1. A	7. A	13. B	19. B	25. C
2. C	8. D	14. C	20. A	26. A
3. D	9. A	15. E	21. A	27. C
4. B	10. B	16. B	22. B	28. C
5. C	11. E	17. E	23. A	29. D
6. D	12. A	18. B	24. B	30. C

Solutions

1. If $a_1 = 3$, then $a_4 = 3 + 3d = 14 \Rightarrow d = \frac{14}{3}$. Thus, $a_{50} = 3 + 49d = 3 + 49 \cdot \frac{14}{3} = \frac{695}{3}$ $\boxed{\textbf{A}}$

2. By turning 3 into $\frac{9}{3}$, it can be found that the sequence is defined by $b_n = \frac{n+8}{3^n}$, so $b_{50} = \frac{50+8}{3^{50}}$ $\boxed{\textbf{C}}$

3. $c_4 = 16 = 2r^3 \Rightarrow r = 2, -1 + i\sqrt{3}, -1 - i\sqrt{3}$. Since $c_3 = 2r^2$, c_3 could be 8, $-4 - i\sqrt{3}$, or $-4 + 4i\sqrt{3}$ $\boxed{\textbf{D}}$

4. Using the ratio test gives you $\lim_{n\to\infty} \left|\frac{-x}{3}\right| \leq 1 \Rightarrow |x| \leq 3$, meaning the radius of convergence is 3. $\boxed{\textbf{B}}$

5. Using the ratio test gives $\lim_{n\to\infty} \left|\frac{\frac{(-n-1)^{n+1}x^{n+1}}{(n+1)!}}{\frac{(-n)^n x^n}{n!}}\right| = \lim_{n\to\infty} |-x(\frac{n+1}{n})^n| = ex \leq 1 \Rightarrow \frac{-1}{e} < x < \frac{1}{3}$. To know if the bounds converge or diverge, we can use Raabe's Test. $\lim_{n=1} n(\frac{a_n}{a_{n+1}-1})$ for $x = \frac{-1}{e}$ evaluates to $\frac{1}{2}$. The same limit for $x = \frac{1}{e}$ evaluates to $-\infty$. Since both limits are less than 1, they both diverge, so the interval is open on both ends. $\boxed{\textbf{C}}$

6. Using the hockey stick identity, we have $\binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \binom{5}{2} + \binom{6}{2} + \binom{7}{2} + \binom{8}{2} + \binom{9}{2} = \binom{10}{3} = 120$ $\boxed{\textbf{D}}$

7. This question can be solved easily using Vandermonde's Identity. However it can also be solves by using smaller cases. For example, if we replace 2020 with 2m the sum becomes $\binom{2}{0}\binom{2}{2} + \binom{2}{1}\binom{2}{1} + \binom{2}{2}\binom{2}{0} = 6 = \binom{4}{2} = \binom{2+2}{2}$, implying the original sum equals $\binom{4040}{2020}$ $\boxed{\textbf{A}}$

8.Since both i and n are always positive in this problem, the absolute value bars can be ignored. Thus the expression inside the limit is equivalent to $\frac{i}{n}(\frac{i}{n} + 1)^2$. Evaluating the limit gives $\int_0^2 (x+1)^2 dx = \frac{26}{3}$ $\boxed{\textbf{D}}$

9.By using the Taylor Series of e^x, we can turn the expression into the following infinite sum: $\frac{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}{1} + \frac{\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}}{1} + \frac{\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}^2}{2} + \cdots = \sum_{n=0}^{\infty} \frac{\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}^n}{n!}$. By experimentation, it is easy to see that $\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}^n = \begin{bmatrix}2^n & 0\\0 & 1\end{bmatrix}$, thus the sum evaluates to $\sum_{n=0}^{\infty} \frac{\begin{bmatrix}2^n & 0\\0 & 1\end{bmatrix}}{n!} = \begin{bmatrix}e^2 & 0\\0 & e\end{bmatrix}$ $\boxed{\textbf{A}}$

10. Since a_n is always less than or equal to 1 and is never negative, the maximum value for each sum occurs when $a_n = 1$, for all n. In this case only III converges. $\boxed{\textbf{B}}$

11.The sum diverges as $\dfrac{1}{x^2 - 8x + 15}$ is not defined for $x = 3$, or $x = 5$. $\boxed{\textbf{E}}$

