropped into a sorting machine. It has 5 levels. The first level splits the balls into two equal groups of 64. The second level splits each group into two equal groups, and combines the middle groups, so the groups are now 32, 64, 32. The third level does the same thing, splitting the groups into 16, 48, 48, 16. After the balls have finished sorting, how many balls are in the second group from the outside?

A. 4 B. 8 C. 20 D. 24 E. NOTA

26. Given the sequence $\frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}, \frac{1}{16}, \frac{1}{19}$, ... What kind of sequence could it be?

A. Geometric B. Cubic C. Harmonic D. Arithmetic E. NOTA

27. A sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, … is written so that each number appears that number of times. What is the 2022nd term in the sequence?

A. 63 B. 64 C. 65 D. 66 E. NOTA

28. The angles in a nonagon form an arithmetic sequence. If the second smallest angle is equal to 125, what is the largest angle?

A. 135 B. 140 C. 150 D. 165 E. NOTA

29. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, what is $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$?

A. $\frac{\pi^2}{24}$ B. $\frac{\pi^2}{12}$ C. $\frac{\pi^2}{8}$ D. Infinite (does not converge) E. NOTA

30. Compute: $2+\cfrac{1}{1+\cfrac{2}{1+\cfrac{1}{1+\cfrac{2}{\ddots}}}}$

A. $1+\sqrt{2}$ B. $-1+\sqrt{2}$ C. $2\sqrt{2}$ D. 3 E. NOTA