For this test, E) NOTA means "None Of These Answers". All inverse trigonometric functions will be restricted to their traditional domains and ranges. The constant $i = \sqrt{-1}$, and unless otherwise stated, logarithms will only accept positive inputs and output a real number. Define $cis(\theta) = \cos \theta + i \sin \theta$. Good luck! _____ 1) Find the sum of the solutions of $\sin 2x = \frac{2}{2}$ for $0 \le x < 2\pi$. A) π B) 2π C) 6π D) 8π E) NOTA 2) Which of the following is equivalent to $(1 + i\sqrt{3})^5 (2 - 2i)^6$? A) $2^{11} cis \frac{7\pi}{6}$ B) $2^{11} cis \frac{\pi}{6}$ C) $2^{14} cis \frac{7\pi}{6}$ D) $2^{14} cis \frac{\pi}{6}$ E) NOTA 3) Solve for the 2 × 2 matrix *M* if $\begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} M = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$ A) $\begin{bmatrix} -3 & -39 \\ 1 & 16 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 39 \\ -1 & -16 \end{bmatrix}$ C) $\begin{bmatrix} 13 & -9 \\ -1 & 0 \end{bmatrix}$ D) $\begin{bmatrix} -13 & 9 \\ 1 & 0 \end{bmatrix}$ E) NOTA 4) Find the domain of $f(x) = \frac{\sqrt{x+2}}{\sqrt{x-2}}$ **B)** $(-\infty, -2]$ A) [-2, 2]**C**) (2,∞) **D**) $(-\infty, -2] \cup (2, \infty)$ **E**) NOTA 5) Simplify $4 \sec^2 x + 4 \csc^2 x$. A) $\frac{1}{4} \csc^2 2x$ B) $\csc^2 2x$ C) $4 \csc^2 2x$ D) $16 \csc^2 2x$ E) NOTA 6) Find the sum of the squares of the roots for $f(x) = x^3 - 4x^2 + 7x - 2$ A) 2 B) 16 C) 20 D) 30 E) NOTA 7) Compute $(1, 2, 3) \cdot ((3, 2, 1) \times (1, 1, 1))$ A) –8 **B) 0** E) NOTA **C) 8** D) 10

8) The sum of the solutions for $\log_2(3x - 4) = 2 + \frac{1}{2}\log_2 x$ can be expressed as $\frac{m}{n}$ for relatively prime positive integers *m* and *n*, find the value of m + n.

A) 5 B) 13 C) 25 D) 49 E) NOTA

Alpha Individual

9) A bag contains 4 red, 6 green, and 5 blue marbles. 3 marbles are drawn out of the bag at random and without replacement. What is the probability that all 3 are of different colors?

A) $\frac{4}{91}$	B) $\frac{12}{91}$	C) $\frac{24}{91}$	D) ⁵⁷ / ₉₁	E) NOTA
21	71	21	91	

10) Identify the polar graph $r = 2 + 3 \cos \theta$

A) Looped Limacon B) Cardioid C) Dimpled Limacon D) Convex Limacon E) NOTA

11) A triangle contains a 120° angle. The two shorter sides measure 3 and 5. Compute the length of the longest side.

n_{j_2} n_{j_1} n_{j_2} n_{j_1} n_{j_2}	A) $\frac{\sqrt{166}}{2}$	B) 7	C) $\frac{\sqrt{226}}{2}$	D) 8	E) NOTA
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12) Convert the base 6 repeating decimal $0.\overline{25}$ into a fraction in base 10.

	$\frac{5}{11}$ C) $\frac{17}{35}$	D) 5 7	E) NOTA
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13) The hypotenuse of a right triangle has length of 8, and the area of the right triangle is 8. Find the tangent of the smallest angle of the triangle.

A) $8 - \sqrt{63}$ B) $4 - \sqrt{15}$ C) $2 - \sqrt{3}$ D) 1 E) NOTA

14) Find the number of asymptotes on the graph of $(x) = \frac{x+1}{\sqrt{x^2-4}}$ A) 2 B) 3 C) 4 D) 5 E) NOTA

15) For all ordered pairs of integers (a, b) such that 12345a + 54321b is divisible by 11, it can also be said that $123456a + n \cdot b$ is also divisible by 11. Which of the following is a possible value of n?

A) 321 B) 4321 C) 54321 D) 654321 E) NOTA

16) Find the area of the polar graph $r = 8 \sin \theta + 6 \cos \theta$.

A) 25π	B) 49π	C) 100π	D) 198π	E) NOTA
j	j	- ,	j	j -

17) Let (x, y, z) be an ordered triple of integers that satisfies $\begin{cases} 2x + y + 2z = 28 \\ 5x + 3y + z = 25 \end{cases}$. Which of the following primes is guaranteed to divide into 8x + 5y?

