

For this test, E) NOTA means "None Of These Answers". All inverse trigonometric functions will be restricted to their traditional domains and ranges. The constant  $i = \sqrt{-1}$ , and unless otherwise stated, logarithms will only accept positive inputs and output a real number. Define  $\text{cis}(\theta) = \cos \theta + i \sin \theta$ . Good luck!

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1) Find the sum of the solutions of  $\sin 2x = \frac{2}{3}$  for  $0 \leq x < 2\pi$ .

- A)  $\pi$                       B)  $2\pi$                       C)  $6\pi$                       D)  $8\pi$                       E) NOTA

2) Which of the following is equivalent to  $(1 + i\sqrt{3})^5 (2 - 2i)^6$ ?

- A)  $2^{11} \text{cis} \frac{7\pi}{6}$               B)  $2^{11} \text{cis} \frac{\pi}{6}$               C)  $2^{14} \text{cis} \frac{7\pi}{6}$               D)  $2^{14} \text{cis} \frac{\pi}{6}$               E) NOTA

3) Solve for the  $2 \times 2$  matrix  $M$  if  $\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} M = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$

- A)  $\begin{bmatrix} -3 & -39 \\ 1 & 16 \end{bmatrix}$               B)  $\begin{bmatrix} 3 & 39 \\ -1 & -16 \end{bmatrix}$               C)  $\begin{bmatrix} 13 & -9 \\ -1 & 0 \end{bmatrix}$               D)  $\begin{bmatrix} -13 & 9 \\ 1 & 0 \end{bmatrix}$               E) NOTA

4) Find the domain of  $f(x) = \frac{\sqrt{x+2}}{\sqrt{x-2}}$

- A)  $[-2, 2]$                       B)  $(-\infty, -2]$                       C)  $(2, \infty)$                       D)  $(-\infty, -2] \cup (2, \infty)$                       E) NOTA

5) Simplify  $4 \sec^2 x + 4 \csc^2 x$ .

- A)  $\frac{1}{4} \csc^2 2x$                       B)  $\csc^2 2x$                       C)  $4 \csc^2 2x$                       D)  $16 \csc^2 2x$                       E) NOTA

6) Find the sum of the squares of the roots for  $f(x) = x^3 - 4x^2 + 7x - 2$

- A) 2                                  B) 16                                  C) 20                                  D) 30                                  E) NOTA

7) Compute  $\langle 1, 2, 3 \rangle \cdot (\langle 3, 2, 1 \rangle \times \langle 1, 1, 1 \rangle)$

- A) -8                                  B) 0                                  C) 8                                  D) 10                                  E) NOTA

8) The sum of the solutions for  $\log_2(3x - 4) = 2 + \frac{1}{2} \log_2 x$  can be expressed as  $\frac{m}{n}$  for relatively prime positive integers  $m$  and  $n$ , find the value of  $m + n$ .

- A) 5                                  B) 13                                  C) 25                                  D) 49                                  E) NOTA

9) A bag contains 4 red, 6 green, and 5 blue marbles. 3 marbles are drawn out of the bag at random and without replacement. What is the probability that all 3 are of different colors?

- A)  $\frac{4}{91}$                       B)  $\frac{12}{91}$                       C)  $\frac{24}{91}$                       D)  $\frac{57}{91}$                       E) NOTA

10) Identify the polar graph  $r = 2 + 3 \cos \theta$

- A) Looped Limacon    B) Cardioid                      C) Dimpled Limacon    D) Convex Limacon    E) NOTA

11) A triangle contains a  $120^\circ$  angle. The two shorter sides measure 3 and 5. Compute the length of the longest side.

- A)  $\frac{\sqrt{166}}{2}$                       B) 7                      C)  $\frac{\sqrt{226}}{2}$                       D) 8                      E) NOTA

12) Convert the base 6 repeating decimal  $0.\overline{25}$  into a fraction in base 10.

