For all questions below, the answer E. NOTA means "None of these answers".

1. C Let m be the number of minutes either plan is used. The two plans cost the same amount when $0.02m + 12 = 0.03m + 10$. Thus $0.01m = 2$ and so $m = 200$.

2. D We need $\left(1 - \frac{p}{100}\right)\left(1 + \frac{p}{100}\right) = 1 - \frac{36}{100}$, so $1 - \left(\frac{p}{100}\right)^2 = 1 - \frac{36}{100}$. This means $\frac{p}{100} = \sqrt{\frac{36}{100}}$ so $p = 100\left(\frac{6}{10}\right) = 60$.

3. A The number of gallons required to travel the given distance is $\frac{D}{G}$. Multiplying this by the cost of gas per gallon gives $\frac{DC}{G}$ dollars.

4. C The maximum height occurs when $t = -\frac{b}{2a} = -\frac{48}{-32} = 1.5$ seconds. The maximum height is $h(1.5) = h\left(\frac{3}{2}\right) = -16\left(\frac{9}{4}\right) + 48\left(\frac{3}{2}\right) + 4 = -36 + 72 + 4 = 40$ feet.

5. D The 4-liter salt solution is made up of 1 liter of salt and 3 liters of water. By adding x liters of pure water to this, there will be 1 liter of salt and $3+x$ liters of water for a total of $4+x$ liters. Since the final solution is only 5% salt, then $\frac{1}{4+x} =$ so $x = 16$.

6. B We want $0.96\ell_0 = \ell_0\sqrt{1 - v^2/c^2}$ or $\frac{96}{100} = \frac{24}{25} = \sqrt{1 - v^2/c^2}$. Squaring both sides, $\left(\frac{24}{25}\right)^2 = 1 - v^2/c^2$ so $1 - \left(\frac{24}{25}\right)^2 = v^2/c^2$. This means $\frac{25^2 - 24^2}{25^2} = \frac{7^2}{25^2} = \frac{v^2}{c^2}$ so $\frac{7}{25} = \frac{v}{c}$. This means $v = 0.28c$ or 28% of the speed of light.

7. B Let d be the desired distance. Then $\frac{850}{2.5} = \frac{d}{1.5}$ so $d = \frac{1.5(850)}{2.5} = \frac{3(850)}{5} = 3(170) = 510$ meters.

8. D Let S be the sum of the 16 numbers. Then $\frac{S}{16} = 20$. Let M be the max and N the min. When the maximum is removed, $\frac{S-M}{15} = 20 - 2$ so $S - M = 15(18)$. When the minimum is removed, $\frac{S-N}{15} = 20 + 1$ so $S - N = 15(21)$. Subtracting the two equations, $M - N = 15(21) - 15(18) = 15(21 - 18) = 15(3) = 45$, which is the range.

9. C The perimeter of the irregular shape is equivalent to the perimeter of the circumscribed rectangle, namely, $2(2x + 10) + 2(4x - 10) = 2(6x) = 12x$, which equals 120. So $x = 10$.

10. B Let h, s, w be the respective costs of 1 hammer, 1 screwdriver, and 1 wrench. We are given $3h + 1s + 1w = 25, 1h + 3s + 1w = 20, 1h + 1s + 3w = 17.50$. Adding all 3 equations, $5h + 5s + 5w = 25 + 20 + 17.50 = 62.50$. Dividing by 5, we have $1h + 1s + 1w = 12.50$, the desired cost.

11. C By drawing a diagonal, the desired area is twice the area of one triangle, which we can find as $\frac{1}{2}\left(2\sqrt{3}\right)\left(3\sqrt{3}\right)\sin(150^\circ) = 9 \cdot \frac{1}{2} = \frac{9}{2}$. Doubling, the area of the parallelogram is 9. Alternatively, one could draw in an altitude to find the area of a triangle.

12. C We can get to 1 as follows: $2020 \rightarrow 404 \rightarrow 403 \rightarrow 402 \rightarrow 401 \rightarrow 400 \rightarrow 80 \rightarrow 16 \rightarrow 15 \rightarrow 3 \rightarrow 2 \rightarrow 1$, namely we divide by 5 for a multiple 5 and subtract 1 otherwise. This requires 11 minimal button presses.

