NOTA denotes "None of the Above."

1. When written in closed form, the expression $1^{2022} + 2^{2022} + \cdots + n^{2022}$ will be a polynomial in n with leading term an^{2023} for some a. Find a.

   (A) $\frac{1}{2021}$ (B) $\frac{1}{2022}$ (C) $\frac{1}{2023}$ (D) $\frac{1}{2024}$ (E) NOTA

2. Evaluate $\int_{-\infty}^{\infty} \frac{x}{1+5x^4}\,dx$.

   (A) 5 (B) 10 (C) -5 (D) -10 (E) NOTA

3. Evaluate $\int_0^1 \frac{x^2}{x^6+1}\,dx$.

   (A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) NOTA

4. Evaluate $\lim_{n\to\infty} n \int_1^{2022} \frac{1}{1+x^n}\,dx$ *Hint: Try $u = x^{-1}$ and converting to a sum.*

   (A) 1 (B) $\frac{\pi^2}{6}$ (C) $\ln(2)$ (D) e (E) NOTA

5. Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{n!x^n}{n^n}$.

   (A) $\frac{1}{e\sqrt{2\pi}}$ (B) $\frac{1}{e}$ (C) e (D) $e\sqrt{2\pi}$ (E) NOTA

6. A solution to the differential equation $\frac{dy}{dx} = 3x^2y + 9x^2 + y + 3$ passes through the origin and $(1, k)$. Find k.

   (A) $e^2 - 3$ (B) $e^2 - 1$ (C) $3e^2 - 3$ (D) $3e^2 - 1$ (E) NOTA

7. What is the area of the region bounded by $r = 4 + 3\cos\theta$ in the polar plane?

   (A) $\frac{25\pi}{2}$ (B) $\frac{41\pi}{2}$ (C) 25π (D) 41π (E) NOTA

8. Find the slope of the line tangent to $x^2y + 2xy^3 - x^4 = 2$ at the point $(1, 1)$.

   (A) 0 (B) $\frac{2}{7}$ (C) $\frac{4}{7}$ (D) $\frac{6}{7}$ (E) NOTA

9. Evaluate $\lim_{x \to 0} \dfrac{\sin(x^2 \sin(x^2)) + x\sin(\sin(2x))}{\sin(2x\sin(x^2)) + \sin(\sin(\sin(x\sin(x))))}$.

(A) 1 (B) $\dfrac{5}{4}$ (C) $\dfrac{3}{2}$ (D) 2 (E) NOTA

10. Find the volume of the solid that results when the area between the curve $y = e^{2x}$ and the lines $y = 0$, $x = 1$, and $x = 2$ is rotated around the x-axis.

(A) $\dfrac{\pi(e^4 - e^2)}{4}$ (B) $\dfrac{\pi(e^4 - e^2)}{2}$ (C) $\dfrac{\pi(e^8 - e^4)}{4}$ (D) $\dfrac{\pi(e^8 - e^4)}{2}$ (E) NOTA

11. Evaluate $\displaystyle\int_1^{2021} (x-1)(x-2)\cdots(x-2021)\, dx$.

(A) 2020! (B) 2021! (C) 2022! (D) 0 (E) NOTA

12. Determine the convergence or divergence of the infinite series

$$\frac{1}{\ln(4)\ln(2)} + \frac{1}{\ln(27)\ln(3)} + \frac{1}{\ln(256)\ln(4)} + \frac{1}{\ln(3125)\ln(5)} + \frac{1}{\ln(46656)\ln(6)} + \cdots.$$

(A) Absolutely Convergent (B) Conditionally Convergent (C) Divergent
(D) Impossible to Determine (E) NOTA

13. Evaluate $\displaystyle\int_1^2 \frac{3x^2 - 4}{x^3 - 4x + 5}\, dx$.

(A) $\ln(2)$ (B) $\ln\left(\dfrac{5}{2}\right)$ (C) $\ln(3)$ (D) $\ln\left(\dfrac{9}{2}\right)$ (E) NOTA

14. Find the length of the polar curve $r = \sqrt{1 + \cos(2\theta)}$ over the interval $[0, 2\pi]$.

(A) 2π (B) $2\pi\sqrt{2}$ (C) 4π (D) $4\pi\sqrt{2}$ (E) NOTA

15. Evaluate $\displaystyle\int_0^1 \arctan\sqrt{x}\, dx$.

(A) $\dfrac{\pi - 2}{4}$ (B) $\dfrac{\pi - 1}{4}$ (C) $\dfrac{\pi - 2}{2}$ (D) $\dfrac{\pi - 1}{2}$ (E) NOTA

