Answers

1. 1530150
2. 1
3. 25
4. 12
5. 9
6. 90
7. $96 + 6\pi$
8. 178
9. 336
10. $\frac{3\sqrt{3}}{4}$
11. $\frac{3}{5}$
12. $6\sqrt{3} + \pi$
13. 400
14. 2π
15. 12π
16. $\sqrt{74} + 7$
17. 27
18. 84
19. 137
20. $5\sqrt{13}$
21. $\frac{37}{2}$
22. $\frac{1}{3}$
23. $(0, 0)$
24. $\frac{1}{4}$
25. $3\sqrt{41}$

Solutions

1. This is a multiple of a 3-4-5 triangle, so the area is $\frac{1}{2}(1515)(2020) = \frac{1}{2}(3)(4)(505^2) = \frac{1}{2}(3)(4)(25)(10201) = \frac{1}{2}(3060300) = 1530150$.
2. $\sin 30° = \cos 60° = \frac{1}{2}$, so the answer is 1.
3. The area is $\frac{1}{2}(10)(h)$ where we want to maximize h, the distance from a point on the semicircle to the diameter. This is maximized by the point in the center of the arc, making the triangle a 45-45-90 triangle with area $\frac{1}{2}(10)(5) = 25$. (To further convince yourself, this is equivalent to maximizing $y = \sqrt{25 - x^2}$, which clearly reaches its maximum at $(0, 5)$.)
4. The points are $(\pm 3, \pm 4), (\pm 4, \pm 3), (0, \pm 5), (\pm 5, 0)$. This is $4 + 4 + 2 + 2 = 12$ lattice points.
5. The possible angle measures x are solutions to the equation $x^2 + x - 90 = 0$, which are $x = -10, 9$, so the answer is 9.
6. This describes SSA congruence, which only holds when the congruent angle is 90 degrees because then it becomes HL congruence.
7. The answer is (surface area of cube) – (area of two circular bases) + (lateral area of cylinder), which is $6(4^2) - 2\pi(1)^2 + 2\pi(1)(4) = 96 + 6\pi$.
8. Since the sum of exterior angles is 360°, each exterior angle must measure $\frac{360}{180} = 2°$, and so each interior angle must then measure 178°.
9. The diagonals of a rhombus are perpendicular bisectors of each other, so the half-diagonal is 7 and thus the other half-diagonal is 24, as part of a 7-24-25 right triangle. The area is then $\frac{1}{2}(14)(48) = 336$.
10. Dropping altitudes from the shorter base to the longer base creates a rectangle and two right triangles with altitude $\frac{\sqrt{3}}{2}$, so the area is $\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)(1 + 2) = \frac{3\sqrt{3}}{4}$. Alternatively, this is half a regular hexagon with side length 1.
11. The total number of ways to choose three stones is $\binom{6}{3} = 20$. The number of ways to choose a right triangle can be counted as follows: there are 3 ways for the first two stones to be endpoints of a diameter, and for each diameter, there are 4 possible locations for the third stone to form a right triangle, for 12 in total. So the answer is 3/5.
12. The area is (area of hexagon) + (three 240° arcs) − (three 120° arcs), which is $\frac{3(2)^2\sqrt{3}}{4} + 3\left(\frac{2}{3}\right)(\pi) - 3\left(\frac{1}{3}\right)(\pi) = 6\sqrt{3} + \pi$.
13. The height of the lateral faces is $\sqrt{194 - \left(\frac{10}{2}\right)^2} = 13$, so the height of the pyramid is $\sqrt{13^2 - 5^2} = 12$, and the volume is $\frac{1}{3}(10^2)(12) = 400$.
14. We are increasing the distance traveled from $2\pi r$ feet to $2\pi(r + 1) = 2\pi r + 2\pi$ feet, so the answer is 2π.
15. The chord's perpendicular bisector is a diameter, so the two segments' lengths are $\frac{3}{2}r, \frac{1}{2}r$. Drawing a radius from the center to an endpoint of the chord gives that $\left(\frac{r}{2}\right)^2 + 3^2 = r^2 \to r^2 = 12$, so the area is 12π.
16. This distance is along the line connecting the centers of the regions. The distance is then (distance between centers) + (radius 1) + (radius 2), which is $\sqrt{74} + 7$.
17. We use a parallelogram area formula: $\left(\frac{1}{2}(9)(12)\sin 30°\right) = 27$ since $\sin 30° = \frac{1}{2}$. Alternatively, we can avoid taking the sine of an obtuse angle by noting that all the four triangles formed by the

diagonals have the same area since the diagonals bisect each other and form adjacent pairs of triangles with the same height and base; the area is then $4\left(\frac{1}{2}\left(\frac{9}{2}\right)\left(\frac{12}{2}\right)\sin 30^\circ\right) = 27$.

18. An altitude can be drawn to split this into a $6-8-10$ triangle and a $8-15-17$ triangle. The sum of these areas is $\frac{6\cdot 8}{2} + \frac{8\cdot 15}{2} = 24 + 60 = 84$.
19. Draw radius OR and it is clear that $\angle ROP = 43^\circ$, so $\angle ROQ = 137^\circ$, which is also the measure of arc QR.
20. The height of the trapezoid is 10. The lateral distance from the left edge of the shorter base to the right edge of the longer base is $12 + \frac{18-12}{2} = 15$. The length of the hypotenuse of the right triangle with these legs is $\sqrt{325} = 5\sqrt{13}$.
21. Using the Shoelace Theorem, $\frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ 0 & 6 & 1 \\ -4 & -1 & 1 \end{vmatrix} = \frac{37}{2}$.
22. The medians of a triangle divide it into six equal-area pieces. Hence the sum of two of them form $\frac{1}{3}$ of the triangle's area.
23. This is a square centered on the origin, so the answer is $(0,0)$.
24. The first two conditions mean that $DE \parallel AC$ and $DE = \frac{1}{2}AC$, so the altitude from B to AC is also split in half by DE. This means any triangle with DE as a base and a third vertex on AC will have area fourth that of ΔABC.
25. The fly has a choice of all points in a circle halfway up the curved lateral face of the cone, but the farthest distance would be if the fly's downward shadow onto the base followed a diameter. The cross section is shown below, with the path in bold. Using similar triangles, the length of the path is then $\sqrt{\left(10 + \frac{10}{2}\right)^2 + \left(\frac{24}{2}\right)^2} = 3\sqrt{41}$.