	A) 7	B) 11	C) 13	D) 17	E) NOTA
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MAO National Convention

Alpha Individual

18) Let (x, y, z) be an ordered triple of integers that satisfies $\begin{cases} 2x + y + 2z = 28\\ 5x + 3y + z = 25 \end{cases}$. Which of the following primes is guaranteed to divide into 5x + 8y? A) 7 **B) 11** C) 13 D) 17 E) NOTA 19) A circle of radius 5 divides a larger concentric circle into two regions of equal area. A chord of the larger circle is tangent to the smaller circle. Find the length of the chord. B) 5√3 D) $10\sqrt{3}$ A) 5 C) 10 E) NOTA 20) The graph of $6x^2 - 13xy + 6y^2 - 4x + y - 2 = 0$ consist of two intersecting lines. Find the sum of the coordinates of the intersection of the lines. B) $-\frac{1}{6}$ C) $\frac{1}{6}$ A) –3 **D)** 3 E) NOTA 21) The graph of $6x^2 - 13xy + 6y^2 - 4x + y - 2 = 0$ consist of two intersecting lines. Find the sine of an angle formed by the two lines. B) $\frac{2}{3}$ C) $\frac{12}{13}$ A) $\frac{5}{13}$ D) 1 E) NOTA 22) Anna is pedaling her bicycle at a rate of 1 revolution per second. Attached to the pedals is a gear with 72 teeth. With a chain, it is connected to a gear on the rear wheel with 36 teeth. The wheels on her bike has a radius of $\frac{100}{\pi}$ centimeters. How fast, in kilometers per hour, is Anna riding at? A) 3.6 B) 7.2 C) 14.4 E) NOTA D) 28.8

23) $(H + E + L + E + N + A)^6$ is expanded, and the like terms are collected. How many terms are in the final expression?

A) 126 B) 210 C) 252 D) 462 E) NOTA

24) Milaan, Kasra, Alex, and Bailey are at the same point on a circular path. Milaan first starts to walk around the path at 5 ft/s. One minute later, Kasra starts to jog around the track at 7ft/s. After another minute, Alex starts to run around the track at 10ft/s. Finally, 3 minutes after Milaan starts, Bailey starts to sprint around the path at 15ft/s. They all travel in the same direction, and 11 minutes after Milaan started, they are all at the same point on the path. What is the maximum possible length (in feet) of the path?

A) 60	B) 120	C) 300	D) 600	E) NOTA
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MAO National Convention

Alpha Individual

25) Amy and Eddie are working together on a set of flash cards to quiz themselves in preparation of an AP exam. If they each work alone, it would take Amy 3 hours and 20 minutes to finish and Eddie 4 hours to finish. When they work together, it takes them 2 hours and 30 minutes to finish. They talk as they are working together, so between the two of them, they get 18 fewer cards done each hour than the theoretic sum of their individual work. How many flash cards are in the set?

A)	80 B) 120 (C) 160	D) 240	E) NOTA
	00 D	j120 (J 100	<i>D j L</i> 10	LJNOIM

26) *ALICE* is a regular pentagon. *KIRA* is a rectangle. If AI = 40, find the minimum possible length for *KL*. For the purpose of this question, use the approximation $\sin 36^\circ = 0.6$.

A) 4 B) 5 C) 6 D) 8 E) NOTA

27) Jack and Andrew are playing chess. Jack always takes 15 seconds for each move he makes. Andrew starts quickly, taking only 1 second for his first move, but every move thereafter takes him one more second than the previous move. Andrew goes first. *T* seconds after the start of the game, Andrew and Jack spent an equal amount of time making or thinking about their moves. Find the number of positive values of *T*, assuming they are evenly matched, and the game goes on for at least half an hour. The smallest is T = 2, where Andrew has spent 1 second making his first move, and Jack has spent 1 second thinking about his first move.

A) 2	B) 3	C) 4	D) 5	E) NOTA
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28) William is building a playpen for his pet rabbit. He would like the playpen to be quadrilateral in shape, with a wall as one of the sides. If William has 12 feet of fencing available, what is the area, in square feet, of largest playpen (by area) he can build?

A) 16	B) 18	C) 12√3	D) 16√3	E) NOTA

29) Find the number of distinguishable permutations of the 8 characters in JEFFREY! under the following two conditions. First, ! must come between the two Es, but it does not have to be immediately adjacent to the Es. Second, there are an even number of characters between the two Fs, possibly 0. For example, JEFF!REY is valid. JEFFERY! violates the first condition. JEF!FREY violates the second condition.

A) 1920	B) 1968	C) 2016	D) 3360	E) NOTA

30) Zach provides Eric with a choice of two random number generators. For the first, it has a probability of p to generate a 5 and a probability of (1 - p) to generate a 0. For the second, it has a probability of (p - 0.3) to generate a 10 and a probability of (1.3 - p) to generate a 0. Eric must pick one of the two random number generators, generate two numbers, and add them together. His goal is to produce a sum of at least 10 with the highest probability. Zach is generous, so he selected a value $p \ge 0.5$. Zach is also a troll, so he selected the value of p such that Eric's choice does not actually matter. Which of the following is the closest to the value Zach chose?

	A) 0. 54	B) 0 .64	C) 0.74	D) 0.84	E) 0.94
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