2022 MU ALPHA THETA NATIONAL CONVENTION
ALPHA BOWL - CONDENSED

Question 0

Find the value of $10(x+y)$ using the x- and y-values which satisfy $\begin{cases} \lfloor x \rfloor + \lfloor y \rfloor + y = 19.8 \\ x + y - \lfloor x \rfloor = 8.5 \end{cases}$.

Question 1

Let A = the number of solutions to $\csc x \sin 2x = \tan\frac{x}{2} + 2$ over $[0, 2\pi)$.

Let B = the sum of the solutions to $\csc x \sin 2x = \tan\frac{x}{2} + 2$ over $[0, 2\pi)$.

Let C = the value of $\text{Arcsin}\frac{2}{5} + \text{Arccos}\frac{2}{5}$. (Use radians.)

Let D = the sum of the solutions to $\begin{cases} 2\sin^2 x - \cos 2x = 0 \\ 2\cos^2 x - 3\sin x = 0 \end{cases}$ over $[0, 2\pi]$.

Find $AC + B + D$.

Question 2

A hyperbola has its center at the origin, passes through the point (4, 2), and has transverse axis of length 4 along the x-axis. Let A = the square of the eccentricity.

The centers and intersections of $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ form the vertices of a quadrilateral. Let B = the area of the quadrilateral.

Find $\frac{B}{A}$.

Question 3

For all parts, $i = \sqrt{-1}$.

Let A = the number of nonzero integral solutions to $|1-i|^x = 2^x$.

Let B = the sum of the four least positive radian values of θ for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

Let C = the sum of all $\theta, -\frac{\pi}{2} < \theta < \pi$, for which $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary.

Let D = the value of $x^2 + y^2$ if $x + iy = \frac{a+bi}{a-bi}$.

Find $\frac{A+B}{CD}$.

2022 MU ALPHA THETA NATIONAL CONVENTION
ALPHA BOWL - CONDENSED

Question 4

Let A = the value of k such that $\begin{cases} 9x-6y=21 \\ 6x-4y=k \end{cases}$ has infinitely many solutions.

Let B = the value of k such that $\begin{cases} 2x+8y=5 \\ kx+36y=8 \end{cases}$ has no solutions.

Let C = the value of k such that $\begin{cases} -3x+2y-4z=6 \\ 7x+6y+4z=k \\ -5x-4y-3z=-1 \end{cases}$ has infinitely many solutions.

Find $A-B-C$.

Question 5

Let N = the number of integers between $\sqrt{x^2+2x+4}$ and $\sqrt{4x^2+2x+1}$ if $x=2021^{2021}$. Let A = the last two digits of N.

Let K = the coefficient of the x^{2020} term in the expansion of $(x+3)^{2021}$. Let B = the sum of the digits in the base-8 representation of K.

Find $B-A$.

Question 6

Use radians in all parts.

Let A = the value of $\sin\left[\text{Arccos}\frac{5}{13}-\frac{\pi}{2}\right]$.

Let B = the value of $\tan\left[\text{Arccos}\frac{3}{5}-\text{Arcsin}\left(-\frac{5}{13}\right)\right]$.

Let C = the value of $\sin\left[\text{Arctan}\left(-\frac{3}{4}\right)+\text{Arccot}\frac{12}{5}\right]$.

Let D = the value of x such that $\text{Arctan}\,x+\text{Arctan}\frac{1}{3}=\text{Arctan}\,1$.

Find $\frac{100BCD}{A}$.

Question 7

Let $A = \left(\left(\log_2 9\right)^2\right)^{\frac{1}{\log_2(\log_2 9)}}\left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$.

One thousand unit cubes are fastened together to form a large cube with side length 10 units. This is painted and then separated into the original 1000 cubes. Let B = the number of unit cubes that have at least one face painted.

Find $\frac{B}{A}$.

Question 8

Let A = the number of different possible remainders when any integer is divided by 6.

Let B = the sum of all n, $0 \le n \le 20$, for which $3n \equiv 2 \bmod 5$.

Let C = the value of x (least residue) for which $13^{20} \equiv x \bmod 11$.

Let D = the sum of all x, $0 < x < 10$, for which $2x + 3 \equiv -1 \bmod 4$.

Find $A + B + C + D$.

Question 9

Let A = the square of the focal width of an ellipse whose foci are 6 units apart and whose directrices are 12 units apart.

Let B = the sum of the values of p for which the circle $x^2 + y^2 + 2x + 4y - p = 0$ intersects the coordinate axes exactly three times.

Find $A + B$.

Question 10

The curve $y = x^3 - 5x^2 + 2x + 5$ intersects $y = 2x + 1$ at three points, one being (1, 3). Let A = the sum of the y-coordinates of the other two points.

Let B = the value of $r(p+q)$, given that $x^3 + 3x^2 + 9x + 3$ is a factor of $x^4 + 4x^3 + 6px^2 + 4qx + r$.

Find $A + B$.

Question 11

Gob Bluth buys a Segway for $22,000. He pays $4000 cash and agrees to pay the balance in annual installments of $1000 plus 10% interest of the unpaid amount. (For example, if he owed $20,000, he would pay $1000 + $2000 interest.) What will be the total amount of money he has spent on the Segway when he has made his final payment?

Question 12

Let $A = f(5)$, if $\left[f(x)\right]^2 = \left[f(x-1)\right]\left[f(x+1)\right]$, $f(1)=1$, and $f(3)=9$.

Let $B = g(26)$, if $2g(x)=g(x+1)+g(x-1)$, $g(2)=6$ and $g(6)=2$.

Let $C = h(11)$, if $h(x+1)h(x-1)=\left[h(x)\right]^2-\left(-1\right)^x$ for $x \ge 1$ and $h(0)=h(1)=1$.

Find $\cos\left[5\left(B+C-A\right)\right]^\circ$.

Question 13

Let A = the value of $k+d$ if the line $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ lies in the plane $4x+4y-kz-d=0$.

Let B = the value of $\left\lfloor \frac{D}{3} \right\rfloor$, if the distance from the point $(-1,-5,-10)$ to the intersection of the line $\frac{x-2}{2}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$ is D.

Let C = the sum of the values of k for which the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar.

Find $\frac{B^{-C}}{A}$.

Question 14

Company records show that 85% of the new workers who attend the training program meet their production quota the first month on the job, while 33% of the new workers who do not attend the training program meet their production quota during the first month. Assume that 60% of all new workers attend the training program.