Open Comprehensive Answers and Solutions

ANSWER KEY: BDDAC-CABBA-DDEBA-CCABD-CABCD-BCEAD

SOLUTIONS:

1.) B

By the circle chord theorem, we know that (AE)(BE) = (CE)(DE). Thus, (4)(6) = (8)(DE). DE = 3. Now, to find the area of the quadrilateral we can use the fact that chords AB and CD are perpendicular to each other they form 4 right angles at point E resulting with 4 right triangles. To find the area of a right triangle we use $\frac{1}{2}bh$. Thus,

Area of quadrilateral = area of 4 right triangles

Area of quadrilateral = $\frac{1}{2}(3)(4)+\frac{1}{2}(4)(8)+\frac{1}{2}(8)(6)+\frac{1}{2}(3)(6)$

Area of quadrilateral = $6+16+24+9 \ = \ 55$

2.) D

Since the volume and surface area is the same, set the equations for both equal to each other. Thus,

Surface area of tetrahedron = $\sqrt{3}s^2$ and the volume of a tetrahedron $= \frac{s^3}{6\sqrt{2}}$

$$\sqrt{3}s^2=\frac{s^3}{6\sqrt{2}}$$

$$6\sqrt{6}=s$$

6+6 = 12

3.) D

Call the vertices of the equilateral triangle A,B, and C. We know that the center of the circle and the center of the equilateral triangle lie on the same point, call this point D. We know that the length between AD is 4 as this is the radius. Drop a perpendicular from point D to one of the sides of either AB or AC (it does not matter as we will still obtain the same solution as the triangle is equilateral). Now, we are left with a right triangle with a hypotenuse of 4. Realize that the radius bisects the angle A into two equal parts meaning that its value is now 30° within triangle with the right angle. The last angle must be 60°. Thus, we are left with a 30°-60°-90° where the side across from the 90° is 4. Since we are looking for the side length, which is across the 60°, we find that its value is $2\sqrt{3}$ and therefore the value of the entire side length of the equilateral triangle is double that since the perpendicular creates two equal lengths. Thus, $s=4\sqrt{3}$. Area = $s^2\frac{\sqrt{3}}{4}= \ (4\sqrt{3})^2\frac{\sqrt{3}}{4}=12\sqrt{3}$. 12 + 3 = 15

4.) A

To find the solutions for x, we need to realize that the triangle must uphold the Pythagorean theorem of $a^2 + b^2 = c^2$. We need to construct triangles where 21 is the hypotenuse and x is the hypotenuse. We don't include 20 in this case as the hypotenuse must be larger that both legs and 20 < 21. Thus,

$20^2 + x^2 = 21^2$

$x^2 = 441 - 400$

$x^2 = 41$

$x = \sqrt{41}$

$20^2 + 21^2 = x^2$

$841 = x^2$

$x = 29$

Thus, $n = 41, m = 29$. 41 + 29 = 70

5.) C

The area of a rectangle is bh. Thus,

63 = (x+1)(x-1)

63 = $x^2 - 1$

$x^2 = 64$

6.) C

I. True, for example, if an angles intercepts half of the circle it intercepts 180° and the angle itself is a straight angle of 180°.

II. True, as the area they cut off is equal.

III. False, adjacent angles are supplementary because if 2 lines intersect, they create 2 pairs of adjacent angles which add up to 180° each which add up to 360°.

Thus, I and II are true.

7.) A

To solve for matrix multiplication, we multiply rows by columns. Thus,

$$\begin{bmatrix} 2 & -1 & 0 \\ 6 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 6 & -4 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} (2)(1)+(-1)(6)+(0)(-2) & (2)(0)+(-1)(-4)+(0)(-1) \\ (6)(1)+(2)(6)+(5)(-2) & (6)(0)+(2)(-4)+(5)(-1) \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 6 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 6 & -4 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 8 & -13 \end{bmatrix}$$

