1. A
2. A
3. C
4. C
5. B
6. E
7. A
8. B
9. B
10. B
11. B
12. A
13. C
14. C
15. C
16. B
17. A
18. B
19. D
20. C
21. B
22. E
23. A
24. D
25. A
26. C
27. A
28. A
29. A
30. C

1. A $\log 30^{2020} = 2020\log 30 \approx 2020(1.477) < 3030$
2. A John Napier coined the term.
3. C This will be when the exponents in the binomial expansion cancel, so the term is $\binom{10}{5}x^5x^{-5} = 252$.
4. C As $\log_a b \cdot \log_b c = \log_a c$, the product telescopes to $\log_7 343 = 3$
5. B $x = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ is a solution to $x^3 + 1 = 0$. Thus $x^3 = -1$. $x^{2020} = (x^3)^{673} \cdot x = -x$
6. E We need to solve $e = e^{.2t}$, so $t = 5$. 5 years is 60 months.
7. A B>A: 2020! (multiplying 2020 numbers, 2018 of which are larger than 2) is clearly significantly larger than 2^{2020} (multiplying 2020 2's).
   C>A: $10^{1000} > 8^{1000} = 2^{3000} > 2^{2020}$.
   D>A: $e^{e^{e^{e^{e}}}} > 2^{2^{2^{2^{2}}}} = 2^{2^{2^4}} = 2^{2^{16}} > 2^{2020}$.
8. B Let S be the desired sum. Then we have
   $$2S = 3S - S = \sum_{n=1}^{\infty}\frac{n}{3^{n-1}} - \sum_{n=1}^{\infty}\frac{n}{3^n} = \sum_{n=0}^{\infty}\frac{n+1}{3^n} - \sum_{n=0}^{\infty}\frac{n}{3^n} = \sum_{n=0}^{\infty}\frac{1}{3^n} = \frac{3}{2}$$
   So $S = \frac{3}{4}$, and the answer is 7.
9. B Solving: $e^x + 1 = 3e^x - 3 \rightarrow 2e^x = 4 \rightarrow x = \ln 2$
10. B Let $x = 10^a$, then $10^{a^2} = 10^{100}$, or $a = \pm 10$. Only $a = 10$ gives an integer solution to x, so $\log x = 10$.
11. B Note that every 4 consecutive terms of the sum cancel to 0. It leaves $i^0 = 1$.
12. A $S = \frac{(-2)-(-2)^{2021}}{1-(-2)} \approx \frac{2^{2021}}{3}$
    $\log S \approx 2021\log 2 - \log 3 \approx 2021(0.301) - 0.477 \approx 606$
13. C $ROB = 6^6 = a^3r^3$ and $O - R = ar - a$ or $a(r-1) = k^2$ where k is some integer. From the first equation, we cube root both sides to isolate $6^2 = ar$, which tells us that $36 - a = k^2$. A quick check by changing k^2 to different perfect squares less than 36 yields $k^2 = 9$ since $k^2 = 4$ makes a far too large to work with the original equation and the other squares won't work in the original prime factorization. So $a = 27$ and $r = \frac{4}{3}$, yielding. $(R,O,B) = (27,36,48) = 111$
14. C $2^{20} = 1024^2 > 1000000$, so $\lfloor \log_2 1000000 \rfloor = 19$
15. C Last digits on powers of integers always cycle every 4, so it is sufficient to look at powers of all 10 ending digits raised to the 4th power. Starting at 1, they are 1, 6, 1, 6, 5, 6, 1, 6, 1, 0. They add to 33. There are 202 sets of 10 final digits, so $202 \cdot 3$ ends in a 6.
16. B Square both sides, subtract $x + y$, then square again, getting
    $$4xy = (2020 - x - y)^2$$
    Let $d = \gcd(x, y)$, then let $x = ad$ and $y = bd$, then
    $$(2d)^2ab = (2020 - x - y)^2$$
    This implies as $\gcd(x, y) = 1$ that both a and b are squares. Let $a = m^2$ and $b = n^2$ The original equation is then $(m + n)\sqrt{d} = 2\sqrt{505}$. Thus $m = n = 1, d = 505$, and there is only one solution.

