The acronym "NOTA" stands for "None Of The Above".

1. If $a, b,$ and c are non-zero differentiable functions, then the derivative of $\frac{ab}{cd}$ is

A. $\frac{a'bcd+ab'cd-abc'd-abcd'}{cd}$

C. $\frac{a'bcd+ab'cd+abc'd+abcd'}{c^2d^2}$

E. NOTA

B. $\frac{a'bcd+ab'cd-abc'd-abcd'}{c^2d^2}$

D. $\frac{-a'bcd-ab'cd+abc'd+abcd'}{c^2d^2}$

2. If $x + 2022y = 2022$ is normal to the graph of f at the point $(2022,0)$, then $f'(2022)=$

A. -2022

C. $-\frac{1}{2022}$

E. NOTA

B. 2022

D. $\frac{1}{2022}$

3. Let $g(x) = \frac{1}{f^{-1}(x)}$. Given the following data, determine $g'(2)$.

x	0	1	2	3	4
$f(x)$	1	2	3	4	5
$f'(x)$	-5	-4	-3	-2	-1

A. $-\frac{1}{9}$

C. $-\frac{1}{27}$

E. NOTA

B. $\frac{1}{18}$

D. $\frac{1}{36}$

4. Evaluate

$$\lim_{x\to\infty} \ln(2022) - \ln(x^{2022}) - \ln\left(tan^{-1}\left(\frac{1}{x^{2022}}\right)\right)$$

A. $\ln 2022 - 1$

C. $\ln 2022$

E. NOTA

B. $\ln 2022 + 1$

D. DNE

5. Find the equation of the tangent line to the following curve at $(0,2022)$

$$x = \ln(t), y = 2021 + t^2$$

A. $y = 2x + 2022$

C. $y = 2x - 2022$

E. NOTA

B. $y = -2x + 2022$

D. $y = 2022x + 2022$

6. Compute

$$\frac{d}{dx}\left[\frac{60(-x^2+2)^{\frac{1}{5}}\arctan(x)}{x^{\frac{1}{3}}\cdot\sqrt{2x^2-1}}\right]\Bigg|_{x=-1}$$

A. $26\pi + 30$ C. $41\pi + 30$ E. NOTA

B. $26\pi - 30$ D. $41\pi - 30$

7. Evaluate

$$\lim_{x\to 0}\frac{\cos x^2 - e^{x^4}}{\sin x^4}$$

A. $\frac{1}{2}$ C. $\frac{3}{2}$ E. NOTA

B. $-\frac{1}{2}$ D. DNE

8. Consider the definition: if $\lim_{x\to a}f(x) = \infty$ and $\lim_{x\to a}x^n f(x) = L$, such that $n \in \mathbb{R}, 0 < L < \infty$,

then $f(x)$ goes to ∞ like x^{-n}. Which of the following does $f(x)$ goes to ∞ like if

$$f(x) = \frac{1}{e^{x^{2022}}-1}$$

A. x^{-2022} C. x^{2022} E. NOTA

B. x^{-2021} D. x^{2021}

9. What is the radius of curvature of the function $f(x) = \cosh x$ at $x = \ln(2)$?

A. $\frac{4}{3}$ C. $\frac{4}{5}$ E. NOTA

B. $\frac{16}{9}$ D. $\frac{16}{25}$

10. Compute

$$\frac{d}{dx}\left[2022!^{2022!} + \ln(2022!^x) + \ln 2022 - e^{2022} - 2022^x\right]\Bigg|_{x=0}$$

A. $\ln 2022!$ C. $\ln 2023!$ E. NOTA

B. $\ln 2021!$ D. 0

11. Evaluate

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i^2}{n^3} + \frac{8i}{n^2} + \frac{2}{n} \right)$$

A. $\frac{8}{3}$

C. 8

E. NOTA

B. $\frac{26}{3}$

D. 26

12. Let $g(x)$ be a differentiable function with the properties that

$$\lim_{x\to 0} g(x^2 + 3x) = 0 \quad and \quad \lim_{x\to 0} g'(x^2 + 3x) = 4.$$

Evaluate

$$\lim_{x\to 0} \frac{g(x^2 + 3x)}{\sin x}.$$

A. 0

C. 4

E. NOTA

B. 3

D. 12

13. The value of k that will make the function f below continuous at $x = 0$ can be written as the reduced fraction a/b, where a and b are integers. Compute $a + b$.

$$f(x) = \begin{cases} \dfrac{9x - 3\sin(3x)}{5x^3}, & x < 0 \\ ke^x - \sqrt[3]{x} + 1, & x \geq 0 \end{cases}$$

