ANSWERS

1) D
2) D
3) B
4) C
5) A
6) C
7) D
8) E (diverges)
9) A
10) A
11) B
12) B
13) B
14) C
15) B
16) E (DNE)
17) A
18) D
19) D
20) B
21) C
22) B
23) B
24) A
25) C
26) D
27) D
28) C
29) A
30) B

SOLUTIONS

1) The limits of integration are the points of intersection between $k(x)$ and the *x*-axis. In this case, $x=\pm 2$.

$$Total\ Area=\int_{-2}^{2}\left|\left(x^2-4\right)\right|dx=\left[\left|\frac{x^3}{3}-4x\right|\right]_{-2}^{2}=\frac{32}{3}.$$ **D**

2) The limits of integration are the points of intersection between $f(x)$ and $g(x)$. In this case, $x=-2,3$. To see which function is the upper boundary, plug in an *x* value in the interval. Plugging in $x=0$ reveals that $g(x)$ is the upper function.

$$Area=\int_{-2}^{3}\left(\left(2x^3+4x^2+13\right)-\left(2x^3+5x^2-x+7\right)\right)dx=\int_{-2}^{3}\left(-x^2+x+6\right)dx=-\int_{-2}^{3}\left(x^2-x-6\right)dx$$

$$Area=-\left[\frac{x^3}{3}-\frac{x^2}{2}-6x\right]_{-2}^{3}=\frac{125}{6}$$ **D**

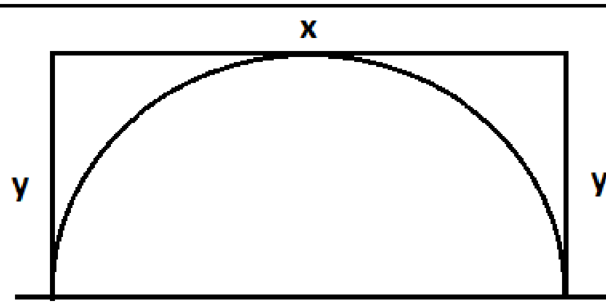
3) Choice A is not correct. Using the washer method to revolve around a vertical line requires the functions and the integral to be in terms of *y*.

Choice B is correct. This is a set up using the shell method. The radius in this case is $(3+x)$ and the height is the upper function less the lower function: $\left(-x^2+x+6\right)$. Additionally, the integrand is in terms of *x* and simplified using algebra.

Choice C is not correct. This is a shell set up but incorrectly subtracts the radius, *x*, of the functions from the distance of the axis of rotation to the *y*-axis. It assumes radius is $(3-x)$.

Choice D is not correct. This is a shell set up but it assumes the radius of the rotated shape is indeed the 'hollow' space created by the shape and the rotational axis: $(1+x)$. **B**

4) See the image to the right. The rectangular enclosure will have a fencing perimeter of $x+2y=100 \rightarrow x=100-2y$



Using optimization, we can solve for maximal fenced area, xy.

$Area=-2y^2-100y$

$\frac{DA}{Dx}=-4y-100 \rightarrow y=25 \therefore x=50$

We are solving for the area of the maximum possible semicircle, which would have a radius equal to *y* and a diameter equal to *x* in this specific case. The area of the semicircular pool is therefore $\frac{625}{2}\pi$. **C**

5) Because the velocity is positive for all time values greater than 0 seconds, the total distance traveled is the same as the net displacement. Absolute values do not need to be considered in the calculations.

$$displacement = \int_0^{27} e^{\sqrt[3]{t}}\,dt$$

To begin, make a substitution to rewrite the integral:

$$x = t^{\frac{1}{3}} \;\rightarrow\; t = x^3 \;\rightarrow\; dt = 3x^2dx \;\rightarrow\; \int_0^3 3x^2e^x dx.$$

From here, perform integration by parts with $u = 3x^2$ and $dv = e^x dx$:

$$\int_0^3 3x^2e^x dx = 3x^2e^x - \int_0^3 6xe^x dx$$

Do integration by parts again with $u = 6x$ and $dv = e^x dx$:

$$\int_0^3 3x^2e^x dx = 3x^2e^x - 6xe^x + \int_0^3 6e^x dx = \left[3x^2e^x - 6xe^x + 6e^x\right]_0^3 = 15e^3 - 6$$

A

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6) Answer choices A, B, and D do not make sense. **<u>Choice C is the most plausible answer</u>** because it correctly identifies a restriction in integration, in which the function being integrated must not exhibit discontinuities on the interval of calculation. However, while the floor function is discontinuous and integrable on the theory of summation, choice C also takes care of this by noting *x* values that may not be defined for $f'(x)$. The floor function is discontinuous but defined at all values of *x*.