13. C Let the radius of the semicircle be *r* and the side length of the square be *s*. We are told that $2r + \frac{1}{2}(2\pi r) = 4s$ so $r(2+\pi) = 4s$. Squaring both sides, $r^2(2+\pi)^2 = 16s^2$. Multiplying by $\pi, \pi r^2(2+\pi)^2 = 16\pi s^2$ and thus, $\frac{\frac{1}{2}\pi r^2}{s^2} = \frac{8\pi}{(2+\pi)^2}$.

14. C To make the largest prime, we consider a number of the form 98_. Since 987 is divisible by 3, 986 is even, 985 is divisible by 5, and 984 is even, we consider 983. One can check that this is indeed prime, so the sum of digits is 9+8+3 = 20.

15. B This is a convergent geometric series with ratio $r = \frac{3}{4} < 1$. The total distance the ball travels downward is $\frac{20}{1-\frac{3}{4}} = 80$ cm. The total distance the ball travels upward is $\frac{15}{1-\frac{3}{4}} = 60$ cm. So the total distance traveled is 80 + 60 = 140 cm. Alternatively, the total distance can be computed as $\frac{35}{1-\frac{3}{4}}$ or $2 \cdot \frac{20}{1-\frac{3}{4}} - 20$.

16. D There are two ways that Jack and Jill can occupy the ends and there are 4! ways to rearrange the remaining players in between. This yields $2 \cdot 4!$ possible photographs with the given constraint out of 6! possible photos for a probability of $\frac{2\cdot 4!}{6!} = \frac{2}{5\cdot 6} = 1/15$.

17. C Note the first three figures have area $1 \cdot 3,\ 2 \cdot 5,$ and $3 \cdot 7$. Generally, the n^{th} figure has $n(2n+1)$ unit squares so the 100^{th} has $100(2 \cdot 100 + 1) = 100(201) = 20100$.

18. B Since lcm(2,3,4) = 12, consider the first 12 coins, initially PPPPPPPPPPPP. After the nickels replace every second coin, we have PNPNPNPNPNPN. After we replace every third coin with a dime, we have PNDNPDPNDNPD. Finally, the quarters are placed at every fourth coin: PNDQPDPQDNPQ. These 12 coins have value 4(1)+2(5)+3(10)+3(25) or 119 cents. Since 101=8(12)+5, we have 119 cents 8 times plus the value of the 5 coins of the next sequence. This yields a total value of 8(119) + (1+5+10+25+1) = 994 cents = $9.94.

19. D The smaller pyramid is half the height and half the side length of the base of the larger pyramid. Thus, its volume is $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ that of the larger volume. Once removed, the remaining solid has volume $1 - \frac{1}{8} = \frac{7}{8}$ that of the original pyramid.

20. C Since $44 < \sqrt{1980} < 45$, we can guess that the two pages are 44 and 45. Surely, (44)(45) = 1980 so the sum of the two page numbers is 44+45=89. Alternatively, call the two page numbers x and $x + 1$ for which $x(x + 1) = 1980$ so $x^2 + x - 1980 = 0$. Factoring, $(x + 45)(x - 44) = 0$ so $x = 44$ and $x + 1 = 45$ are the two page numbers as before. Finally, we can prime factorize: $1980 = 198 \cdot 10 = 99 \cdot 20 = (3 \cdot 3 \cdot 11)(2 \cdot 2 \cdot 5)$. We can rearrange the factors to be the product of two consecutive integers, 44 and 45, as before.

21. E Let there be n total rows and so there are $20 + 2(n - 1)$ seats in the last row. The total number of seats is an arithmetic series: $S_n = \left(\frac{a_1+a_n}{2}\right)n$. In this case, we have $840 = \left(\frac{20+20+2(n-1)}{2}\right)n$. Simplifying, $n^2 + 19n - 840 = 0 = (n - 21)(n + 40)$ so either $n = 21$ or $n = -40$, the latter being extraneous. Thus, there are $n = 21$ rows.

