Mu Ciphering Answers and Solutions

ANSWERS

0. y=2x
1. $\frac{17}{6}$
2. $3\sqrt{3}$
3. $\frac{6-\pi\sqrt{3}}{6} = 1 - \frac{\pi\sqrt{3}}{6}$
4. $2\sqrt{3}$
5. 6
6. $\frac{23}{8}$
7. $\frac{3}{2}$
8. $2+3e^{12}$
9. $\frac{5}{24}$
10. $\frac{22}{3}$
11. $\frac{16}{3}$
12. -1080

SOLUTIONS

0. $y' = \cos x + 1 \rightarrow y' = 2 \rightarrow y = 2x$

1. $\int_1^3 \sqrt{1+\left(\frac{x^2}{4}-\frac{1}{x^2}\right)^2}dx = \int_1^3 \sqrt{\left(\frac{x^2}{4}+\frac{1}{x^2}\right)^2}dx = \int_1^3 (\frac{x^2}{4}+\frac{1}{x^2})dx = \frac{x^3}{12}-\frac{1}{x}\Big|_1^3 = \frac{17}{6}$

2. $$A = 2\left[\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\sin^2\theta d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(2-\sin\theta)^2 d\theta\right] = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(-4\cos 2\theta + 4\sin\theta)d\theta$$

$$-2\sin 2\theta - 4\cos\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 3\sqrt{3}$$

3. Use a trig sub!! $a = \sqrt{3}$ and $x = \sqrt{3}\sec\theta$

$$\int_{\sqrt{3}}^{2}\frac{\sqrt{x^2-3}}{x}dx = \int_0^{\frac{\pi}{6}}\frac{(\tan\theta\sqrt{3})(\sec\theta\tan\theta\sqrt{3})}{\sec\theta\sqrt{3}} = \sqrt{3}\int_0^{\frac{\pi}{6}}\tan^2\theta d\theta = \sqrt{3}\int_0^{\frac{\pi}{6}}(\sec^2\theta - 1)d\theta$$

$$\sqrt{3}(\tan\theta - \theta)\Big|_0^{\frac{\pi}{6}} = \frac{6-\pi\sqrt{3}}{6}$$

4. Draw a picture and call the radius of the cylinder r. the volume is $\pi r^2 h = 2\pi r^2\sqrt{9-r^2}$ Take the

$$2\pi r^2 \bullet \frac{1}{2}(-2r)\bullet\frac{1}{\sqrt{9-r^2}} + \sqrt{9-r^2}(4\pi r) = 0$$

derivative and set equal to 0. $-2\pi r^3 + (9-r^2)(4\pi r) = 0 \rightarrow -r^2 + 18 - 2r^2 = 0$

$$r = \sqrt{6} \rightarrow h = 2\sqrt{3}$$

5. $t = \frac{d}{r} = \frac{\sqrt{4^2+x^2}}{6} + \frac{4-x}{10} \rightarrow dt = \frac{1}{2}\frac{(16+x^2)^{\frac{-1}{2}}(2x)}{6} - \frac{1}{10} = 0 \rightarrow \frac{x}{6\sqrt{16+x^2}} - \frac{1}{10} = 0$

$$5x = 3\sqrt{16+x^2} \rightarrow 25x^2 = 144 + 9x^2 \rightarrow x = 3 \rightarrow d = 5 + 4 - 3 = 6$$

$$y=-2k\left(x-k\right)+6-k^2=-2kx+k^2+6$$

6. $$\left(0,k^2+6\right)\left(\frac{k^2+6}{2k},0\right)\to A=\frac{\left(k^2+6\right)^2}{4k}\to A^{'}=0=\frac{16k^2\left(k^2+6\right)-4\left(k^2+6\right)^2}{16k^2}$$

$$4k^2=1\to k=\frac{1}{2}\to\left(\frac{1}{2},\frac{23}{4}\right)\to\frac{23}{8}$$

7. $$\lim_{x\to\infty}\sum_{i=1}^{n}\frac{\frac{i}{n}}{\frac{i}{n}+1}\left(\frac{1}{n}\right)=\int_0^1\frac{x}{x+1}dx=\int_0^1 1-\frac{1}{x+1}dx=x-\ln\left(x+1\right)\Big|_0^1=1-\ln 2=1+\ln\frac{1}{2}$$

$$1+\frac{1}{2}=\frac{3}{2}$$

$$\frac{dy}{dx}=x^2y-2-2x^2+y=\left(y-2\right)\left(x^2+1\right)$$

8. $$\int\frac{1}{y-2}dy=\int\left(x^2+1\right)dx\to\ln\left|y-2\right|=\frac{x^3}{3}+x+C$$

$$C=\ln 3\to y=2+3e^{\frac{x^3}{3}+x}\to 2+3e^{12}$$

9. $$\int_0^{\frac{\pi}{3}}\frac{\sin^3 x}{\cos^4 x}dx=\int_0^{\frac{\pi}{3}}\left(1-\cos^2 x\right)\cos^{-4}x\sin x dx=\frac{\sec^3 x}{3}-\sec x\Big|0^{\frac{\pi}{3}}=\frac{1}{24}-\frac{1}{2}-\left(\frac{1}{3}-1\right)$$

$$\frac{1-12+16}{24}=\frac{5}{24}$$

10. $$\frac{x}{1+y}=y\to y^2+y-x=0\to y=\frac{-1+\sqrt{1+4x}}{2}\to$$

$$\int_0^6\frac{-1+\sqrt{1+4x}}{2}dx=\frac{-x}{2}+\frac{\left(1+4x\right)^{\frac{3}{2}}}{12}\Big|_0^6=-3+\frac{125}{12}-\frac{1}{12}=\frac{88}{12}=\frac{22}{3}$$

11. $\frac{dV}{dt}=\frac{\pi}{3}(2rh\frac{dr}{dt}+r^2\frac{dh}{dt})$. $\frac{r}{h}=\frac{3}{4}$

and $4\frac{dr}{dt}=3\frac{dh}{dt}$. $4=\frac{\pi}{3}(6\frac{dr}{dt}4+\frac{4}{3}\frac{dr}{dt}9)\to\frac{dr}{dt}=\frac{1}{3\pi}$

Then

$S=\pi r^2+\pi rl\to l=\frac{5}{3}r\to S=\frac{8\pi}{3}r^2\to\frac{dS}{dt}=\frac{16\pi}{3}r\frac{dr}{dt}=\frac{16}{3}$

12. ${}_5C_2\left(-3x\right)^3\left(\frac{2}{x}\right)^2=10\bullet-27\bullet 4x\to-1080$