12.The first few turns of the sequence are 3, $\sqrt{6-3}$, $\sqrt{6-\sqrt{6-3}}$, $\sqrt{6-\sqrt{6-\sqrt{6-3}}}$, ... As n gets infinitely large, $a_n = \sqrt{6-\sqrt{6-....}}$ This infinite sequence can be solved by making the substitution $x = \sqrt{6-x}$. Solving for x gives $x = 2$. Plugging $a_n = 2$ back into its definition gives that $a_{n+1} = a_n = 2$ so it is a stable solution. **A**

13.The right side of the equation is equivalent to $(1+\frac{1}{2})(1+\frac{1}{2^2})(1+\frac{1}{2^4})(1+\frac{1}{2^8})$... Multiplying both sides by $(1-\frac{1}{2})$ creates a cascading effect on the right side. $(1-\frac{1}{2})(1+\frac{1}{2})(1+\frac{1}{2^2})(1+\frac{1}{2^4})(1+\frac{1}{2^8})... = (1-\frac{1}{2^2})(1+\frac{1}{2^2})(1+\frac{1}{2^4})(1+\frac{1}{2^8})... = (1-\frac{1}{2^4})(1+\frac{1}{2^4})(1+\frac{1}{2^8})... = (1-\frac{1}{2^8})(1+\frac{1}{2^8})...... = (1-\frac{1}{2^\infty}) = 1 = (1-\frac{1}{2})P \Rightarrow P = 2$ **B**

14. The power series expansion of $\sec^{-1}(x)$ can be determined from the power series expansion of $\sin^{-1}(x)$, based on the relation that $\sec^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(\frac{1}{x}) = \frac{\pi}{2} - \frac{1}{x} - \frac{1}{6x^3} - \frac{3}{40x^5}$... Since the series converges for x greater than 1, we can use the first 4 terms of the series to evaluate that the limit converges to $\frac{3}{40}$. **C**

15. The left expression is $\sinh^2(x)$ while the right expression is $\cosh^2(x)$. Thus the final expression is $\sinh^2(x) + \cosh^2(x) = -1$. **E**

16. By cleverly using the binomial theorem, the question becomes much simpler. Let $f(x) = (x+1)^{10} = \sum_{i=1}^{10} \binom{10}{i} x^i$. Then, $\frac{df}{dx} = 10(x+1)^9 = \sum_{i=1}^{10} i\binom{10}{i} x^{i-1}$. Thus to make it equivalent to the question, let $x = 1$. Thus the answer is $10(1+1)^9 = 5120$ **B**

17.The first thing to notice is that the sum is in terms of n, but the denominator is completely in terms of x. So to find the interval of convergence, you first have to find what values make $(-\frac{x}{8})^n$ converge. This interval is from $(-8, 8)$. Now, notice that the denominator is not defined at $x = 4$, so there is a gap in the interval, giving $(-8, 4) \cup (4, 8)$ **E**

18. To find at what rate something converges, it is often useful to compare it to $\frac{1}{n^p}$. Evaluating the limit $\lim_{n->\infty} \frac{\frac{1}{n^p}}{\ln(\frac{n^p+1}{n^p})} = 1 + \frac{1}{n^p} \Rightarrow$ both functions grow at the same rate. Thus the sum converges when the sum of $\frac{1}{n^p}$ converges, which is the interval $p \in (1, \infty)$. **B**

19. The sequence limits to 0, so the limit is $\lim_{n\to\infty} \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^n}} = \frac{1}{2} = \mu \Rightarrow$ the sequence converges linearly. **B**

20. The sequence limits to 0, so the limit is $\lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = 1 = \mu \Rightarrow$ the sequence converges sublinearly. **A**

21.The easiest way to find what this sequence limits to is to realize that it is equivalent to the perimeter of a regular polygon inscribed in a circle of radius 1. Thus the limit is the perimeter of a circle, 2π. (You can also find this limit in a more standard way, but it is not the prettiest limit). Once you have what the limit evaluates to, you can evaluate the limit to find its rate of converges. This limit evaluates to 1, thus the sequence converges sublinearly. **A**

22. This sequence can be proven to converge, at least conditionally, easily by using the integral test. If the integral test is too tricky, the alternating series test also works. To find if it absolutely converges, you can use the integral test again. $\int_0^\infty \frac{|\sin(x)|}{x} dx = \sum_{i=0}^\infty (-1)^n \int_{2\pi i}^{2\pi(i+1)} \frac{\sin(x)}{x} dx \leq \sum_{i=0}^\infty \frac{(-1)^n}{2\pi i} \int_{2\pi i}^{2\pi(i+1)} \sin(x) dx =$

$\sum_{i=0}^{\infty} \frac{(-1)^n}{2\pi i} 2(-1)^n = \sum_{i=0}^{\infty} \frac{1}{\pi i} \Rightarrow$ The sequence doesn't converge absolutely, so it only converges conditionally. **B**

