1	B	11	C	21	D
2	A	12	C	22	D
3	D	13	C	23	D
4	E	14	D	24	B
5	A	15	A	25	D
6	C	16	A	26	E
7	B	17	C	27	C
8	E	18	A	28	B
9	A	19	A	29	C
10	D	20	D	30	E

1) $+$ is the `OR` operator in Boolean algebra. `True OR True = True`, so $1+1=1$. $\boxed{\text{B}}$

2) The first statement is the contrapositive of the given statement and is thus logically equivalent. Choices B) and D) are the converse and inverse of the original statement, respectively, and choice C) is irrelevant. $\boxed{\text{A}}$

3) Create a truth table.

A	B	C	$A' \vee B$	$A \vee C'$	$(A' \vee B) \wedge (A \vee C')$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

$\frac{4}{8} = \frac{1}{2}$ of the cases are true. $\boxed{\text{D}}$

4) $\mathbb{Q}$ has the same cardinality as $\mathbb{Z}$ and $\mathbb{N}$ by the classic diagonalization argument. This argument can be slightly modified to include $\mathbb{Q}^2$. I, II, and IV are correct. $\boxed{\text{E}}$

5) $(A' \cup B)' = A \cap B'$, and $(A \cap B) \cup (A \cap B') = A$. $\boxed{\text{A}}$

6) $|A \cup B| = |A| + |B| - |A \cap B| = 20 + 22 - 9 = 33$. $\boxed{\text{C}}$

7) $\frac{1}{14} = \frac{1}{2} \cdot \frac{1}{7} = \frac{1}{2}\left(\frac{1/8}{1-1/8}\right) = \frac{1}{2}\left(\frac{1}{8} + \frac{1}{64} + \frac{1}{512} + \cdots\right)$. This is equivalent to $0.0\overline{001}_2$. $\boxed{\text{B}}$

8) The first relation is not a function. The second relation is bijective, so it is also injective and surjective. The third relation is surjective but not injective. $B = 1$, $I = 1$, and $S = 2$, so $4B + 2I + S = 8$. $\boxed{\text{E}}$

9) The tallest tree would have every node on its own level, giving it a height of 2022. The shortest tree would be as close to complete as possible. A complete tree has $2^h - 1$ nodes, where h is the height of the tree. Note that $h = 11$ gives a maximum of 2047 nodes. The difference between its maximum and minimum height is $2022 - 11 = 2011$. $\boxed{\text{A}}$

10) $1\ 3\ +\ 3\ 7\ +\ \times = 4\ 10\ \times = 40$. $\boxed{\text{D}}$

11) The inverse of $3x + 7$ mod 26 is $9(x - 7)$ mod 26.

e	A	F	N	L	N	C	X	F	N	T	N
$\#e$	0	5	13	11	13	2	23	5	13	19	13
$9(\#e - 7)$	-63	-18	54	36	54	-45	144	-18	54	108	54
mod 26	15	8	2	10	2	7	14	8	2	4	2
d	P	I	C	K	C	H	O	I	C	E	C

The message spells `PICK CHOICE C`. $\boxed{\text{C}}$

12) Observe that $x \equiv -1$ mod 5 and $x \equiv -1$ mod 7, so by the Chinese Remainder Theorem, $x \equiv -1$ mod 70. By inspection, combining this with $x \equiv 1$ mod 4 gives $x \equiv 69$ mod 140 and $x = 140n + 69$ for $n \in \mathbb{Z}$. $n = 14$ gives $x = 2029$. $2 + 0 + 2 + 9 = 13$. $\boxed{\text{C}}$

13) Note that 55 and 89 are consecutive Fibonacci numbers, so by the Euclidean algorithm, this will be the most longest continued fraction for a denominator of 89 or less.

	r	$s_n = \lfloor r \rfloor$	$f = \text{frac}(r)$	$\frac{1}{f}$
s_0	$\frac{55}{89}$	0	$\frac{55}{89}$	$\frac{89}{55}$
s_1	$\frac{89}{55}$	1	$\frac{34}{55}$	$\frac{55}{34}$
s_2	$\frac{55}{34}$	1	$\frac{21}{34}$	$\frac{34}{21}$
s_3	$\frac{34}{21}$	1	$\frac{13}{21}$	$\frac{21}{13}$
s_4	$\frac{21}{13}$	1	$\frac{8}{13}$	$\frac{13}{8}$
s_5	$\frac{13}{8}$	1	$\frac{5}{8}$	$\frac{8}{5}$
s_6	$\frac{8}{5}$	1	$\frac{3}{5}$	$\frac{5}{3}$
s_7	$\frac{5}{3}$	1	$\frac{2}{3}$	$\frac{3}{2}$
s_8	$\frac{3}{2}$	1	$\frac{1}{2}$	2
s_9	2	2	0	STOP