-4 + 4 + 8 + -13 = -5

8.) B

$$\frac{\left(\frac{x}{x+1} - x\right) - \left(\frac{1}{x} + \frac{x+1}{x}\right)}{\left(\frac{x-1}{x+1} + \frac{x+1}{x+1}\right) + \left(\frac{1}{x-1} - 1\right)}$$

$$\frac{\left(\frac{x}{x+1} - \frac{x^2+x}{x+1}\right) - \left(\frac{x+2}{x}\right)}{\left(\frac{2x}{x+1}\right) + \left(\frac{1}{x-1} - \frac{x-1}{x-1}\right)}$$

$$\frac{\frac{-x^2}{x+1} - \frac{x+2}{x}}{\frac{2x}{x+1} + \frac{-x+2}{x-1}}$$

$$\frac{\frac{-x^3-(x^2+3x+2)}{x^2+x}}{\frac{(2x)(x-1)+(-x+2)(x+1)}{x^2-1}}$$

$$\frac{\frac{-x^3-x^2-3x-2}{x^2+x}}{\frac{2x^2-2x-x^2-x+2x+2}{x^2-1}}$$

$$\frac{(x^2-1)(-x^3-x^2-3x-2)}{(x^2+x)(x^2-x+2)}$$

$$\frac{(x+1)(x-1)(-x^3-x^2-3x-2)}{(x)(x+1)(x^2-x+2)}$$

$$\frac{-x^4-2x^2+x+2}{x^3-x^2+2x}$$

Thus, a=0, b=-2, c=1,d=2,f=0,g=1,h=-1,j=2,k=0

-2+1+2+0+1+-1+2 = 3

9.) B

To find the y-intercept of a function set x = 0 then evaluate. Thus,

$f(0) = 3(0+2)^2 - 9$

$f(0) = 3(4) - 9$

$f(0) = 3$

Thus, the y-intercept is 3.

10.) A

Utilizing the kinetic energy formula, we find that

$40000 = \frac{1}{2}(m)(20)^2$

$80000 = 400m$

$m = 200$

11.) D

Since we know that potential energy and kinetic energy is equal then we know that

$\frac{1}{2}mv^2 = mgh$

$\frac{1}{2}v^2 = gh$

$\frac{1}{2}v^2 = (10)(10)$

$v^2 = 200$

$v = 10\sqrt{2}$

12.) D

Firstly, put the ellipse in its standard form.

$x^2 + 2y^2 - 4x + 12y - 14 = 0$

$(x^2 - 4x + y) + 2(y^2 + 6y + 9) = 14 + (1)(4) + (2)(9)$

$(x-4)^2 + 2(y+3)^2 = 36$

$\frac{(x-4)^2}{36} + \frac{(-1+3)^z}{18} = 1$

Since we know that the length of the minor axis is $2b$ as it is the distance between the center to a co-vertex is b then the distance between the co-vertices is $2b$. Since $a^2 > b^2$ we know that $b^2 = 18$. $b = 3\sqrt{2}$. $2b = 6\sqrt{2}$

13.) E

For any real x, $sin^2 x + cos^2 x$ will always equal 1.

14.) B

Since $\sin x$ produces a positive value and is within the domain $\frac{-\pi}{2} \leq x < \frac{\pi}{2}$, we know that x lies in the first quadrant. Constructing a right triangle with the opposite side having a length of 3 with a hypotenuse of 5, we can see that the length of the adjacent side is 4 (as this is a Pythagorean triplet). Thus, $\cos x = \frac{4}{5}$. Using the half angle formula for tan x we find that

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{\frac{3}{5}}{\frac{9}{5}} = \frac{3(5)}{9(5)} = \frac{3}{9} = \frac{1}{3}$$

15.) A

When written in the form $r = a + b\cos\theta$ where $a > b$ the polar is a Limacon with dimpled loop.

16.) C

To find the absolute value of a complex number, we need to find its length which is $\sqrt{a^2+b^2}$. Thus,

$\sqrt{3^2 + x^2} = 4$

$3^2 + x^2 = 4^2$

$x^2 = 16 - 9$

$x = \pm\sqrt{7}$

17.) C

Firstly, to find the trailing zeroes of n! we divide n by the factors of 5 and find the sum of all of the quotients and disregarding the remainder. The word comprehensive has 13 letters and therefore its permutations are 13!. However, there are 3 repeating e's so we must divide that by 3!. The number of trailing zeroes in 13! is $\frac{13}{5} = 2.6 = 2$. Therefore, 13! has 2 trailing zeroes. The amount of trailing zeroes of 3! is $\frac{3}{5} = 0.6 = 0$. Therefore, 3! has 0 trailing zeroes. Since we are finding the trailing zeroes of $\frac{13!}{3!}$ and since when dividing the trailing zeroes will cancel out, we find 2-0 = 2. Thus, there are 2 trailing zeroes in the distinct permutations in the word comprehensive.

