

1) A	7) B	13) B	19) A	25) A
2) D	8) D	14) E	20) C	26) D
3) A	9) A	15) D	21) B	27) C
4) B	10) D	16) B	22) C	28) C
5) D	11) D	17) B	23) C	29) A
6) A	12) B	18) C	24) B	30) A

1) **A** - From Euler's formula, $e^{i\frac{\pi}{2}}$. Plugging in we have $i^{-i} = (e^{i\frac{\pi}{2}})^{-i} = e^{-i^2\frac{\pi}{2}} = e^{\frac{\pi}{2}}$

2) **D** - $I = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$, $II = 1 - i$, $III = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$, $IV = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$, and $V = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$. I, IV, and V are equivalent

3) **A** - One must simplify the negative underneath the square root before combining together into one square root. $\sqrt{-6} \times \sqrt{-6} = i\sqrt{6} \times i\sqrt{6} = i^2 6 = -6$

4) **B** - The complex conjugate of a real number is itself.

5) **D** - This is equal to $(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}))^6 = \text{cis}(\frac{\pi}{3})^6 = \text{cis}(2\pi) = 1$

6) **A** - Multiplying the fraction by $\frac{(1+2i)(5-i)}{(1+2i)(5-i)}$ will 'realize' the denominator, giving $\frac{(2+i)(1+2i)(5-i)}{(5)(26)} = \frac{1+5i}{26}$.

7) **B** - Since the cubic has only real coefficients, $2+i$ must also be a root. The quadratic with roots $2+i$ and $2-i$ is $x^2 - 4x + 5$ since the roots add to 4 and multiply to 5. Therefore our cubic is $(x-5)(x^2 - 4x + 5)$. To find the sum of the coefficients, just plug in $x = 1$, to get -8

8) **D** - $|z|^2 = z\bar{z}$. We have $(z\bar{z})(z\bar{z})(\bar{z}) = (2)(2)(\bar{z}) = 4 + 4i\sqrt{3} \Rightarrow \bar{z} = 1 + i\sqrt{3}$ which has argument $\frac{\pi}{3}$, so z has argument $\frac{5\pi}{3}$.

9) **A** - $\sqrt{-i} = cis(\frac{3\pi}{2})^{\frac{1}{2}} = cis(\frac{3\pi}{4}) = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$. The other value is the negative of this, $\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

10) **D** - $(\frac{1}{2} + \frac{i\sqrt{3}}{2})^{12} = cis(\frac{\pi}{3})^{12} = cis(4\pi) = 1$

11) **D** - $\frac{(2-2i)^4(1+i\sqrt{3})^4}{e^{i\pi/3}} = \frac{[(2\sqrt{2})(e^{i\frac{7\pi}{4}})]^4 \cdot [(2)(e^{i\frac{\pi}{3}})]^4}{e^{i\frac{\pi}{3}}} = \frac{(64)(e^{7i\pi})(16)(e^{i\frac{4\pi}{3}})}{e^{i\frac{\pi}{3}}} = 1024$

12) **B** - This equation describes the locus of points whose distance from $3 - 3i$ is equal to the distance to the line $Im(z) = -4$, which is a parabola.

13) **B** - This describes an ellipse with major axis length 8. The minor axis can be found by sketching a picture, where $0+0i$ and $4+0i$ are foci, and the z being plugged in has $Re(z) = 2$ (over the center of the ellipse). Connecting the foci to the point gives a hypotenuse of 4 and leg along the major axis length 2, which by Pythagorean Theorem gives $2\sqrt{3}$ as the semi-minor axis. The area is then $2\sqrt{3} \times 4 \times \pi = 8\pi\sqrt{3}$

14) **E** - A base case (with inspiration from Euler's Formula) would be $x = e^{2\pi}$, however due to the periodic nature of complex exponentiation, $e^{4\pi}, e^{6\pi}, \dots$ all also work, meaning there are infinitely many solutions.

15) **D** - Manipulating the exponentials we get two points in the complex plane with magnitude e^5 , one with angle 0 and the other with angle π , meaning that they are just along the x-axis. The distance between them is $2e^5$.

