1. D	7. C	13. E	19. B	25. C
2. C	8. B	14. A	20. D	26. B
3. B	9. C	15. D	21. C	27. B
4. B	10. A	16. B	22. A	28. C
5. C	11. D	17. D	23. B	29. B
6. A	12. A	18. D	24. D	30. A

1. If $y = t^2 + t$, at what rate does y increase when $t = 2$?

A. 1 B. 2 C. 4 $\boxed{\text{D. 5}}$ E. NOTA

Solution: $y' = 2t + 1$.

2. What is the maximum volume of a rectangular prism with surface area 24?

A. 2 B. 6 $\boxed{\text{C. 8}}$ D. 12 E. NOTA

Solution: While this can be solved with tedious calculus, we can use the AM-GM inequality to see that $\sqrt[3]{(xy)(yz)(xz)} = (xyz)^{\frac{2}{3}} \leq \frac{xy+yz+xz}{3}$. For prism side lengths of x, y and z, we know the surface area is $2xy + 2yz + 2xz = 24$, and the volume is $V = xyz$. Substituting this into the inequality, we get $V^{\frac{2}{3}} \leq \frac{12}{3}$, and so $V \leq 8$, so 8 is the maximum volume. This intuitively occurs when $x = y = z$, or the prism is a cube.

3. Konwoo's yard is shaped like the region below $y = \sin(x)$ and above the x axis from 0 to π. What is the area of Konwoo's yard?

A. 1 $\boxed{\text{B. 2}}$ C. π D. 2π E. NOTA

Solution: $\int_0^\pi \sin(x)\,dx = -\cos(\pi) + \cos(0) = 1 + 1 = 2$.

4. Find the equation of the line that is tangent to $y = x^3$ and passes through the point $(3, 27)$.

A. $y = 18x - 27$ $\boxed{\text{B. } y = 27x - 54}$ C. $y = 9x$ D. $y = 18x + 54$ E. NOTA

Solution: The derivative of x^3 at 3 is 27, so the slope is 27. To pass through (3, 27), then, the line must have an intercept of -54, which gives B.

5. Find $f'(1)$, where

$$f(x) = \sum_{i=0}^{50} x^{2i}$$

A. 5050 B. 2019 $\boxed{\text{C. 2550}}$ D. 5100 E. NOTA

Solution: $f'(x) = \sum_1^{50}(2i)x^{2i-1}$, so $f'(1) = \sum_1^{50}(2i) = 2\frac{(50)(51)}{2} = 2550$

6. Starting at the point $(0, 0)$, Daniel needs to walk to the point $(1, 1)$ while always moving to the right. The fact that Daniel must cross every y value between 0 and 1 along his trip is a result of which theorem?

$\boxed{\text{A. Intermediate Value}}$ B. Extreme Value
C. Mean Value D. Ham Sandwich E. NOTA

Solution: This is the definition of IVT, though slightly reworded.

7. Evaluate

$$\lim_{x\to 0} \frac{\sin(12x)}{3x} - \lim_{x\to\infty} \frac{\sin(12x)}{3x}$$

A. 0 B. $\frac{15}{4}$ $\boxed{\text{C. } 4}$ D. $-\frac{15}{4}$ E. NOTA

Solution: Using L'Hopital's rule on the first limit gives $\frac{12\cos(0)}{3} = 4$, while the second is trivially 0.

8. A rock is dropped from a height of 20m. If the acceleration due to gravity is $10\frac{\text{m}}{\text{s}^2}$, how many seconds does it take the rock to hit the ground?

A. 1 $\boxed{\text{B. } 2}$ C. 4 D. 8 E. NOTA

Solution: $20 = \frac{1}{2}(10)t^2$ gives $t = 2$.

9. The radius of a sphere increases at 1 unit per second starting at $t = 0$, where t is measured in seconds. If the radius of the sphere is 3 at $t = 2$, how fast is the volume of the sphere increasing at $t = 3$, in units cubed per second?

A. 4π B. 16π $\boxed{\text{C. } 64\pi}$ D. 256π E. NOTA

Solution: We know that $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. At $t = 3$, the radius has grown to $3 + 1(3 - 2) = 4$ units, and $\frac{dr}{dt}$ is always 1. Thus, we get $\frac{dV}{dt} = 4\pi(4)^2(1) = 64\pi$.

10. Find the volume of the figure formed when the area bound by $y = x^2$ and $y = 4$ is revolved around the y-axis.

$\boxed{\text{A. } 8\pi}$ B. 16π C. 24π D. 36π E. NOTA

Solution: We can integrate circles over y using disc method to get $\int_0^4 \pi(\sqrt{y})^2\, dy = 8\pi$.