C

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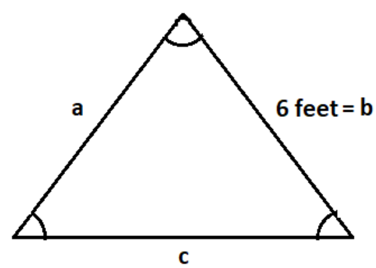
7) See the image to the right for the equilateral triangle in the scenario. Here, the length of each ladder is 6 feet, and now the stretch of ground between the ladders is 6 feet because the bases of the ladders are being pulled apart. There are many ways to find the area of a triangle, but is solution will show $A = ab\sin(\theta)$, where θ is the angle opposite of side *c*.



Differentiating with respect to time gives:

$$\frac{dA}{dt} = \frac{da}{dt}b\sin(\theta) + a\frac{db}{dt}\sin(\theta) + ab\cos(\theta)\frac{d\theta}{dt}$$

Note that the ladder lengths *a* and *b* are not changing, so:

$$\frac{dA}{dt} = ab\cos(\theta)\frac{d\theta}{dt} = (6)(6)\cos\left(\frac{\pi}{3}\right)\left(\frac{1}{4}\right) = \frac{9}{2}$$

D

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8) In the integrand, the constant term does not change the limit of the denominator so we can use the comparison test with an integral that is known to diverge by the *p*-series test:

$$\int_2^{\infty} \frac{2x^4}{x^5 - 3}dx \;\rightarrow\; 2\int_2^{\infty}\left(\frac{x^4}{x^5}\right) = 2\int_2^{\infty}\left(\frac{1}{x}\right)dx$$

Furthermore, because $x^5 - 3 < x^5$ for all $x > 2$, we can conclude that $\int_2^{\infty} \frac{2x^4}{x^5 - 3}dx > \int_2^{\infty}\left(\frac{1}{x}\right)dx$ and diverges. **E**

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9) $\int_0^2 (-3x+6)^2 dx = 9\int_0^2 (x-2)^2 dx = 3(2-2)^3 - 3(0-2)^3 = 24$. The volume is 24.

A

10) Begin with a substitution:

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\cos(2x)}dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos(u)}du$$

Then via the half angle identity for cosine:

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos(\theta)}{2}} \quad\rightarrow\quad \cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos(\theta)}{2} \quad\rightarrow\quad \sec^2\left(\frac{\theta}{2}\right) = \frac{2}{1+\cos(\theta)} \quad\rightarrow\quad \frac{1}{2}\sec^2\left(\frac{\theta}{2}\right) = \frac{1}{1+\cos(\theta)}$$

Plugging in:

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos(u)}du = \frac{1}{4}\int_0^{\frac{\pi}{2}} \sec^2\left(\frac{u}{2}\right)du = \frac{1}{2}\left[\tan\left(\frac{u}{2}\right)\right]_0^{\frac{\pi}{2}} = \frac{1}{2}(1) = \frac{1}{2}$$

A

11) If the incircle has a circumference of $2\sqrt{3}\pi$ inches, then the inradius is $\sqrt{3}$ inches. This is the apothem of the hexagon. Because it is a regular hexagon, the side lengths of the hexagon are 2 inches (can be found with 6 equilateral triangles in the hexagon and the Pythagorean theorem). The area of the hexagon is therefore $6\sqrt{3}$ inches squared.

B

12) Upon recognition that this is an odd function being integrated on an even interval, the answer is 0. The answer can also be obtained by using trigonometric identities and *u*-substitution. **B**

13) In a triangle given by coordinates *ABC*, the area is represented by $Area = \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$, where $\overrightarrow{AB}$ is the vector from point *A* to point *B*, and $\overrightarrow{AC}$ is the vector running from point *A* to point *C*.

$\overrightarrow{AB} = <3,0,-6>$ and $\overrightarrow{AC} = <4,-3,-9>$.