16) **B** - To rotate by 45° clockwise, we must multiply by a complex number with modulus 1 and angle to the positive x-axis of -45° , that number being exactly $\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$. We have $(3+i)(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}) = 2\sqrt{2} - i\sqrt{2}$

17) **B** - Multiplying z by any purely imaginary power of e will not change the magnitude, only the argument, so the magnitude is still 4.

18) **C** - This is a geometric series with first term $4 + 4i$ and common ratio $(.5 + .5i)$. Using the sum of an infinite geometric series formula $\frac{a}{1-r}$, we get $\frac{4+4i}{1-(\frac{1}{2}+\frac{1}{2}i)} = 8i$ as our answer.

19) **A** - $\ln(-20) = \ln((-1)(20)) = \ln(e^{i\pi}) + \ln(20) = i\pi + \ln(20)$

20) **C** - The arguments of the roots have the form $\frac{2k\pi}{47}$ where k is a positive integer less than or equal to 47. To be in the second quadrant, the argument has to lie between $\frac{\pi}{2}$ and π . We then should solve the inequalities $\frac{2k\pi}{47} > \frac{\pi}{2} \Rightarrow k > 11.75$ and $\frac{2k\pi}{47} < \pi \Rightarrow k < 23.5$. This corresponds to 12 values of k .

21) **B** - The roots are all symmetric about both the x and y axes, so they must sum up to 0.

22) **C** - The tangent of the angle between the two lines is given by the formula $\tan(\theta) = \frac{m_1 - m_2}{1 + m_1 m_2} \Rightarrow \frac{5-4}{1+5 \cdot 4} = \frac{1}{21}$. The area of one sector is $\theta r^2 = \theta$ since $r = 1$ here. The ratio then would be the area of one sector

23) **C** - Since $e^{ix} = \cos(x) + i \sin(x)$, $Re(f(x)) = \sin(x)$, and $Im(f(\frac{\pi}{2} - x)) = \cos(\frac{\pi}{2} - x) = \sin(x)$. So the sum is $2 \sin(x)$ which has amplitude 2.

24) **B** - This is the peak of the sin wave, and by symmetry and no calculus involved the tangent must be a flat line with slope 0.

25) **A** - This corresponds to traversing around the unit circle once, which has a circumference of 2π .

26) **D** - Since the fifth roots of unity of the number have arguments differing by $\frac{2\pi}{5}$, shifting one root over by half of that will result in a root of unity of the negative of the original number, so taking it to the power of 10 will be equal to squaring the negative of 1, which is still 1.

27) **C** - The slope of a line is equal to the tangent of the argument of the complex number. We're given that $\tan(\theta) = 2$ and we're searching for $\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)} = \frac{2(2)}{1 - 2^2} = -\frac{4}{3}$

28) **C** - The description provided gives the 49 roots of unity of 1 shifted to the right by 1. Let a_1, a_2, \dots, a_{49} refer to the 49 roots of unity of 1. Therefore these points are $a_1 + 1, a_2 + 1, \dots, a_{49} + 1$. Multiplying them all, we get, by symmetry, we get all of Vieta's sums for the polynomial $x^{49} = 1$. All terms besides the first and last is equal to zero, and all we are left with is the product of all the terms, 1, added by 1, which is 2.

29) **A** - The function factors to $f(x) = \frac{(x-1)(x-2)(x-3)}{2(x-2)(x+2)}$. The removable discontinuity is at the factor that cancels, $x - 2$, so $a = 2$. The slope of the slant asymptote of a rational function $q(x) = \frac{rx^{n+1}+blah}{sx^n+blah}$ is $\frac{r}{s}$, in this case it's $\frac{1}{2} = b$. $|2 + \frac{1}{2}i| = \sqrt{2^2 + \frac{1}{2}^2} = \frac{\sqrt{17}}{2}$

30) **A** - Add the first two equations to get $z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 = 19$, then subtract the third equation to get $z_3 + z_5 + z_6 + z_7 = 3$