11. Find the volume of the figure formed when the square with vertices $(-2, 0), (0, 2), (2, 0)$, and $(0, -2)$ is revolved around the line $x = 1$.

A. 8π B. $4\pi\sqrt{2}$ C. $\frac{46}{3}\pi$ $\boxed{\text{D. } \frac{50}{3}\pi}$ E. NOTA

Solution: This can be done with integration or Pappus' theorem, but I suggest envisioning the object as two cones, each of which have two cone shaped sections missing. Consider just the top half ($y > 0$). This is a cone with radius 3 and height 3, missing two cones of radii 1 and height 1. This gives a volume of $\frac{1}{3}\pi(3)^2(3) - 2\frac{1}{3}\pi(1)^2(1) = \frac{25}{3}\pi$. This is the volume of the top half, which we double to find the total volume, D.

12. Buffy can be approximated as a sphere with radius 2. If Buffy's head is the locus of points within Buffy that are at least 3 units from the ground he rests on, what is the volume of his head?

$\boxed{\text{A. } \frac{5}{3}\pi}$ B. $\frac{7}{3}\pi$ C. $\frac{13}{12}\pi$ D. $\frac{11}{12}\pi$ E. NOTA

Solution: At a height h above the center of Buffy, the radius of the cross section of Buffy's head at that height is $r = \sqrt{4 - h^2}$, as seen in the right triangle between the central axis and a radius. Thus, we can integrate using disc method over h as $\int_1^2 \pi r^2 \, dh = \int_1^2 \pi(4 - h^2)\, dh = \frac{5}{3}\pi$.

13. Starting at $(0, 1)$, approximate $f(3)$ using Euler's method with three equal subintervals, where $f'(x) = x^2 - 2$.

A. -5 B. -4 C. -3 D. -2 $\boxed{\text{E. NOTA}}$

Solution: The first step from (0, 1) is $1 + 1(f'(0))$ to (1, -1). The second is $-1 + 1(f'(1))$ to (2, -2). The final step leaves us at $-2 + 1(f'(2))$ to (3, 0), so the answer is E.

14. Approximate the positive root of $y = x^2 - 2x - 1$ using 2 iterations of Newton's method starting at the point (3, 2).

$\boxed{\text{A. } \frac{29}{12}}$ B. $\frac{25}{12}$ C. $\frac{31}{12}$ D. $\frac{35}{12}$ E. NOTA

Solution: The derivative is $2x - 2$, so the first step takes us to $x = 3 - \frac{2}{4} = \frac{5}{2}$, or the point $(\frac{5}{2}, \frac{1}{4})$. The next step then takes us to $x = \frac{5}{2} - \frac{\frac{1}{4}}{3} = \frac{29}{12}$

15. Approximate $\int_0^4 x^4 + x + 1\, dx$ using Simpson's method with four equal subintervals.

A. 130 B. $\frac{640}{3}$ C. 210 $\boxed{\text{D. } \frac{652}{3}}$ E. NOTA

Solution: The calculation is $\frac{1}{3}\left[1f(0) + 4f(1) + 2f(2) + 4f(3) + 1f(4)\right] = \frac{652}{3}$.

16. A non-ideal spring is modeled by the equation $F = -2x^2$ where F is the restoring force in newtons and x is the displacement in meters. Find the amount of work in Joules it takes to pull the spring from equilibrium to a displacement of 3 meters.

A. 12 $\boxed{\text{B. } 18}$ C. 24 D. 30 E. NOTA

Solution: As stated in the directions, $W = \int F\, dx$. Thus the work required is $\int_0^3 2x^2\, dx = 18$ Joules.

17. What is the smallest area that can be bound by a line segment in the first quadrant passing through the point (4, 8)?

A. 32 $\boxed{\text{B. } 64}$ C. 128 D. 256 E. NOTA

Solution: Let the b-axis intersect the line at b. Then the line has the equation $y = \frac{8-b}{4}x + b$, and so the x intercept is $(\frac{4b}{b-8}, 0)$, which gives the triangle an area of $\frac{1}{2}(b)(\frac{4b}{b-8})$. Taking derivative, we find that this is minimized at $b = 16$, so the area is 64.

18. The function $y = x^2$ is revolved around the y-axis to create a reservoir to hold liquid. Water is poured in at a rate of π cubic units per second starting at $t = 0$ seconds. At what rate is the water level in the reservoir rising at $t = 8$ seconds?