Cross product (using any method) is: $\overrightarrow{AB} \times \overrightarrow{AC} = <-18,3,-9>$. $\frac{1}{2}\sqrt{18^2+3^2+9^2} = \frac{\sqrt{414}}{2} \approx 10$. **B**

14) $2\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + 2\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + 2\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4+3+2}{12} = \frac{9}{12} = \frac{3}{4}$. **C**

15) Begin with a *u*-substitution:

$$u = e^x - 3 \quad\rightarrow\quad dx = \frac{du}{e^x} \quad\rightarrow\quad dx = \frac{du}{u+3}$$

$$\int_0^{\ln(2)} \left(\frac{3}{e^x - 3}\right)dx = 3\int_{-2}^{-1} \frac{1}{u(u+3)}du$$

From here, partial fractions or another substitution can be used.

$\int_{-2}^{-1}\left(\frac{3}{u(u+3)}\right)du=\int_{-2}^{-1}\left(\frac{A}{u}-\frac{B}{u+3}\right)du$	$3\int_{-2}^{-1}\left(\frac{1}{u(u+3)}\right)du=3\int_{-2}^{-1}\left(\frac{1}{u^2+3u}\right)du=3\int_{-2}^{-1}\left(\frac{1}{u^2\left(1+\frac{3}{u}\right)}\right)du$
$3=A(u+3)+B(u) \quad \therefore \quad A=1 \quad \therefore \quad B=-1$	$k=\frac{3}{u}+1 \rightarrow dk=-\frac{3}{u^2}du \rightarrow du=-\frac{u^2dk}{3}$
$\ln\lvert u\rvert\Big\|_{-2}^{-1}-\ln\lvert u+3\rvert\Big\|_{-2}^{-1}=-\ln(2)-\ln(2)=-2\ln(2)$	$-\int_{-\frac{1}{2}}^{-2}\left(\frac{1}{k}\right)dk=\ln\lvert k\rvert\Big\|_{-2}^{-\frac{1}{2}}=\ln\left(\frac{1}{2}\right)-\ln(2)=-2\ln(2)$
Final Answer: $=-\ln(4)$	**B**

16) The conditions for the MVTI are not met because there is a discontinuity in $g(x)=\frac{3}{e^x-3}$ at $x=\ln(3)$. This is not a definite integral. The answer to this question is does not exist or E. **E**

17) The graphs $f(x)=\frac{1}{2}x^3$, $g(x)=\sqrt{8x}$, and $h(x)=2x$ all intersect each other at the origin and at the point $(2,4)$.

Because we are finding the positive difference between *S* and *K*, it does not matter which area we label as larger. For this solution, let *S* be the region bounded by $f(x)$ and $h(x)$, and let *K* be the region bounded by $g(x)$ and $h(x)$. So, compute two separate integrals for area.

$$\underbrace{\int_0^2\left(2x-\frac{1}{2}x^3\right)dx}_{S}-\underbrace{\int_0^2\left(\sqrt{8x}-2x\right)dx}_{K}=\left[x^2-\frac{1}{8}x^4\right]_0^2-\left[\frac{1}{12}(8x)^{\frac{3}{2}}-x^2\right]_0^2=(4-2)-\left(\frac{16}{3}-\frac{12}{3}\right)=2-\frac{4}{3}=\frac{2}{3}$$

To help confirm the answer, the total area bound by *f* and *g* should be the sum of *S* and *K* $\rightarrow$ $2+\frac{4}{3}=\frac{10}{3}$

$$\int_0^2\left(\sqrt{8x}-\frac{1}{2}x^3\right)dx=\left[\frac{1}{12}(8x)^{\frac{3}{2}}-\frac{1}{8}x^4\right]_0^2=\frac{16}{3}-2=\frac{10}{3}\rightarrow\text{checks out.}$$

A

18) For this situation, it would be helpful to imagine *S* centered at the origin. This allows for a function that can be used in relation to the *x* and *y*-axis. Further, the vertical lines, because they are tangent to *S*, can be represented by the equations $x=\pm1$. Because we are minimizing area, a function to only represent the shaded area needs to be created in terms of integrals. However, because the horizontal line is at height *h* above and parallel to the *x*-axis, it would be more advantageous to integrate in terms of *y*, not *x*, to solve for the soon-to-be limit of integration, *h*.