$8 \cdot 1 + 2 = 10$. $\boxed{\text{C}}$

14) Setting up the same chart as before gives this.

	r	$s_n = \lfloor r \rfloor$	$f = \text{frac}(r)$	$\frac{1}{f}$	Simplified
s_0	$\sqrt{41}$	6	$\sqrt{41} - 6$	$\frac{1}{\sqrt{41}-6}$	$\frac{\sqrt{41}+6}{5}$
s_1	$\frac{\sqrt{41}+6}{5}$	2	$\frac{\sqrt{41}-4}{5}$	$\frac{5}{\sqrt{41}-4}$	$\frac{\sqrt{41}+4}{5}$
s_2	$\frac{\sqrt{41}+4}{5}$	2	$\frac{\sqrt{41}-6}{5}$	$\frac{5}{\sqrt{41}-6}$	$\sqrt{41} + 6$
s_3	$\sqrt{41} + 6$	12	$\sqrt{41} - 6$	STOP	

The sequence of $\{2, 2, 12\}$ will continue on forever. $2 + 2 + 12 = 16$. $\boxed{\text{D}}$

15) Note that $3^{210} = 9^{105} = (-1+10)^{105}$. By binomial expansion, this is equal to $\binom{105}{105}(-1)^{105}(10)^0 + \binom{105}{104}(-1)^{104}(10)^1 + \binom{105}{103}(-1)^{103}(10)^2 + \cdots$, and all future terms will be divisible by 1000. This simplifies to $-1 + 105 \cdot 10 - \frac{105 \cdot 104}{2} \cdot 100 + \cdots = 1049 + 1000n + \cdots$, whose last three digits is 049. $\boxed{\text{A}}$

16) Obviously, $x \not\equiv 0 \bmod 29$. By Fermat's Little Theorem, $x^28 \equiv 1 \bmod 29$, so $x^{84} \equiv 1 \bmod 29$ and thus $x^2 \equiv 6 \bmod 29 \equiv 64 \bmod 29$. Therefore, $x \equiv \pm 8 \bmod 29$ and $x \in \{8, 21, 37\}$. $8 + 21 + 37 = 66$. $\boxed{\text{A}}$

17) The primes one more than a factor of 64 are 2, 3, 5, and 17. Let $n = 2^a 3^b 5^c 17^d$. Because of properties of the totient function, $b, c, d \leq 1$ and $a \leq 7$. Note that $\phi(3) = 2^1$, $\phi(5) = 2^2$, and $\phi(17) = 2^4$, so $\phi(3^b 5^c 17^d = 2^{b+2c+4d})$. Perform casework on a, noting $\phi(2^7) = 2^6 = 64$ and $\phi(2k) = 2\phi(k)$ if $2|k$.

- $a = 0$: $b + 2c + 4d = 6$, so $(a, b, c, d) = (0, 0, 1, 1)$ and $n = 85$
- $a = 1$: $b + 2c + 4d = 6$, so $(a, b, c, d) = (1, 0, 1, 1)$ and $n = 170$
- $a = 2$: $b + 2c + 4d = 5$, so $(a, b, c, d) = (2, 1, 0, 1)$ and $n = 204$
- $a = 3$: $b + 2c + 4d = 4$, so $(a, b, c, d) = (3, 0, 0, 1)$ and $n = 136$
- $a = 4$: $b + 2c + 4d = 3$, so $(a, b, c, d) = (4, 1, 1, 0)$ and $n = 240$
- $a = 5$: $b + 2c + 4d = 2$, so $(a, b, c, d) = (5, 0, 1, 0)$ and $n = 160$
- $a = 6$: $b + 2c + 4d = 1$, so $(a, b, c, d) = (6, 1, 0, 0)$ and $n = 192$
- $a = 7$: $b + 2c + 4d = 0$, so $(a, b, c, d) = (7, 0, 0, 0)$ and $n = 128$