18.) A

Firstly, factor the function getting $\frac{x(x+5)}{(x+5)^2} = \frac{x}{x+5}$. To find the vertical asymptote we set the denominator equal to 0 then evaluate. x + 5 = 0, x = -5. Thus, n = -5. To find the horizontal asymptote we divide the coefficients of the highest degree of both the numerator and denominator. Thus, $y = \frac{1}{1} = 1$. So, m = 1

-5 + 1 = -4

19.) B

Plugging in 0 we get $\frac{\cos 0 - 1}{\sin 0} = \frac{1-1}{0} = \frac{0}{0}$. Now, we can apply L'Hopital's rule by taking the derivative of both the numerator and the denominator. Thus getting $\frac{-\sin\theta}{\cos\theta}$. Now, we can plug in 0 for theta resulting in $\frac{-\sin 0}{\cos 0} = \frac{-0}{1} = 0$

20.) D

Realize that one of the applications of integration is trying to find the area under the curve and in our case, we are trying to find the area under the curve of |x| from -3 to 2. Looking at this from a geometrical perspective we see that we create two right triangles. One with a base and height of 3 and the other with a base and height of 2. The area of which is $\frac{1}{2}bh + \frac{1}{2}bh = \frac{1}{2}(3)(3) + \frac{1}{2}(2)(2) = \frac{9}{2} + \frac{4}{2} = \frac{13}{2}$.

Alternate Solution: we can break up |x| into x and -x for when x>0 and x<0 respectively. Thus,

$$\int_{-3}^{2} |x| dx = \int_{-3}^{0} -x dx + \int_{0}^{2} x dx$$

$$\int_{-3}^{2} |x| dx = \left.\frac{-x^2}{2}\right|_{-3}^{0} + \left.\frac{x^2}{2}\right|_{0}^{2}$$

$$\int_{-3}^{2} |x| dx = \frac{9}{2} + \frac{4}{2} = \frac{9}{2} + 2 = \frac{13}{2}$$

21.) C

Since we know that $\int_a^d f(x)dx = 30$ we know that the sum entire sections of between a and b, b and c, and c and d is 30. Let $\int_a^b f(x)dx = y$ and therefore $\int_b^c f(x)dx$ is also equal to y. Furthermore, $\int_d^c f(x)dx = -\int_c^d f(x)dx$ and therefore

$-\int_c^d f(x)dx = 3\int_b^c f(x)dx$

$-\int_c^d f(x)dx = 3y$

$\int_c^d f(x)dx = -3y$

$\int_a^b f(x)dx + \int_b^c f(x)dx + \int_c^d f(x)dx = 30$

$y + y - 3y = 30$

$-y = 30$

$y = -30$

Thus, $\int_c^a f(x)dx + \int_c^b f(x)dx = -(-30-30) + (-30) = 60 - 30 = 30$

22.) A

Writing the first few terms of the sequence will help us see the bigger picture. Let the entire sum of the series equal S. Note: the formula for an infinite geometric series is $\frac{a}{1-r}$ where a is the first term and r is the common ration. Thus,

$\sum_{n=0}^{\infty} \frac{n}{2^n} = 0 + \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots$ writing sequence

$S = \frac{1}{2} + \frac{1+1}{4} + \frac{1+2}{8} + \frac{1+3}{16} + \cdots$ rearranging

$S = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right) + \frac{1}{2}\left(\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots\right)$ manipulating

$S = \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{1}{2}(S)$ using infinite geometric series formula and substitution

$\frac{1}{2}S = 1$

$S = 2$

23.) B

Using the product rule, we have to find F(S') + S(F') where F and S are the first and second multiplicand respectively. Thus,

$f'(x) = (x^3)\left(\frac{1}{x}\right) + (\ln x)(3x^2) = x^2(1 + 3\ln x)$

$f'(2) = 2^2(1 + 3\ln 2) = 4(1 + 3\ln 2)$

Thus, $a + b + c + d = 4 + 1 + 3 + 2 = 10$

24.) C

In order to solve these first-order differential equations we have to separate the variables, integrate, solve with the initial condition, then solve with the last condition. Thus,