This semicircle centered at the origin can be represented by $y=\sqrt{1-x^2}$. Because this is an even function, we can write this function in terms of *y* and use symmetry to make the calculations easier. In terms of *y*, the circle only in quadrant I can be represented by $x=\sqrt{1-y^2}$ for $y>0$. There are now two shaded regions created:

Let A_1 be bounded by the line at *h* units from the *x*-axis, $x=1$, and $x=\sqrt{1-y^2}$. $\rightarrow A_1=\int_0^h\left(1-\sqrt{1-y^2}\right)dy$

Let A_2 be bounded by the y-axis, the line at h units from the x-axis, and $x=\sqrt{1-y^2}$. $\rightarrow$ $A_2=\int_h^1\left(\sqrt{1-y^2}\right)dy$

Let A_3 and A_4 be the symmetric regions of A_1 and A_2 respectively (because we are not just working with a quarter circle, but a semicircle). The area of the two shaded regions in quadrant I can be represented by the expression:

$$A_1+A_2=\int_0^h\left(1-\sqrt{1-y^2}\right)dy-\int_1^h\left(\sqrt{1-y^2}\right)dy$$

The area of all shaded regions in quadrants I and II for S can be represented by:

$$A(h)=A_1+A_2+A_3+A_4=2\left[\int_0^h\left(1-\sqrt{1-y^2}\right)dy-\int_1^h\left(\sqrt{1-y^2}\right)dy\right]$$

Because area is being minimized, differentiate with respect to h using the second fundamental theorem and set it equal to 0 with intent to solve for h:

$$\frac{dA(h)}{dh}=2\left[\left(1-\sqrt{1-h^2}\right)-\left(\sqrt{1-h^2}\right)\right]=0 \;\rightarrow\; 1-2\sqrt{1-h^2}=0 \;\rightarrow\; 1-h^2=\frac{1}{4} \;\therefore\; h=\frac{\sqrt{3}}{2}$$ **D**

19) In this question, total distance is not related to velocity. The total distance the kite has flown is represented as an arc length because the height (vertical distance) is given as a function of horizontal distance, x. Use the formula for arc length to assist. Further, this arc length calculation can be approximated with Simpson's rule using infinitesimally small intervals on the specified domain.

$arc=\int_a^b\sqrt{1+\left(f'(x)\right)^2}dx$ where $f(x)$ is continuous on $[a,b]$ and $a<x<b$.

$$h(x)=\frac{x^3}{6}+\frac{1}{2x}$$

$$\int_{\frac{1}{2}}^{1}\sqrt{1+\left(\frac{1}{2}x^2-\frac{1}{2x^2}\right)^2}dx \;\rightarrow\; \int_{\frac{1}{2}}^{1}\sqrt{\frac{1}{2}+\left(\frac{1}{4}x^4+\frac{1}{4x^4}\right)}dx \;\rightarrow\; \frac{1}{2}\int_{\frac{1}{2}}^{1}\sqrt{x^4+2+\frac{1}{x^4}}dx \;\rightarrow\; \frac{1}{2}\int_{\frac{1}{2}}^{1}\sqrt{\left(x^2+\frac{1}{x^2}\right)^2}dx$$

$$\frac{1}{2}\int_{\frac{1}{2}}^{1}\sqrt{\left(x^2+\frac{1}{x^2}\right)^2}dx=\frac{1}{2}\int_{\frac{1}{2}}^{1}\left(x^2+\frac{1}{x^2}\right)dx=\frac{1}{2}\left[\frac{x^3}{3}-\frac{1}{x}\right]_{\frac{1}{2}}^{1}=\frac{1}{2}\left(\left(\frac{1}{3}-1\right)-\left(\frac{1}{24}-\frac{48}{24}\right)\right)=\frac{1}{2}\left(-\frac{16}{24}+\frac{47}{24}\right)=\frac{31}{48}$$ **D**

20) Radius of circle is $\frac{1}{2}$ so area is $\frac{1}{4}\pi$. Area between square and circle is $1-\frac{\pi}{4}\approx 0.2$. **B**

21) Essentially, the question is asking to find another family of curves that always intersects the family $y=kx^2$ at right angles. To begin, it is worth noting that the family $y=kx^2$ is a series of parabolas that are even with respect to the y-axis and have vertices located at the origin.

The slopes can be found by differentiating and substituting k out of the equation. Note that $k=\frac{y}{x^2}$.

$$y=kx^2 \;\rightarrow\; \frac{dy}{dx}=2kx \;\rightarrow\; \frac{dy}{dx}=2\left(\frac{y}{x^2}\right)x \;\rightarrow\; \frac{dy}{dx}=\frac{2y}{x}$$

This means that the tangent lines at point (x, y) on any of the parabolas in the family have slopes $\frac{dy}{dx} = \frac{2y}{x}$. The orthogonal (perpendicular) family will need to have slopes $\frac{dy}{dx} = -\frac{x}{2y}$ at any point (x, y). This is a separable differential equation.