22. B Due to the angle, let $(a\sqrt{3}, a)$ be a point on $y = 4 - x^2$ due to special right triangles. Then $a = 4 - \left(a\sqrt{3}\right)^2$ so $3a^2 + a - 4 = 0$. Factoring, $(3a + 4)(a - 1) = 0$ so $a = 1$ and $(\sqrt{3}, 1)$ is our point. The base of the rectangle is $2\sqrt{3}$ and the height is 2 so the area of the rectangle is $2\left(2\sqrt{3}\right) = 4\sqrt{3}$.

23. A Ship A is $(20 - 10t)$ miles north of O while Ship B is $(50 + 4t)$ east of O. By the Pythagorean Theorem, the ships are a distance $\sqrt{(20 - 10t)^2 + (50 + 4t)^2}$ away. Simplifying, $\sqrt{400 - 400t + 100t^2 + 2500 + 400t + 16t^2} = \sqrt{2900 + 116t^2}$ or $\sqrt{4(725 + 29t^2)} = 2\sqrt{725 + 29t^2}$.

24. C Since each of the four entries has two possible values (0 or 1) there are $2^4 = 16$ possible matrices. To be invertible, the determinant should be non-zero. There are only 6 such non-zero determinants: $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}, \begin{vmatrix}0 & 1\\1 & 0\end{vmatrix}, \begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\1 & 0\end{vmatrix}, \begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}, \begin{vmatrix}0 & 1\\1 & 1\end{vmatrix}$. The desired probability is $1 - \frac{6}{16} = \frac{10}{16} = \frac{5}{8}$.

25. C Since $-1 \le \sin(x) \le 1$, the range of blood pressures is $[120 - 15, 120 + 15]$ so $M = 135$ and $N = 105$ and $M - N = 30$. Note this is just twice the amplitude of the wave.

26. C We can write the model that $P(t) = A(1.20)^{\frac{t}{10}}$ where A is the initial population of the deer and t is in years. Then $P(20) = A(1.20)^{\frac{20}{10}} = A(1.2)^2 = 1.44A$ so the population has grown by 44%.

27. D The original surface of the cube before removing pieces is $6(3)^2 = 54$ square units. Note that by removing any one of the 8 corner cubes, three faces that could be seen are traded for three faces that were hidden that now can be seen. Thus, the total surface area won't change and is still 54 square units.

28. B There is a total of 12 letters: 1 A, 2 I's, 2 O's, 1 C, 1 M, 1B, 2N's, 1 T, 1 S. We group the 5 vowels together. They can be arranged in $\frac{5!}{2!2!}$ ways (note the repeats). There are then 7 other letters and the vowel group, yielding 8 total objects that can be arranged in $\frac{8!}{2!}$ ways due to the repeat of N's. There are $\frac{5!}{2!2!} \cdot \frac{8!}{2!}$ ways to have the vowels together, In total, there are $\frac{12!}{2!2!2!}$ ways to scramble all the letters, noting repeats. The desired probability is then

$(\frac{5!}{2!2!} \cdot \frac{8!}{2!})/\frac{12!}{2!2!2!} = (5!\,8!)/12!$. Simplifying, this is $\frac{5\cdot4\cdot3\cdot2\cdot1}{12\cdot11\cdot10\cdot9} = \frac{1}{99}$ so $m + n = 100$.

29. B Suppose the company decreases the cost by x increments of 50 cents. Then we want to maximize $(10 - .5x)(36 + 3x) = -1.5x^2 + 12x + 360$. The zeros occur at $x = \frac{-12\pm\sqrt{144-4(-1.5)(360)}}{2(-1.5)} = \frac{-12\pm48}{-3} = -12, 20$ which means the maximum occurs at $x = 4$. This requires we drop the cost by 4 increments of 50 cents or by \$2 for a final cost of \$8.

30. C Call the distinct side lengths of the box x, y, z. We are told $xy = 40, yz = 60, xz = 96$. Multiplying all 3 equations, $(xy)(yz)(xz) = 40 \cdot 60 \cdot 96$ or $(xyz)^2 = 24 \cdot 96 \cdot 100$ or $(xyz)^2 = (2)^2(24)^2(10)^2$ and so $xyz = 2 \cdot 24 \cdot 10 = 480$