- A)  $\frac{25}{99}$                       B)  $\frac{5}{11}$                       C)  $\frac{17}{35}$                       D)  $\frac{5}{7}$                       E) NOTA

13) The hypotenuse of a right triangle has length of 8, and the area of the right triangle is 8. Find the tangent of the smallest angle of the triangle.

- A)  $8 - \sqrt{63}$                       B)  $4 - \sqrt{15}$                       C)  $2 - \sqrt{3}$                       D) 1                      E) NOTA

14) Find the number of asymptotes on the graph of  $(x) = \frac{x+1}{\sqrt{x^2-4}}$

- A) 2                      B) 3                      C) 4                      D) 5                      E) NOTA

15) For all ordered pairs of integers  $(a, b)$  such that  $12345a + 54321b$  is divisible by 11, it can also be said that  $123456a + n \cdot b$  is also divisible by 11. Which of the following is a possible value of  $n$ ?

- A) 321                      B) 4321                      C) 54321                      D) 654321                      E) NOTA

16) Find the area of the polar graph  $r = 8 \sin \theta + 6 \cos \theta$ .

- A)  $25\pi$                       B)  $49\pi$                       C)  $100\pi$                       D)  $198\pi$                       E) NOTA

17) Let  $(x, y, z)$  be an ordered triple of integers that satisfies  $\begin{cases} 2x + y + 2z = 28 \\ 5x + 3y + z = 25 \end{cases}$ . Which of the following primes is guaranteed to divide into  $8x + 5y$ ?

- A) 7                      B) 11                      C) 13                      D) 17                      E) NOTA

18) Let  $(x, y, z)$  be an ordered triple of integers that satisfies  $\begin{cases} 2x + y + 2z = 28 \\ 5x + 3y + z = 25 \end{cases}$ . Which of the following primes is guaranteed to divide into  $5x + 8y$ ?

- A) 7                      B) 11                      C) 13                      D) 17                      E) NOTA

19) A circle of radius 5 divides a larger concentric circle into two regions of equal area. A chord of the larger circle is tangent to the smaller circle. Find the length of the chord.

- A) 5                      B)  $5\sqrt{3}$                       C) 10                      D)  $10\sqrt{3}$                       E) NOTA

20) The graph of  $6x^2 - 13xy + 6y^2 - 4x + y - 2 = 0$  consist of two intersecting lines. Find the sum of the coordinates of the intersection of the lines.

- A) -3                      B)  $-\frac{1}{6}$                       C)  $\frac{1}{6}$                       D) 3                      E) NOTA

21) The graph of  $6x^2 - 13xy + 6y^2 - 4x + y - 2 = 0$  consist of two intersecting lines. Find the sine of an angle formed by the two lines.

- A)  $\frac{5}{13}$                       B)  $\frac{2}{3}$                       C)  $\frac{12}{13}$                       D) 1                      E) NOTA

22) Anna is pedaling her bicycle at a rate of 1 revolution per second. Attached to the pedals is a gear with 72 teeth. With a chain, it is connected to a gear on the rear wheel with 36 teeth. The wheels on her bike has a radius of  $\frac{100}{\pi}$  centimeters. How fast, in kilometers per hour, is Anna riding at?

- A) 3.6                      B) 7.2                      C) 14.4                      D) 28.8                      E) NOTA

23)  $(H + E + L + E + N + A)^6$  is expanded, and the like terms are collected. How many terms are in the final expression?

- A) 126                      B) 210                      C) 252                      D) 462                      E) NOTA

24) Milaan, Kasra, Alex, and Bailey are at the same point on a circular path. Milaan first starts to walk around the path at 5 ft/s. One minute later, Kasra starts to jog around the track at 7ft/s. After another minute, Alex starts to run around the track at 10ft/s. Finally, 3 minutes after Milaan starts, Bailey starts to sprint around the path at 15ft/s. They all travel in the same direction, and 11 minutes after Milaan started, they are all at the same point on the path. What is the maximum possible length (in feet) of the path?

- A) 60                      B) 120                      C) 300                      D) 600                      E) NOTA

