For this test, E) NOTA means "None Of These Answers". $\text{cis}(\theta) = \cos(\theta) + i\sin(\theta) = e^{i\theta}$. If $z = a + bi$, define $Re(z) = a$ and $Im(z) = b$.

1) Which of the following is equivalent to i^{-i}?

A) $e^{\frac{\pi}{2}}$ B) $e^{-\frac{\pi}{2}}$ C) $ie^{\frac{\pi}{2}}$ D) $ie^{-\frac{\pi}{2}}$ E) NOTA

2) How many of the following are equivalent?

I. $e^{i\frac{3\pi}{4}}$

II. $1 + i\tan\left(\frac{3\pi}{4}\right)$

III. $\sin\left(\frac{\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

IV. $\text{cis}\left(\frac{3\pi}{4}\right)$

V. $\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)$

A) 0 B) 1 C) 2 D) 3 E) NOTA

3) What is the value of $\sqrt{-6} \times \sqrt{-6}$

A) -6 B) 6 C) 6i D) -6i E) NOTA

4) Which of the following is the complex conjugate of 5?

A) -5 B) 5 C) $-5i$ D) $5i$ E) NOTA

5) What is $\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)^6$?

A) -1 B) $\frac{1}{2} - i\frac{\sqrt{3}}{2}$ C) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ D) 1 E) NOTA

6) Simplify the denominator: $\dfrac{2+i}{(1-2i)(5+i)}$

A) $\dfrac{1+5i}{26}$ B) $\dfrac{1-5i}{26}$ C) $\dfrac{1+5i}{65}$ D) $\dfrac{1-5i}{65}$ E) NOTA

7) If 5 and $2-i$ are the roots of a cubic polynomial with real coefficients and lead coefficient 1, what is the sum of the coefficients of the cubic?

A) -12 B) -8 C) 0 D) 9 E) NOTA

8) Let $|z| = 2$. If $z^2\overline{z}^3 = 4 + 4i\sqrt{3}$, what is the argument of z?

A) $\dfrac{\pi}{6}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{2\pi}{3}$ D) $\dfrac{5\pi}{3}$ E) NOTA

9) Which of the following is a value of $\sqrt{-i}$?

A) $-\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$ B) -1 C) $\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$ D) DNE E) NOTA

10) What is $\left(\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}\right)^{12}$?

A) -1 B) $\dfrac{1}{2} - \dfrac{i\sqrt{3}}{2}$ C) $\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2}$ D) 1 E) NOTA

11) What is the value of $\dfrac{(2-2i)^4(1+i\sqrt{3})^4}{e^{i\pi/3}}$?

A) -1024 B) $512-512i$ C) $512+512i$ D) 1024 E) NOTA

12) $|z-3+3i| - |Im(z)+4| = 0$ represents what graph?

A) Line B) Parabola C) Ellipse D) Hyperbola E) NOTA

13) What is the area of the locus of points such that $|z| + |z - 4| = 8$?

A) 8π B) $8\pi\sqrt{3}$ C) $12\pi\sqrt{6}$ D) 64π E) NOTA

14) How many real numbers x exist such that $x^i = i$?

A) 0 B) 1 C) 2 D) 4 E) NOTA

15) What is the distance between $e^{(5+i)\pi}$ and $e^{(5+4i)\pi}$ on the complex plane?

A) $e^{(5+3i)\pi}$ B) $\sqrt{2}e^{(5+3i)\pi}$ C) $e^{5\pi}$ D) $2e^5$ E) NOTA

16) Given that $z = 3 + i$, what complex number results from rotating z by $45°$ clockwise along the complex plane?

A) $-\sqrt{2}+2\sqrt{2}i$ B) $2\sqrt{2}-\sqrt{2}i$ C) $-2\sqrt{2}+\sqrt{2}i$ D) $-2\sqrt{2}+2\sqrt{2}i$ E) NOTA

17) Given that $|z| = 4$, what is the largest possible value of $|ze^{i(2x+5)}|$, where x is a real number?.