A. 2 B. 1 C. $\frac{1}{2}$ $\boxed{\text{D. } \frac{1}{4}}$ E. NOTA

Solution: First we must find out how deep the water is at $t = 8$. We know that the volume when it it h deep is $V = \int_0^h \pi y\, dy = \frac{h^2\pi}{2}$. At $t = 8$, this is equal to the 8π cubic units of water that have flowed in, so we get $h = 4$. Then, we can use the fact that $\frac{dV}{dt} = \pi h \frac{dh}{dt}$ to find that $\pi = \pi(4)(\frac{dh}{dt})$, and the answer is thus $\frac{dh}{dt} = \frac{1}{4}$.

19. A population of p river rats grows with time t according to the equation $\frac{dp}{dt} = 30p(50 - p)$. If there are initially 100 river rats, find

$$\lim_{t\to\infty} p$$

A. 1500 $\boxed{\text{B. } 50}$ C. 30 D. $\frac{5}{3}$ E. NOTA

Solution: We seek the carrying capacity of the river rats. We put the equation in the equivalent form $\frac{dp}{dt} = 1500(p)(1 - \frac{p}{50})$ to see that this is 50.

20. The radius of a loop of wire varies according to the equation $r = 2t$ for $t > 1$. If a constant magnetic field of 10T is applied perpendicular to the plane of the wire loop, what is the magnitude of the electromotive force induced in the wire at $t = 2$?

A. 80 B. 160 C. 80π $\boxed{\text{D. } 160\pi}$ E. NOTA

Solution: As stated in the instructions, we must find the time derivative of the flux through the loop. The flux is $\Phi = 10(\pi r^2) = 40\pi t^2$, so the derivative at time 2 is 160π.

21. Find $f(0)$, where

$$f(x) = \int_2^x te^{f(t)}\, dt$$

A. $3e$ B. $2e$ $\boxed{\text{C. } -\ln(3)}$ D. $\ln(2)$ E. NOTA

Solution: We can derive both sides w.r.t. x to receive $\frac{df}{dx} = xe^f$. We can separate this as $e^{-f}df = xdx$ and integrate to find $-e^{-f} = \frac{x^2}{2} + C$. To find C, we can utilize the fact that $f(2)$ must be zero because, looking at the original integral, the interval is from 2 to 2 and thus 0. Then we see that f passes through (2, 0) and so C is found to be -3. We then plug in 0 to find $-e^{-f(0)} = 0 - 3$, or $f(0) = -\ln(3)$.

22. Andy is riding a very unsafe roller coaster which travels along the curve $y = x^3$. Being very unsafe, the coaster may at any time dismount from the graph and continue in the direction it was traveling. If Andy's ride begins at (-2, 8) and travels upwards, find the area of the intersection of [the locus of points which Andy could possibly reach] and [the locus of points within the rectangle bounded by (-2, -8), (-2, 8), (2, -8), and (2, 8)].

$\boxed{\text{A. } \frac{76}{3}}$ B. $\frac{86}{3}$ C. $\frac{32}{3}$ D. $\frac{64}{3}$ E. NOTA

Solution: The left boundary is marked by the tangent line to the roller coaster at (-2, -8), where it starts. The lower boundary is the graph itself for negative x, and $y = 0$ for positive x. The right boundary is the graph itself for negative x, and $x = 2$ for positive x. The upper boundary is $y = 8$.

The easiest way to calculate this area is to take the entire top half, add the part below $y = 0$ and above $y = x^3$ on the left, and subtract the triangle formed by the tangent line and the rectangle. The first part is trivially $(4)(8) = 32$, the second is $-\int_{-2}^{0} x^3 = 4$. To find the third, we see that the tangent line at -2 has slope 12, so it intersects the top at a distance of $\frac{16}{12}$ from the left. Thus the triangle has area $\frac{1}{2}(\frac{4}{3})(16) = \frac{32}{3}$. The final area is then $32 + 4 - \frac{32}{3} = \frac{76}{3}$

23. Andy has abandoned the dangerous roller coaster and is now sitting *on the edge* of a merry-go-round, ready to pounce. The merry-go-round has a radius of 1 and is rotating at $\frac{1}{4}$ revolutions per second. At any time, Andy can pounce from the merry-go-round and will travel at 1 foot per second in a direction normal to where he jumped off (directly away from the center of the merry-go-round). Andy has 3 seconds total to move; after 3 seconds, he stops no matter what. What is the area of the locus of points Andy can reach?