23. $\sum_{n=1}^{\infty} \frac{\sin(n)}{n} = \sum_{n=1}^{\infty} \frac{e^{in} - e^{-in}}{2in} = \frac{1}{2i} \sum_{n=1}^{\infty} (\frac{e^{in}}{n} - \frac{e^{-in}}{n}) = \frac{\ln(\frac{1-e^{-i}}{1-e^{i}})}{2i} = \frac{\ln(-e^{-i})}{2i} = \frac{\ln(-1)-i}{2i} = \frac{\pi i - i}{2i} = \frac{\pi - 1}{2}$ **A**

24. The smallest complex zero of the denominator is $x = i$ which has a magnitude of 1, so the radius of convergence is 1. **B**

25. The smallest complex zero of the denominator is $x = 2\pi i$ which has a magnitude of 2π, so the radius of convergence is 2π. **C**

26. By replacing $f_n = r^n$, we change the recurring sequence with a polynomial as now $r^n = 3r^{n-2} + 2r^{n-3} \Rightarrow r^3 - 3r - 2 = 0 \Rightarrow r = 2, -1, -1$. Since -1 has multiplicity of 2, $f_n = 2^n + n(-1)^n$. $\sum_{n=0}^{\infty} \frac{f_n}{3^n} = \sum_{n=0}^{\infty} \frac{2^n + n(-1)^n}{3^n} = \frac{45}{16}$. **A**

27. First we must realize that the decimal expansion of $\frac{100}{341}$ in base 4 must be a repeating infinite decimal. For it to have been a finite decimal expansion, 341 must be a power of 2. The decimal expansion can be written as the infinite sum in base ten of $\sum_{n=1}^{\infty} \frac{a_n}{4^n}$ where a_n represents the n^{th} digit after the decimal place, and thus can only be 0,1,2 or 3. Furthermore, since we know it has to repeat after m digits, $a_n = a_{n+m}$. To find the value of m we will need to experiment. For example, when m is 3, the sum becomes 3 different infinite sums with the same common ratio of 4^3. The sum evaluates to $\frac{\frac{a_1}{4} + \frac{a_2}{16} + \frac{a_3}{64}}{1 - \frac{1}{64}} = \frac{16a_1 + 4a_2 + a_3}{63}$. Since the denominator of the sum is less than 341, it is too small for it to be the answer. But now we know that the value for m will occur when $4^m - 1$ is a multiple of 341. When m is 5, $4^5 - 1 = 1023 = 3 * 341$, so the decimal expansion will be equivalent to $\frac{256a_1 + 64a_2 + 16a_3 + 4a_4 + a_5}{1023} = \frac{300}{1023}$. Now we can solve for the digits using trial and error and the fact that they can either be 0,1,2, or 3. Eventually you will solve that $a_1 = 1, a_2 = 0, a_3 = 2, a_4 = 3, a_5 = 0$, so the final base four expansion is $0.\overline{10230}$. Thus the 63rd digit is 2. **C**

28. The problem becomes more approachable by replacing both $(2n)!!$ with $2^n n!$. The first thing to notice is that the left sum turns into $xe^{-\frac{x^2}{2}}$. From this, we can do a u-sub with $u = \frac{x^2}{2}; du = xdx$. This turns the integral into $\sum_{n=0}^{\infty} \frac{1}{2^n (n!)^2} \int_0^{\infty} e^{-u} u^n du = \sum_{n=0}^{\infty} \frac{-e^{-u}(u^n + nu^{n-1} + n(n-1)u^{n-2} + \ldots + n!u + n!)|_0^{\infty}}{2^n (n!)^2} = \sum_{n=0}^{\infty} \frac{n!}{2^n (n!)^2} = e^{\frac{1}{2}}$ **C**

29. $\sum_{n=1}^{\infty} \frac{4n^2+4n}{(2n+1)^4} = \sum_{n=1}^{\infty} \frac{4n^2+4n+1-1}{(2n+1)^4} = \sum_{n=1}^{\infty} \frac{(2n+1)^2-1}{(2n+1)^4} = \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} - \frac{1}{(2n+1)^4} = \frac{\pi^2}{8} - \frac{\pi^4}{96}$ by the Riemann Zeta Function values for 2 and 4. $\Rightarrow \sum_{n=1}^{\infty} \frac{4n^2+4n}{(2n+1)^4} = \frac{12\pi^2 - \pi^4}{96}$ **D**

30. I'll leave the proof of this as an exercise for the reader ◡̈ **C**