A) 1 B) 4 C) $4\sqrt{2}$ D) DNE E) NOTA

18) Evaluate

$$\sum_{n=0}^{\infty}(4+4i)\left(\frac{1}{2}+\frac{1}{2}i\right)^n$$

A) $16 + 16i$ B) $8 + 8i$ C) $8i$ D) DNE E) NOTA

19) Allowing for complex inputs to the natural logarithm make it possible to take the log of a negative number! In particular, since $e^{i\pi} = -1$, $\ln(-1) = \ln(e^{i\pi}) = i\pi$. Given that information, evaluate $\ln(-20)$

A) $\ln(20) + i\pi$ B) $2\ln(5) + i\pi$ C) $20i\pi$ D) $-20i\pi$ E) NOTA

20) How many of the 47^{th} roots of the hexidecimal number BC_{16} lie in the second quadrant when plotted on the Argand plane?

A) 10 B) 11 C) 12 D) 13 E) NOTA

21) What is the sum of the 16 roots of unity of 16?

A) -16 B) 0 C) 16 D) 32 E) NOTA

22) Two lines with slopes 5 and 4 going through the origin split the unit circle into four pieces. Compute the ratio of the area of one of the smaller pieces to the entire circle.

A) $\frac{\tan^{-1}\left(\frac{1}{19}\right)}{\pi}$ B) $\frac{\tan^{-1}\left(\frac{1}{20}\right)}{\pi}$ C) $\frac{\tan^{-1}\left(\frac{1}{21}\right)}{\pi}$ D) $\frac{\tan^{-1}\left(\frac{1}{22}\right)}{\pi}$ E) NOTA

23) Let $f(x) = e^{ix}$. What is the amplitude of $Re\left(f(x)\right) + Im\left(f\left(\frac{\pi}{2} - x\right)\right)$?

A) 1 B) $\sqrt{2}$ C) 2 D) 4 E) NOTA

24) The derivative of a function at a point is the slope of the line tangent to the function at that point. Using this information, what is the derivative of $Im(e^{ix})$ at $x = \frac{3\pi}{2}$?

A) -1 B) 0 C) 1 D) Not Enough Information E) NOTA

25) The arc length of a function is how long the path is between two points along the function. Given this, what is the arc length of the function $f(t) = e^{it}$ in the complex plane from $t = 0$ to $t = 2\pi$?

A) 2π B) $4\sqrt{3}\pi$ C) $4\sqrt{6}\pi$ D) 4π E) NOTA

26) Let z be a complex number such that $z^5 = 1$, and let θ be the argument of z. What is the value of $\left(\text{cis}\left(\theta + \frac{\pi}{5}\right)\right)^{10}$

A) -1 B) $\frac{2}{\sqrt{5}} - i\frac{1}{\sqrt{5}}$ C) $\frac{2}{\sqrt{5}} + i\frac{1}{\sqrt{5}}$ D) 1 E) NOTA

27) If the slope of the line in the complex plane going through $z = a + bi$ and the origin is 2, what is the slope of the line going through z^2 and the origin?

A) $-2\sqrt{3}$ B) -2 C) $-\frac{4}{3}$ D) $\frac{4}{3}$ E) NOTA

28) Let $z_1, z_2, ..., z_k$ be a set of points such that $\arctan\left(\frac{Im(z)}{Re(z) - 1}\right) = \frac{2k\pi}{49}$ where k is a positive integer less than 50, and $|z - 1| = 1$. What is the value of $\prod_{n=1}^{49} z_n$?

A) -49 B) 1 C) 2 D) 49 E) NOTA

29) Consider the complex number $z = a + bi$. Let a be the x coordinate of the removable discontinuity of the function $f(x) = \frac{x^3 - 6x^2 + 11x - 6}{2x^2 - 8}$, and let b be the slope of the slant asymptote of $f(x)$. Compute $|z|$.

A) $\frac{\sqrt{17}}{2}$ B) $\frac{\sqrt{17}}{4}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) NOTA

30) A set of eight complex numbers, $z_1, z_2, ..., z_8$, has the following properties:

$$z_1 + z_3 + z_5 + z_7 = 9$$

$$z_2 + z_4 + z_6 + z_8 = 10$$

$$z_1 + z_2 + x_4 + z_8 = 16$$

Compute $z_3 + z_5 + z_6 + z_7$.

A) 3 B) 6 C) 7 D) 9 E) NOTA