

Theta

Logs and Exponents

Test #613

Directions:

1. Fill out the top section of the Round 3 Google Form answer sheet and select **Theta- Logs and Exponents** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).
2. Scoring for this test is 5 times the number correct plus the number omitted.
3. TURN OFF ALL CELL PHONES.
4. No calculators may be used on this test.
5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.
6. If a student believes a test item is defective, select “E) NOTA” and file a dispute explaining why.
7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.
8. If a problem has wording like “which of the following could be” or “what is one solution of”, an answer choice providing one of the possibilities is considered to be correct. Do not select “E) NOTA” in that instance.
9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select “E) NOTA” in that instance.
10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

NOTA denotes “None of the Above.”

- Find the number of digits of 30^{2020} rounded to the nearest 100.
A. 3000 B. 3100 C. 3200 D. 3300 E. NOTA
- Which mathematician coined the term “logarithm”?
A. Napier B. Nash C. Newton D. Nibet E. NOTA
- Evaluate the constant term in the expansion of $(x + x^{-1})^{10}$
A. 60 B. 120 C. 252 D. 720 E. NOTA
- Evaluate $\prod_{n=7}^{342} \log_n(n + 1)$
A. 1 B. 2 C. 3 D. 4 E. NOTA
- Evaluate $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{2020}$
A. $\frac{1}{2} - \frac{i\sqrt{3}}{2}$ B. $-\frac{1}{2} - \frac{i\sqrt{3}}{2}$ C. $\frac{1}{2} + \frac{i\sqrt{3}}{2}$ D. $-\frac{1}{2} + \frac{i\sqrt{3}}{2}$ E. NOTA
- Maxwell has a million dollars he invested in Gymshark. His investment is compounded continuously at an annual rate of 20%. After how many months will his investment be worth e million dollars?
A. 1 B. 2 C. 5 D. 10 E. NOTA

7. Which of the following answer choices is the smallest? (Note if there are multiple choices which obtain the minimum, select NOTA).
- A. 2^{2020} B. $2020!$ C. 10^{1000} D. $e^{e^{e^{e^e}}}$ E. NOTA
8. Let $S = \sum_{n=1}^{\infty} \frac{n}{3^n}$. Given S is a positive rational number, find the sum of the numerator and denominator of the reduced fraction.
- A. 5 B. 7 C. 10 D. 12 E. NOTA
9. Find x , where $\frac{e^x+1}{e^x-1} = 3$
- A. 1 B. $\ln 2$ C. $\ln 3 - 1$ D. $\ln 3$ E. NOTA
10. Let x be the integer solution to $x^{\log x} = 10^{100}$. Find $\log x$.
- A. 1 B. 10 C. 100 D. 1000 E. NOTA
11. Evaluate $\sum_{n=0}^{2020} i^n$
- A. 0 B. 1 C. i D. $-i$ E. NOTA
12. Let $S = \sum_{n=1}^{2020} (-2)^n$. Evaluate $[\log S]$ then round to the nearest 10
- A. 610 B. 620 C. 630 D. 640 E. NOTA

13. It is given that $\log_6 R + \log_6 O + \log_6 B = 6$ where $R < O < B$ are integers that form a geometric sequence and $O - R$ is a perfect square. Find $R + O + B$.
- A. 27 B. 36 C. 111 D. 127 E. NOTA
14. Find the characteristic of $\log_2 1,000,000$.
- A. 17 B. 18 C. 19 D. 20 E. NOTA
15. What is the units digit of $\sum_{n=1}^{2020} n^{2020}$
- A. 1 B. 5 C. 6 D. 9 E. NOTA
16. Find the number of ordered pairs (x, y) of natural numbers that satisfy $\sqrt{x} + \sqrt{y} = \sqrt{2020}$
- A. 0 B. 1 C. 2 D. 4 E. NOTA
17. Evaluate $\sum_{n=0}^{\infty} \log_{2^{2^n}} 2020$
- A. $\log_2 4080400$ B. $\log_2 4084441$
C. $\log_2 4088484$ D. $\log_2 4092529$ E. NOTA
18. Evaluate $\sqrt{9900 + \sqrt{9900 + \sqrt{9900 + \dots}}}$
- A. 99 B. 100 C. 101 D. 121 E. NOTA

19. Given $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$, evaluate $\lim_{n \rightarrow \infty} \left(1 + \frac{5}{2n}\right)^{2n}$
A. 1 B. e C. e^2 D. e^5 E. NOTA
20. The cube root of $\frac{535+207\sqrt{7}}{4}$ can be written in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, d are positive integers, with $\gcd(a, b, d) = 1$ and c is not divisible by the square of any prime. Evaluate $a + b + c + d$.
A. 12 B. 14 C. 17 D. 20 E. NOTA
21. How many integral solutions exist to the inequality $\frac{2^x}{2^x-1} < 2$ for $|x| \leq 10$?
A. 18 B. 19 C. 20 D. 21 E. NOTA
22. Evaluate $\sum_{k=2}^{2020} (\lfloor \sqrt[k]{2020} \rfloor - \lfloor \log_k 2020 \rfloor)$
A. 1 B. 2020 C. 2020^2 D. 2020^{2020} E. NOTA
23. Let S be the sum of the coefficients of all terms which have an even power of x in the simplified expansion of $(x + 3y - 2z)^{2020}$. S can be written in the form a^b , where a, b are natural numbers and a is not divisible by the square of any prime. Find b^a .
A. 4076361 B. 4080400 C. 8242408000 D. 8254655261 E. NOTA
24. How many ordered triples of integers (x, y, z) satisfy $x^5 + y^5 = z^5$?
A. 0 B. 1 C. 5 D. Infinite E. NOTA

25. How many solutions are there to the equation $e^x = x$?
A. 0 B. 1 C. 2 D. 3 E. NOTA
26. What is the product of the solutions: $\log_{243} x - \log_x 9 = \frac{3}{5}$?
A. $\frac{1}{9}$ B. 3 C. 27 D. 729 E. NOTA
27. Evaluate $\frac{1}{\log_4 12} + \frac{1}{\log_3 12}$
A. 1 B. $\log_4 36$ C. 3 D. 4 E. NOTA
28. Find x , where x satisfies $4^{x+2} + 2^{x+1} = 3$
A. $-3 + \log_2 3$ B. $3 - \log_2 5$ C. $5 - \log_2 3$ D. $5 - \log_2 5$ E. NOTA
29. How many distinct permutations of the word EXPONENT are there?
A. 10080 B. 21600 C. 43200 D. 86400 E. NOTA
30. Suppose $8^x - 8^{-x} = 1764$ for some real x . Find $2^x - 2^{-x}$
A. 6 B. 10 C. 12 D. 15 E. NOTA