$\frac{dy}{dx} = -\frac{x}{2y}$

$\int (2y)\, dy = -\int (x)\, dx$

$y^2 = -\frac{x^2}{2} + C \rightarrow \frac{x^2}{2} + y^2 = C \rightarrow x^2 + 2y^2 = C$

This family of ellipses satisfy the orthogonal trajectory of the family $y = kx^2$ **C**

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22) Area is $\frac{1}{2}\int_{-\pi/4}^{\pi/4}(6\cos 2\theta)^2 d\theta = \int_0^{\pi/4} 36\cos^2 2\theta \; d\theta = 36\int_0^{\pi/4}\left(\frac{1+\cos 4\theta}{2}\right) d\theta = 18\int_0^{\pi/4}(1+\cos 4\theta)d\theta$
$= 18\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi/4} = 18\left(\frac{\pi}{4} + \frac{1}{4}(0) - 0 - 0\right) = \frac{9\pi}{2}$. **B**

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23) $V = \frac{h\pi}{3}\left(R^2 + Rr + r^2\right)$ where *h* is the height of the cone, *R* is the radius of the larger base, and *r* is the radius of the smaller base. Based on the information in the question: $h = 1,\ R = 3,\ r = 2,\ \frac{dV}{dt} = 2,\ \frac{dr}{dt} = 0$. The values of $\frac{dh}{dt}$ and $\frac{dR}{dt}$ are not yet known. This solution will determine a relationship between *R* and *h* to work with only one unknown variable to keep the differentiation simple. Based on the dimensions of the glass and by creating similar triangles, the following expression can be found:

$R = \frac{1}{4}h + 2$ Use this to rewrite the formula for volume while plugging in $r = 2$:

$V = \frac{h\pi}{3}\left(R^2 + Rr + r^2\right) = \frac{h\pi}{3}\left(\left(\frac{1}{4}h + 2\right)^2 + 2\left(\frac{1}{4}h + 2\right) + 4\right) \rightarrow \frac{h\pi}{3}\left(\left(\frac{h^2}{16} + h + 4\right) + \left(\frac{h}{2} + 4\right) + 4\right)$

$V = \frac{\pi h^3}{48} + \frac{\pi h^2}{2} + 4\pi h$

Differentiating with respect to time and plugging in $\frac{dV}{dt} = 2$ and $h = 1$ gives:

$\frac{dV}{dt} = \left(\frac{\pi h^2}{16} + \pi h + 4\pi\right)\frac{dh}{dt} \rightarrow 2 = \left(\frac{\pi}{16} + \pi + 4\pi\right)\frac{dh}{dt} \rightarrow 2 = \left(\frac{81\pi}{16}\right)\frac{dh}{dt} \;\therefore\; \frac{dh}{dt} = \frac{32}{81\pi}$ **B**

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24) **<u>Choice A is correct</u>** because while the volume of juice entering the glass is constant as time passes, the radius of the surface of juice is increasing as the height increases. Therefore, a loss in vertical change is observed the larger the fill radius becomes. The upper base of the truncated cone has a larger circumference than the lower base touching a table.
Choice B is not correct. If the glass were not truncated, it would have a height of 12 inches. This can be found using the relationship between *R* and *h*.
Choice C is not correct. This can be disproven using the formula for volume of a truncated cone and the relationship between *R* and *h*.
Choice D is not correct. See explanation for choice A. **A**

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25) This is a semicircle with radius 2. **C**

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26) The area bound by the polar curve is given by:

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(2\cos(\theta)+\sin(2\theta)\right)^2 d\theta \;\rightarrow\; \frac{1}{2}\int_0^{\frac{\pi}{2}}\left(4\cos^2(\theta)+4\cos\theta\sin(2\theta)+\sin^2(2\theta)\right)d\theta$$

Split the integral into 3 and solve. Remembering $\sin(2\theta)=2\cos(\theta)\sin(\theta)$ helps with the second one.

$$\frac{1}{2}\left[4\int_0^{\frac{\pi}{2}}\left(\cos^2(\theta)\right)d\theta+8\int_0^{\frac{\pi}{2}}\left(\cos^2(\theta)\sin(\theta)\right)d\theta+\int_0^{\frac{\pi}{2}}\sin^2(2\theta)\,d\theta\right]$$