$\frac{dy}{dx} = \frac{2y}{x}$

$\frac{1}{y}dy = \frac{2}{x}dx$

$\int \frac{1}{y}dy = 2\int \frac{1}{x}dx$

$\ln|y| = 2\ln|x| + c$

$\ln(|1|) = 2\ln(|2|) + c$

$c = -2\ln 2$

$\ln|y| = 2\ln|x| - 2\ln 2$

$\ln|y| = \frac{2\ln|x|}{-2\ln 2}$

$\ln|y| = \frac{\ln x^2}{-\ln 2^2}$

$|y| = \frac{x^2}{-4}$

$y = \frac{x^2}{4}$

$y = \frac{(4)^2}{4}$

$y = 4$

25.) D

$x^2 = \sum \frac{(O-E)^2}{E}$, we know that there is a total of (9+8+6+5+2 = 30) 30 students in his class. To find our expected values we multiply the percentage by the total which in this case is 30. Thus, our expected counts is (.2)(30) A's, (.3)(30) B's, (.2)(30) C's, (.2)(30) D's, and (.1)(30) F's which is 6 A's, 9 B's, 6 C's, 6 D's and 3 F's. Thus,

$$x^2 = \frac{(9-6)^2}{6} + \frac{(8-9)^2}{9} + \frac{(6-6)^2}{6} + \frac{(5-6)^2}{6} + \frac{(2-3)^2}{3}$$

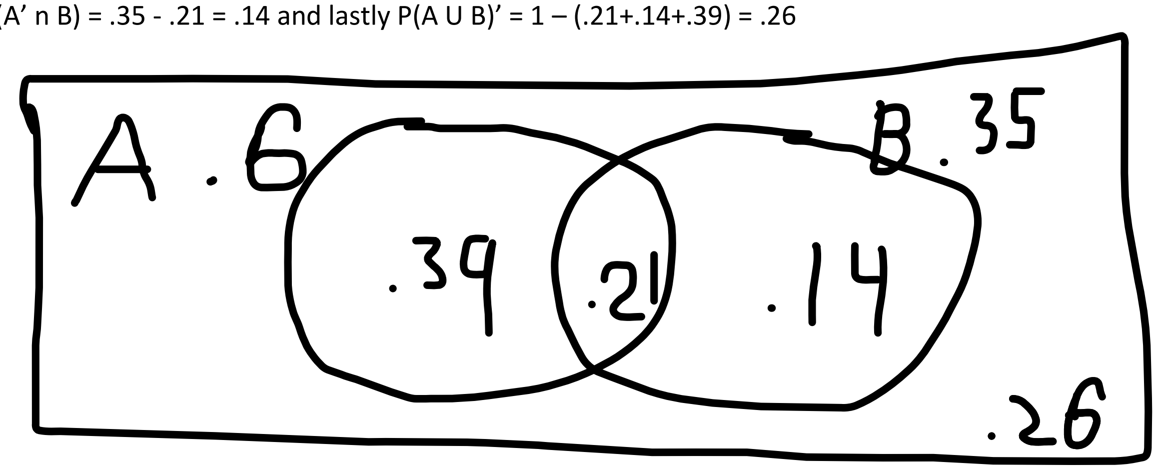
$$x^2 = \frac{9}{6} + \frac{1}{9} + \frac{0}{6} + \frac{1}{6} + \frac{1}{3}$$

$$x^2 = \frac{19}{9}$$

19 + 9 = 28

26.) B

Since A and B are independent we know that P(A n B) = (.6)(.35) = .21 and therefore P(A n B') = .6 - .21 = .39 and P(A' n B) = .35 - .21 = .14 and lastly P(A U B)' = 1 – (.21+.14+.39) = .26



27.) C

To calculate a residual, we take (observed – predicted). We know that our observed height is 70 inches, so we just need to calculate his predicted height which is found when we substitute in 160 pounds for x. Thus,

y = .5(160) – 16

y = 80 – 16

y = 64

Thus, our predicted height is 64 inches. Our residual is 70 – 64 = 6.

28.) E

A is false as when the central limit theorem does not apply, we must draw a histogram to prove that the sample's distribution is approximately normal. B is false as the sample must not necessarily be taken from 2 homogenous groups. C is false as we can use a T-Test for normal distributions. D is false as the degrees of freedom is not the reason why we use T instead of Z. The correct answer is that we use a T-Test over a Z-Test because we do not know the standard deviation of the population. Thus, the answer is E.

29.) A

Firstly, we must realize that $E(x^2) = Var(x) + [E(x)]^2$. We need to substitute accordingly. Note that the square of standard deviation is variance and that E(x) is the expected value and in a binomial situation is its mean. Thus,

$E(x^2) = Var(x) + [E(x)]^2$

$E(x^2) = \left(\sqrt{np(1-p)}\right)^2 + (np)^2$

$E(x^2) = np - np^2 + n^2p^2$

$E(x^2) = np(1 - p + np)$

30.) D

Selecting every k^{th} person is the definition of a systematic sample.