A. 7

C. 47

E. NOTA

B. 27

D. 67

14. Compute the derivative

$$y = \frac{1}{\sqrt{1 + x^{2022}}}$$

A. $-\dfrac{1011x^{2021}}{(1+x^{2022})^{\frac{1}{2}}}$

C. $-\dfrac{1011x^{2021}}{(1+x^{2022})^{\frac{3}{2}}}$

E. NOTA

B. $\dfrac{1011x^{2021}}{(1+x^{2022})^{\frac{1}{2}}}$

D. $\dfrac{1011x^{2021}}{(1+x^{2022})^{\frac{3}{2}}}$

15. Compute the derivative

$$y = x \ln(2022x) + \frac{1}{2022e^{2022x}}$$

A. $\ln 2022x + e^{-2022x}$ C. $1 + \ln 2022x + e^{-2022x}$ E. NOTA

B. $1 + \ln 2022x - e^{2022x}$ D. $\ln 2022x + \ln 2022 - e^{-2022x}$

16. Using Newton's method to find $x^2 = 2022$ starting with $x_0 = 45$ with one iteration to the 4th decimal.

A. 44.9666 C. 44.6666 E. NOTA

B. 44.9667 D. 44.6667

17. Two bikers at the same point begin biking in opposite directions along a circular track of radius 100m at a speed of 5m/s. At what rate is the distance between them changing after 5s?

A. $10 \sin \frac{1}{4}$ C. $10 \cos \frac{1}{4}$ E. NOTA

B. $10 \sin \frac{1}{2}$ D. $10 \cos \frac{1}{2}$

18. Find the tangent line at $(1,1)$

$$9(x + y)^2 + (x - y)^2 = 36$$

A. $y = -x + 2$ C. $y = -2x + 3$ E. NOTA

B. $y = 2x - 1$ D. $y = x$

19. Find a formula for $f^{(2022)}(x)$

$$f(x) = xe^{-x}$$

A. $e^{-x}(x + 2022)$ C. $e^{-x}(2022 - x)$ E. NOTA

B. $e^{-x}(2022x + 2022)$ D. $-e^{-x}(2022x + 2022)$

Evaluate

20.
$$\lim_{x \to 0} \left(\frac{1}{x}\right)^{\csc(x)}$$

A. 0   C. 1   E. NOTA

B. -1 D. DNE

21. Evaluate

$$\lim_{x \to 0^+} \frac{\tanh(x)}{\tan(x)}$$

A. 0   C. 1   E. NOTA

B. -1 D. DNE

22. A box is made from two identical square sheets of cardboard with edge length of 9. A small square is removed from the corner of each piece of cardboard, and the two flaps are folded up. Find the maximum volume of the box.

A. 108   C. $\frac{729}{8}$ E. NOTA

B. 98   D. 54

23. For

$$\lim_{x \to 1} \left(x^{1/2} - 2\right),$$

using the $\delta - \varepsilon$ definition of a limit, if $\varepsilon = 0.01$, what is the maximum value of δ?

A. 0.0001 C. 0.0201 E. NOTA

B. 0.0101 D. 0.9801

24. Evaluate

$$\lim_{(x,y,z) \to (1,2,3)} 3x^2 z + xy \cos(\pi x - \pi z)$$

A. 7   C. 8   E. NOTA

B. 11   D. DNE

25. Compute

$$\frac{d}{dx}\left[(2022^x)^{(2022^x)^{(2022^x)^{\cdots}}}\right]\Bigg|_{x=0}$$

A. $\dfrac{1+\ln 2022}{\ln 2022}$ C. $\dfrac{\ln 2022}{1-\ln 2022}$ E. NOTA

B. $\dfrac{\ln 2022}{1+\ln 2022}$ D. $\dfrac{1-\ln 2022}{\ln 2022}$

26. Evaluate

$$\lim_{n\to\infty}\int_{-n}^{n}\frac{\arctan 2022x}{x(1+x^2)}dx$$

A. $\pi \ln 2023$ C. $\pi \ln 2021$ E. NOTA

B. $\pi \ln 2022$ D. $\dfrac{\pi}{2}\ln 2022$

27. Given a positive integer a, let $B(a)$ be the largest integer b such that

$$\binom{b}{a-1} > \binom{b-1}{a}$$

Evaluate

$$\lim_{a\to\infty}\frac{B(a)}{a}$$

A. $\dfrac{1+\sqrt{5}}{2}$ C. $\dfrac{3+\sqrt{5}}{2}$ E. NOTA

B. $\dfrac{\sqrt{5}-1}{2}$ D. $\dfrac{3-\sqrt{5}}{2}$

28. Minimize $|\sin x + \cos x + \tan x + \cot x + \sec x + \csc x|$

A. $3\sqrt{2}-2$ C. 2   E. NOTA

B. $2\sqrt{2}-1$ D. $2-\sqrt{2}$

29. Let a be a positive integer. The nth derivative of $\frac{1}{x^a-1}$ has the form

$$\frac{P_n(x)}{(x^a-1)^{n+1}}$$

where $P_n(x)$ is a polynomial. Find $P_{2022}(1)$

A. $(a)^{2022}2022!$ C. $(-a)^{2023}2022!$ E. NOTA

B. $(a)^{2022}(2023)!$ D. $(-a)^{2022}(2023)!$

30. Let $F_0(x) = \ln x$. For $n \geq 0$ and $x > 0$, let $F_{n+1}(x) = \int_0^x F_n(t)dt$. Evaluate

$$\lim_{n\to\infty} \frac{n!\, F_n(1)}{\ln n}$$

A. 0   C. 1   E. NOTA

B. -1 D. DNE