A. $\frac{21}{4}\pi$ B. $\boxed{\frac{9}{2}\pi}$ C. $\frac{9}{4}\pi$ D. $\frac{3}{2}\pi$ E. NOTA

Solution: The figure is somewhat of a spiral shape around the merry-go-round. We will integrate in polar coordinates using $A = \frac{1}{2}\int_a^b r^2\,d\theta$. We can quickly see that if Andy jumps at time t, his radius will be $1 + t$ from the center by the time he stops moving; $r = 1 + t$. We also see that the angle at which he jumps at time t is $\frac{1}{4}(2\pi) = \frac{\pi}{2}$; so $\theta = \frac{\pi t}{2}$. Combining these equations yields $r = 1 + \frac{2\theta}{\pi}$. Then, we can integrate $\frac{1}{2}\int_0^{\frac{3\pi}{2}} (1 + \frac{2\theta}{\pi})^2\,d\theta = \frac{21\pi}{4}$. However, this is including the inner $\frac{3}{4}$ of the circle which Andy cannot reach, so we subtract out the $\frac{3\pi}{4}$ area to get the final answer of $\frac{9\pi}{2}$.

24. The line segment from $(0, 1, 0)$ to $(\frac{2}{3}, \frac{1}{2}, 0)$ is rotated in every possible orientation in three dimensions about the origin. That is, a given rotation is valid if and only if every point on the segment has the same distance from the origin as it did initially. What is the volume of the locus of points through which the segment moves?

A. $\frac{122}{123}\pi$ B. $\frac{244}{123}\pi$ C. $\frac{122}{375}\pi$ D. $\boxed{\frac{244}{375}\pi}$ E. NOTA

Solution: The figure formed is a spherical annulus with inner radius being the closest point from the origin to the line, and the outer radius the farthest. The farthest is trivially 1. To find the inner radius, we can use the point-to-line formula (or some derivative calculus). The line between the points has equation $3x + 4y = 4$, so the distance to the origin is $\frac{0+0+4}{\sqrt{3^2+4^2}} = \frac{4}{5}$. The volume is then $\frac{4}{3}\pi(1^3) - \frac{4}{3}\pi(\frac{4}{5})^3 = \frac{244\pi}{375}$

25. Evaluate

$$\int_0^\infty e^{-2t}t^8\,dt$$

A. $\frac{315}{8}$ B. $\frac{330}{7}$ C. $\boxed{\frac{315}{4}}$ D. $\frac{155}{7}$ E. NOTA

Solution: There are a few ways to approach this. The first, for those who are familiar with the gamma function, we can make the substitution $u = 2t$ to get the form $\frac{1}{2^9}\int_0^\infty e^{-u}u^8\,du$, which is $\frac{\Gamma(9)}{2^9}$. Then we can use the fact that $\Gamma(n) = (n-1)!$ to get the answer $\frac{8!}{2^9} = \frac{315}{4}$. Another solution involves visualizing the integral as the Laplace transform of t^8, for which it is hopefully known that $F(s) = \frac{n!}{s^{n+1}}$ when $f(t) = t^n$. Thus, we can use this as $F(2) = [\mathcal{L}(t^8)](2) = \int_0^\infty e^{-2t}t^8\,dt = \frac{8!}{2^9} = \frac{315}{4}$. Finally, it is technically possible to use tabular or perform integration by parts 8 times to find the answer, but this is not exactly optimal.

26. The area in quadrant 1 bound by the graphs of $y = \sin(x)$, $y = 1$, and $x = 0$ is revolved around the y-axis to form a funky shape. Find the volume of this funky shape.

A. $\frac{\pi^3}{2} - \pi$ B. $\boxed{\frac{\pi^3}{4} - 2\pi}$ C. $\frac{\pi^3}{2} - 2\pi$ D. $\frac{\pi^3}{4} - \pi$ E. NOTA

Solution: We can integrate w.r.t. y to find the volume using disc method as $\pi \int_0^1 (\sin^{-1}(y))^2\,dy$. Preferably we can convert back to x to avoid the arcsin as $\pi \int_0^{\frac{\pi}{2}} x^2 \cos(x)\,dx$. This can be integrated using integration by parts twice. The first yields $\pi[x^2 \sin(x) - 2\int_0^{\frac{\pi}{2}} x \sin(x)\,dx]$, and the second (on this new integral) yields $\pi[x^2 \sin(x) - 2(-x\cos(x) + \int_0^{\frac{\pi}{2}} \cos(x)\,dx)]$. Thus the final answer is $\pi[x^2 \sin(x) + 2x\cos(x) - 2\sin(x)]_0^{\frac{\pi}{2}} = \frac{\pi^3}{4} - 2\pi$.

27. An art sculpture is to be made to match a figure with a base of the graph $y = \frac{1}{2+\cos(x)}$ from 0 to 2π with cross sections perpendicular to the x-axis that are squares. Find the volume of this lovely art sculpture.

