For this test, E) NOTA means "None Of These Answers".

1) If a_n is an arithmetic sequence with $a_1 = 3$ and $a_4 = 17$, what is a_{50}?

A) $\frac{695}{3}$ B) $\frac{698}{3}$ C) $\frac{700}{3}$ D) $\frac{701}{3}$ E) NOTA

2) If b_n is an arithmetico-geometric sequence with first three terms $3, \frac{10}{9}, \frac{11}{27}$. What is b_{50}?

A) $\frac{58}{3^{49}}$ B) $\frac{59}{3^{49}}$ C) $\frac{58}{3^{50}}$ D) $\frac{59}{3^{20}}$ E) NOTA

3) If c_n is a geometric sequence with $c_1 = 2$ and $c_4 = 16$, which of the following could be the value of c_3?

A) 4 B) $4 + 4i\sqrt{3}$ C) $4 - 4i\sqrt{3}$ D) $-4 + 4i\sqrt{3}$ E) NOTA

4) Find the radius of convergence of the following Maclaurin series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^n}$$

A) ∞ B) 3 C) 1 D) $\frac{1}{3}$ E) NOTA

5) Find the interval of convergence of:

$$\sum_{n=1}^{\infty} \frac{(-n)^n x^n}{n!}$$

A) $(-1, 1)$ B) $(-1, 1]$ C) $\left(-\frac{1}{e}, \frac{1}{e}\right)$ D) $\left(-\frac{1}{e}, \frac{1}{e}\right]$ E) NOTA

6) Evaluate:

$$\sum_{i=2}^{9} \binom{i}{2}$$

A) 36 B) 45 C) 84 D) 120 E) NOTA

7) Evaluate:

$$\sum_{i=0}^{2020} \binom{2020}{i}\binom{2020}{2020-i}$$

A) $\binom{4040}{2020}$ B) $\binom{2020^2}{2020}$ C) $\frac{4040!}{2020^2}$ D) $\frac{4040!}{2020! \cdot 2}$ E) NOTA

8) Evaluate:

$$\lim_{n\to\infty} \sum_{i=1}^{2n} \left(\frac{i^2+in+n^2}{n^3} + \left| \frac{i}{n^2} \right| \right)$$

A) $\frac{7}{3}$ B) 4 C) $\frac{14}{3}$ D) $\frac{26}{3}$ E) NOTA

9) Evaluate:

$$e^{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}}$$

A) $\begin{bmatrix} e^2 & 0 \\ 0 & e \end{bmatrix}$ B) $\begin{bmatrix} e^2+1 & 1 \\ 1 & e+1 \end{bmatrix}$

C) $\begin{bmatrix} 2e & 0 \\ 0 & e \end{bmatrix}$ D) $\begin{bmatrix} 2e+1 & 0 \\ 0 & e+1 \end{bmatrix}$

E) NOTA

10) Given that the monotonically increasing sequence of non-negative real numbers a_n converges to 1, which of the following must also converge?

I. $\sum_{n=1}^{\infty} \frac{a_n}{n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{a_n}$ III. $\sum_{n=1}^{\infty} \frac{a_n}{e^n}$

A) I, II B) III C) I, II, III D) I, III E) NOTA

11) Evaluate:

$$\sum_{x=1}^{\infty} \frac{1}{x^2 - 8x + 15}$$

A) $-\frac{7}{24}$ B) $\frac{7}{24}$ C) $-\frac{7}{12}$ D) $\frac{7}{12}$ E) NOTA

12) Consider the sequence a_n, defined by $a_{n+1} = \sqrt{6 - a_n}$, and $a_1 = 3$. Evaluate $\lim_{n\to\infty} a_n$.

A) 2 B) 3 C) 4 D) Divergent E) NOTA

13) Evaluate:

$$\prod_{n=0}^{\infty} \left(1 + \frac{1}{2^{2^n}}\right) = \left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{16}\right)\left(1 + \frac{1}{256}\right)\cdots$$

A) 1 B) 2 C) $\frac{4}{3}$ D) 4 E) NOTA

14) Evaluate:

$$\lim_{x\to\infty} \frac{\frac{\pi}{2} - \frac{1}{x} - \frac{1}{6x^3} - \operatorname{arcsec}(x)}{\frac{1}{x^5}}$$

A) $\frac{1}{40}$ B) $\frac{1}{20}$ C) $\frac{3}{40}$ D) $\frac{7}{40}$ E) NOTA

15) Evaluate:

$$\left(\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}\right)^2 - \left(\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}\right)^2$$