16. How many continuous functions f with a domain of $[0,1]$ satisfy this integral equation?

$$\int_0^1 (f(x))^2 \, dx = \int_0^1 (f(x))^3 \, dx = \int_0^1 (f(x))^4 \, dx$$

(A) 0 (B) 1 (C) 2 (D) 4 (E) NOTA

17. Find $\dfrac{d^2y}{dx^2}$ where $x = t^2$ and $y = t^2 + t$.

(A) $-\dfrac{1}{4t^3}$ (B) $2t$ (C) $\dfrac{3t^2-1}{2t}$ (D) $-\dfrac{1}{2t^2}$ (E) NOTA

18. A value of θ is uniformly randomly selected from the range $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$. Find the expected value of $\sec^2\theta$.

(A) $\dfrac{12\sqrt{3}-12}{\pi}$ (B) $\dfrac{12-4\sqrt{3}}{\pi}$ (C) $\dfrac{6\sqrt{2}-6}{\pi}$ (D) $\dfrac{12\sqrt{2}-12}{\pi}$ (E) NOTA

19. Find $f^{(5)}(0)$, where $f(x) = \arctan(x)$.

(A) 24 (B) 120 (C) 144 (D) 720 (E) NOTA

20. Let $f(x) = x^{\ln(x)}$. Evaluate $f'(2)$.

(A) $2^{\ln(2)}\ln(2)$ (B) $2^{\ln(2)}\ln^2(2)$ (C) $2\ln(2)$
(D) $2\ln^2(2)$ (E) NOTA

21. A particle moving in the xy-plane has acceleration vector $\mathbf{a}(t) = (9t^2-4)\mathbf{i} + (4t+1)\mathbf{j}$ for all $t \geq 0$, and it has velocity vector $\mathbf{v}(t) = -\mathbf{i} - 2\mathbf{j}$ at time $t = 0$. What is the speed of the particle at time $t = 2$?

(A) $\sqrt{5}$ (B) $\sqrt{17}$ (C) 5 (D) 17 (E) NOTA

22. Evaluate $\displaystyle\int_0^\infty \frac{\lfloor x \rfloor}{(1+x)^2} \, dx$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) Diverges (E) NOTA

23. Let $f(x) = (x^2+3)^3$. Evaluate $\dfrac{dy}{d\sqrt{x}}$ at $x = 1$.

(A) 64 (B) 96 (C) 192 (D) 384 (E) NOTA

24. Find the y-intercept of the tangent line to the curve defined parametrically by $x = e^{3t} + 2$ and $y = \ln(e^{6t} + 4e^{3t} + 4)$ at the point where $t = \ln 2$.

(A) $2\ln(10) - 96$ (B) $2\ln(10) - 48$ (C) $2\ln(10) - 24$
(D) $2\ln(10) - 2$ (E) NOTA

25. Evaluate $\lim_{x\to 1} \frac{(\sqrt[3]{x}-1)(\sqrt[4]{x}-1)(\sqrt[5]{x}-1)}{(x-1)^3}$.

(A) 0 (B) $\frac{1}{60}$ (C) $\frac{1}{24}$ (D) 1 (E) NOTA

26. Evaluate $\lim_{n\to\infty} \frac{1}{n!} \sum_{k=1}^{\infty} \frac{1}{\binom{n+k}{k} k!}$.

(A) 0 (B) $\frac{e}{4}$ (C) $\frac{e}{2}$ (D) e (E) NOTA

27. Evaluate $\int_1^e \frac{x-1}{x + x^2\ln(x)}\, dx$.

(A) 1 (B) $\ln(1+e) - 1$ (C) $\ln(2+e) - 1$ (D) $\ln(1+e^2) - 1$ (E) NOTA

28. A particle's movement in the coordinate plane is parametrized by $x = \sin^2\theta$ and $y = \cos(2\theta)$. Find the total distance (not displacement) the particle travels as t increases from 0 to 2022π.

(A) $2022\sqrt{2}$ (B) $2022\sqrt{5}$ (C) $4044\sqrt{2}$ (D) $4044\sqrt{5}$ (E) NOTA

29. Everybody knows l'Hôpital's rule. But do you know the namesake mathematician's first name? (Hint: He is French.)

(A) Guillaume (B) Johann (C) Gottfried (D) Colin (E) NOTA

30. Evaluate $\int 2022x^{2021}\, dx$.

(A) $x^{2021} + C$ (B) $x^{2022} + C$ (C) $x^{2023} + C$ (D) $x^{2024} + C$ (E) NOTA