17. A

$$\sum_{n=0}^{\infty} log_{2^{2^n}} 2020 = \sum_{n=0}^{\infty} \frac{1}{2^n} \log_2 2020 = 2 \log_2 2020$$

18. B Let x be the given quantity, then $x = \sqrt{9900 + x}$. Then $x^2 = 9900 + x$, or $x = 100, -99$. Since $x > 0, x = 100$.

19. D

$$\lim_{n\to\infty} \left(1 + \frac{5}{2n}\right)^{2n} = \lim_{n\to\infty} \left(\left(1 + \frac{1}{\frac{2n}{5}}\right)^{\frac{2n}{5}}\right)^5 = e^5$$

20. C Cube the expression gives

$$\frac{a^3 + 3ab^2c + (3a^2b + b^3c)\sqrt{c}}{d^3} = \frac{535 + 207\sqrt{7}}{4}$$

From here, it's clear $c = 7$, and d is a power of 2. Try $d = 2$,

$$\frac{a^3 + 21ab^2 + (3a^2b + 7b^3)\sqrt{7}}{8} = \frac{1070 + 414\sqrt{7}}{8}$$

Now we have

$$a(a^2 + 21b^2) = 1070 = 2 \cdot 5 \cdot 107$$
$$b(3a^2 + 7b^2) = 414 = 2 \cdot 3^2 \cdot 23$$

a, b are both integers, so perform a quick parity check to narrow our options. If a, b are both even, then each product would contain more than a single 2 in its prime factorization. If only one of a, b is even, one of the products must be odd. Thus a, b must be both odd. There aren't many reasonable options left, $a = 5, b = 3$.

$$a + b + c + d = 17$$

21. B $\frac{2^x}{2^x-1} - \frac{2 \cdot 2^x - 2}{2^x - 1} < 0 \rightarrow \frac{2-2^x}{2^x-1} < 0 \rightarrow 2^x > 2 \text{ or } 2^x < 1$

Over $-10 \le x \le 10$, only $x = 0, 1$ fail the condition, so it is a total of 19.

22. E Separating the sum,

$$\sum_{k=2}^{2020} \left(\lfloor \sqrt[k]{2020} \rfloor - \lfloor \log_k 2020 \rfloor\right) = \sum_{k=2}^{2020} \lfloor \sqrt[k]{2020} \rfloor - \sum_{k=2}^{2020} \lfloor \log_k 2020 \rfloor$$

Now, I claim both parts of this expression are equal, giving an answer of 0. To see this, note that both sides of the expression are counting the number of positive integer solutions (a, b) to $a^b \le 2020$, where $a > 1$. The left part counts by case on b and the right counts by case on a.

23. A Let $y = z = 1$ to make all terms with a y or z only have x while maintaining the values of coefficients. So we have $f(x) = (x + 1)^{2020}$. The sum of the coefficients of even exponents is half of the overall sum of coefficients, so $S = 2^{2019}$. The final answer is $2019^2 = 4076361$

24. D $(x, y, z) = (0, k, k)$ satisfies the equation for all integer values of k, so there are infinite solutions.

25. A The graphs of e^x and x never intersect.

26. C Use change of base formula and go to base 3. $\frac{\log_3 x}{\log_3 243} - \frac{\log_3 9}{\log_3 x} = \frac{3}{5}$

$$\frac{\log_3 x}{5}-\frac{2}{\log_3 x}=\frac{3}{5} \qquad (\log_3 x)^2-10=3\log_3 x \to (\log_3 x)^2-3\log_3 x-10=0$$

$$(\log_3 x-5)(\log_3 x+2)=0 \to x=243,\frac{1}{9}$$

27. A $\frac{1}{\log_4 12}+\frac{1}{\log_3 12}=\log_{12} 4+\log_{12} 3=1$
28. A Let $y=2^x$, then $16y^2+2y-3=(8y-3)(2y+1)$, since $2^x>0$, we have $x=\log_2\frac{3}{8}=-3+\log_2 3$
29. A $\frac{8!}{2!2!}=10080$
30. C Let $y=2^x-2^{-x}$, then $y^3=2^{3x}-2^{-3x}-3(2^x+2^{-x})=8^x-8^{-x}=1764-3y$ Thus $y^3+3y=1764$, which has $y=12$ as a root and 2 nonreal roots.