A) 0 B) 1 C) $\cos(2x)$ D) $\sin(2x)$ E) NOTA

16) Evaluate:

$$\sum_{i=1}^{10} i\binom{10}{i}$$

A) 4980 B) 5120 C) 6220 D) 10240 E) NOTA

17) Find the interval of convergence of:

$$\sum_{n=1}^{\infty} \frac{(-1)^n \left(\frac{x}{8}\right)^n}{2^x + x^2 - 5x - 12}$$

A) $(-4, 4)$ B) $(-4, 4]$ C) $(-8, 8)$ D) $(-8, 8]$ E) NOTA

18) For what values of p does the following converge?

$$\sum_{n=1}^{\infty} \ln\left(\frac{n^p + 1}{n^p}\right)$$

A) $(0, \infty)$ B) $(1, \infty)$ C) $(0, 1)$ D) $(-\infty, 0) \cup (0, \infty)$ E) NOTA

Use the following information for questions **19-21**

Consider the sequence a_n which converges to a number L, and $\lim_{n\to\infty} \frac{|a_{n+1} - L|}{|a_n - L|} = \mu$, where μ is defined to be the rate of convergence. This sequence is said to converge *superlinearly* if $\mu = 0$, *linearly* if $0 < \mu < 1$, and *sublinearly* if $\mu = 1$

19) What is the rate of convergence of the sequence $a_n = \frac{1}{2^n}$

A) Sublinearly Convergent B) Linearly Convergent

C) Superlinearly Convergent D) Divergent

E) NOTA

20) What is the rate of convergence of the sequence $a_b = \frac{1}{n}$

A) Sublinearly Convergent

B) Linearly Convergent

C) Superlinearly Convergent

D) Divergent

E) NOTA

21) What is the rate of convergence of the sequence $a_n = n\sqrt{2 - 2\cos\left(\frac{2\pi}{n}\right)}$

A) Sublinearly Convergent

B) Linearly Convergent

C) Superlinearly Convergent

D) Divergent

E) NOTA

22) Determine the convergence of the following:

$$\sum_{n=0}^{\infty} \frac{\sin(n)}{n}$$

A) Absolutely Convergent

B) Conditionally Convergent

C) Linearly Convergent

D) Divergent

E) NOTA

23) Evaluate:

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$$

A) $\frac{\pi - 1}{2}$ B) $\frac{\pi + 1}{2}$ C) $\frac{\pi}{2}$ D) $\ln\left(\frac{\pi}{2}\right)$ E) NOTA

24) Find the radius of convergence for the series representation of $f(x) = \dfrac{x}{x^2+1}$ around $x = 0$.

A) 0 B) 1 C) 2π D) ∞ E) NOTA

25) Find the radius of convergence for the series representation of $f(x) = \dfrac{x}{e^x-1}$ around $x = 0$.

A) 0 B) 1 C) 2π D) ∞ E) NOTA

26) Consider the sequence f_n, defined by $f_n = 3f_{n-2} + 2f_{n-3}$, with $f_0 = 1$, $f_1 = 1$, and $f_2 = 6$. Evaluate

$$\sum_{n=0}^{\infty} \frac{f_n}{3^n}$$

A) $\dfrac{45}{16}$ B) $\dfrac{15}{4}$ C) $\dfrac{9}{4}$ D) $\dfrac{57}{16}$ E) NOTA

27) The base 10 fraction $\dfrac{100}{341}$ is converted to base 4 and written as a decimal. What is the 63rd digit after the decimal? (For reference, the first digit after the decimal for $\pi = 3.14159...$ is 1)

A) 0 B) 1 C) 2 D) 3 E) NOTA

28) Evaluate:

$$\int_0^{\infty} \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!!} \right) \left(\sum_{n=0}^{\infty} \frac{x^{2n}}{((2n)!!)^2} \right) dx$$

A) e B) π C) $\sqrt{e}$ D) $\dfrac{\pi}{2e}$ E) NOTA

29) Evaluate:

$$\sum_{n=1}^{\infty} \frac{4n^2+4n}{(2n+1)^4}$$

A) $\frac{8\pi^2+\pi^4}{64}$ B) $\frac{8\pi^2-\pi^4}{64}$ C) $\frac{12\pi^2+\pi^4}{96}$ D) $\frac{12\pi^2-\pi^4}{96}$ E) NOTA

30) And now for the obligatory easy final question: what is 1+1?

A) 0 B) 1 C) 2 D) ∞ E) NOTA