Next, use properties $\cos^2(\theta)=\frac{1}{2}+\frac{1}{2}\cos(2\theta)$ and $\sin^2(\theta)=\frac{1}{2}-\frac{1}{2}\cos(2\theta)$ for the first and third integral respectively, and solve the second one with a *u*-substitution $u=\cos(\theta)$. Continuing onward:

$$=\frac{1}{2}\left[4\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}\cos(2\theta)\right)d\theta-8\int_1^{0}\left(u^2\right)du+\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}-\frac{1}{2}\cos(4\theta)\right)d\theta\right]$$

$$=\frac{1}{2}\left[4\left[\frac{1}{2}\theta+\frac{1}{4}\sin(2\theta)\right]_0^{\frac{\pi}{2}}+\left[\frac{8u^3}{3}\right]_0^{1}+\left[\frac{1}{2}-\frac{1}{8}\sin(4\theta)\right]_0^{\frac{\pi}{2}}\right]$$

After plugging in the limits of integration, the final answer is $\frac{5\pi}{8}+\frac{4}{3}$ **D**

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27) The side length of a cube circumscribing a sphere is the diameter of the sphere. The side length can therefore be represented as $s=2r$

$$\frac{Vs}{Vc}=\frac{\frac{4}{3}\pi r^3}{(2r)^3}=\frac{\frac{4}{3}\pi r^3}{8r^3}=\frac{\pi}{6}$$ **D**

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28) Volume is $\frac{4}{3}(6)(3)(3)=72\pi$. **C**

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29) When considering all the possible rectangles, one of the corners will be at the origin and the corner opposite will have coordinates $(x,-4x+12)$. The area of a rectangle in quadrant I will be:

$$A=xy=x(-4x+12)=-4x^2+12x$$

Maximizing the downward parabola representing area using the first derivative (with respect to *x*) yields:

$$\frac{dA}{dx}=-2x+3=0 \;\rightarrow\; x=\frac{3}{2} \qquad A=xy=\left(\frac{3}{2}\right)\left(-4\left(\frac{3}{2}\right)+12\right)=9.$$ **A**

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30) This one is like the previous question, but more complicated because the height and base of the triangle, unlike the rectangle above, are not changing at rates proportional to one another. Another complication occurs because the *x*-intercept and *y*-intercept of $L(x)$ need to be represented with respect to the family of possible tangent lines such that $0<a<3$.

$y=(x-3)^2$. But at the point $(a,b): b=(a-3)^2$

The slope of the tangent line at any *a* on $0<a<3$ is $b'(a)=m=2a-6$

Using the point-slope form for a line:

$y-b=m(x-a) \rightarrow y=b+m(x-a) \rightarrow y=(a-3)^2+(2a-6)(x-a)$. The tangent line $L(x)$ for any (a,b) on $0<a<3$ is $y=(a-3)^2+2ax-2a^2-6x+6a$.

Now find the intercepts of this line.

Plugging in $y=0$:

$$0=(a-3)^2+2ax-2a^2-6x+6a$$

$$-(a-3)^2+2a^2+6a=(2a-6)x$$

$$\frac{-(a-3)^2+2a^2-6a}{2a-6}=x$$

$$\frac{-a^2+6a-9+2a^2-6a}{2a-6}=x$$

$$\frac{a^2-9}{2a-6}=\frac{(a+3)(a-3)}{2(a-3)}=x \rightarrow \frac{a+3}{2}=x$$

Plugging in $x=0$:

$$y=(a-3)^2+2a(0)-2a^2-6(0)+6a$$

$$y=(a-3)^2-2a^2+6a$$

$$y=a^2-6a+9-2a^2+6a$$

$$y=-a^2+9$$

These values of *x* and *y* represent the base and height respectively of the triangle as the intercepts change with changing values of *a*.

The area of the triangle is $A=\frac{1}{2}xy=\frac{1}{2}\left(\frac{a+3}{2}\right)(-a^2+9) \rightarrow \frac{1}{4}(-a^3-3a^2+9a+27)$

Differentiating with respect to *a* and finding the value that maximizes area:

$$\frac{dA}{da}=-\frac{3}{4}(a^2+2a-3)=0 \rightarrow (a+3)(a-1)=0 \therefore a=-3,1$$

Note that one of these solutions is outside of the first quadrant, and checking the solution with the first derivative test confirms that $a=1$ is the value that yields max area of the right triangle.

Plugging the value for *a* back into the equation for area gives $A=8$. **B**