A. $\frac{\pi\sqrt{3}}{3}$ B. $\boxed{\frac{4\pi\sqrt{3}}{9}}$ C. $\frac{8\pi\sqrt{3}}{9}$ D. $\frac{8\pi\sqrt{3}}{27}$ E. NOTA

Solution: The integral to find the volume is $\int_0^{2\pi} \frac{1}{(2+\cos(x))^2}\,dx$. A neat trick here is to visualize this as a polar area integral. This is simply twice the area of the polar shape $r = \frac{1}{2+\cos(\theta)}$, which should be known as an ellipse. To find the area of the ellipse, we first find the length of the major axis as $r(0) + r(\pi) = \frac{4}{3}$. Thus, $a = \frac{2}{3}$ for this ellipse. Because the focus is at the pole, we can find c as $a - c = r(0)$, so $\frac{2}{3} - c = \frac{1}{3}$ and $c = \frac{1}{3}$. We can then find b using the known fact that $c^2 = a^2 - b^2$ to get $b = \frac{\sqrt{3}}{3}$. Thus, the area of the ellipse is $\pi ab = \frac{2\pi\sqrt{3}}{9}$. But, this is half the actual value of the integral, so the answer is $\frac{4\pi\sqrt{3}}{9}$.

28. Often in the real world, it is not necessary to find the exact value for certain problems. Assuming the answer is not E, find the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{\ln(1+\sin(x))}{\sin(x)}\,dx$$

A. $\frac{\pi^2}{2}$ B. $\frac{\pi^2}{4}$ C. $\boxed{\frac{\pi^2}{8}}$ D. $\frac{\pi^2}{16}$ E. NOTA

Solution: The question is written in such a way as to prompt an approximation, as the answers are all a factor of two apart and E is known to be incorrect. Thus, we can do any simple Riemann approximation. For example, the simplest way is to just use a trapezoidal with one subinterval. The function has value 1 at $x = 0$, and $\ln(2) \approx .7$ at $x = \frac{\pi}{2}$. This gives an area of $\frac{1+.7}{2}(\frac{\pi}{2}) \approx (.85)(1.5) \approx 1.3$. π^2 is around 10, so the closest answer is this over 8, or C, which is actually the exact value of the integral. However, it is also possible to actually solve the integral if desired. The quickest way to do so is likely the "sneakiest substitution in the world", the Weierstrass substitution $t = \tan(\frac{x}{2})$.

29. Sir Gray D. Yent is standing at the point (2, 4, 4) upon the equation $z = x^3 - 4x - y^2 + 20$. Assuming the positive z is directly upwards, and the positive y direction is north, in what direction should Sir Gray D. Yent walk to ascend as fast as possible with his next step?

A. North B. $\boxed{\text{Southeast}}$ C. Northwest D. South E. NOTA

Solution: Sir Gray D. Yent is very clearly prompting the use of the gradient operator. We can see that the gradient at the point he is standing is $(3(2)^2 - 4, -2(4)) = (8, -8)$. If north is positive y, then this is in the southeast direction.

30. Two real numbers are chosen independently as legs of a right triangle, both following a uniform distribution over (0,5]. What is the expected length of the hypotenuse of this triangle?

$\boxed{\text{A. } \frac{5\sqrt{2}}{3} + \frac{5}{3}\ln(1+\sqrt{2})}$	B. $\frac{5\sqrt{2}}{6} + \frac{5}{3}\ln(1+\sqrt{2})$	
C. $\frac{5\sqrt{5}}{3} + \frac{5}{3}\ln(1+\sqrt{5})$	D. $\frac{5\sqrt{5}}{6} + \frac{5}{3}\ln(1+\sqrt{5})$	E. NOTA

Solution: We need to integrate over x, y from 0 to 5 and take the average: $\frac{1}{25}\int_0^5\int_0^5 \sqrt{x^2+y^2}\,dy\,dx$. This is very clearly a good candidate for polar integration, so we can convert this to $\frac{2}{25}\int_0^{\frac{\pi}{4}}\int_0^{5\sec(\theta)} r^2\,dr\,d\theta$, where we have split the square area in two and double it to avoid domain issues. This becomes $\frac{2}{25}\int_0^{\frac{\pi}{4}} \frac{125}{3}\sec^3(\theta)\,d\theta = \frac{10}{3}[\frac{1}{2}(\sec(\theta)\tan(\theta) + \ln(\sec(\theta)+\tan(\theta)))]_0^{\frac{\pi}{4}} = \frac{5\sqrt{2}}{3} + \frac{5}{3}\ln(1+\sqrt{2})$. If the integral of secant cubed is unknown, consider using integration by parts.