The above is an albeit overly rigorous finding of values of n; note the bijection between three-bit binary and the $a \geq 1$ cases. There are 8 possible values of n. $\boxed{\text{C}}$

18) Most of the power tower doesn't matter, since it's 2 to the power of 1 plus a large power of 2 (which is going to be equivalent to 1 mod 4). Thus, the last digit of the power tower is 2, and adding 1 to the expression gives a last digit of 3. $\boxed{\text{A}}$

19) This is one of the many formulations of the Catalan numbers, $C_n = \frac{1}{n+1}\binom{2n}{n}$. Plugging in $n = 4$ gives $\frac{1}{5}\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{5 \cdot 4 \cdot 3 \cdot 2} = 14$. $\boxed{\text{A}}$

20) $f(3) = 7$ and $f(-2) = -3$. $f(x) = q(x)(x^2 - x - 6) + ax + b$ for some constants a and b. Plugging in $x = 3$ gives $3a + b = 7$, and plugging in $x = -2$ gives $-2a + b = -3$. The solution to this system is $\{a, b\} = \{2, 1\}$, so $r(x) = 2x + 1$. $\boxed{\text{D}}$

21) In terms of truthiness, the choices are A) $F \to F$, B) $T \to T$, C) $F \to T$, and D) $T \to F$. Because a predicate cannot be proven false given a false antecedent, $T \to F$ is the only way to obtain a false implication. $\boxed{\text{D}}$

22) Note that $N = 117^4 + 4(2^4)$, which is factorizable by Sophie-Germain. $N = (117^2 + 2 \cdot 117 \cdot 2 + 2 \cdot 2^2)(117^2 - 2 \cdot 117 \cdot 2 + 2 \cdot 2^2) = 14165 \cdot 13229$. Obviously, the first of these factors, and since there are exactly three prime factors, 13229 is the largest prime factor. $1 + 3 + 2 + 2 + 9 = 17$. $\boxed{\text{D}}$

23) Rearranging, $x_n - 4x_{n-1} - 5_{x-2} = 0$. The characteristic equation for this recursion is $X^2 - 4X - 5 = 0$, which has solutions $X = 5$ and $X = -1$, so $x_n = a5^n + b(-1)^n$ and $K = 5$. $\boxed{\text{D}}$

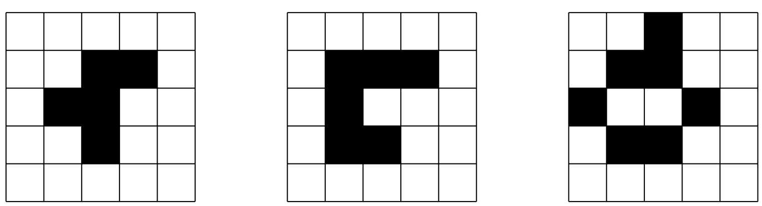
24) Carefully checking the arrows, the adjacency matrix is $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. $\boxed{\text{B}}$

25) The degree matrix is $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, so the Laplacian matrix is $\begin{bmatrix} 4 & -1 & 0 & 0 \\ -2 & 6 & -1 & -1 \\ 0 & 0 & 3 & 0 \\ -1 & -1 & -1 & 4 \end{bmatrix}$. By minors, the determinant of this is $3\begin{vmatrix} 4 & -1 & 0 \\ -2 & 6 & -1 \\ -1 & -1 & 4 \end{vmatrix} = 3(96 - 1 - 4 - 8) = 249$. $\boxed{\text{D}}$

26) This is the traveling salesman problem. Note that an Eulerian circuit and a Hamiltonian cycle do not care about distances between nodes, a knight's tour has fixed length, and a minimum spanning tree is not a loop. $\boxed{\text{E}}$

27) This pseudocode represents binary search, which has a time complexity of $O(\log n)$. $\boxed{\text{C}}$

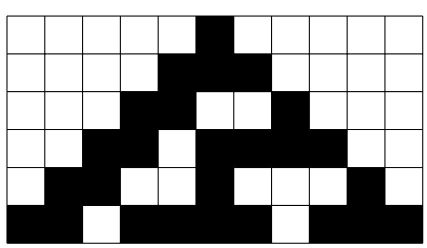
28) The first two iterations of Conway's Game of Life on the F-pentomino are shown here.



Generation 0 Generation 1 Generation 2

Generation 2 has 7 live cells. $\boxed{\text{B}}$

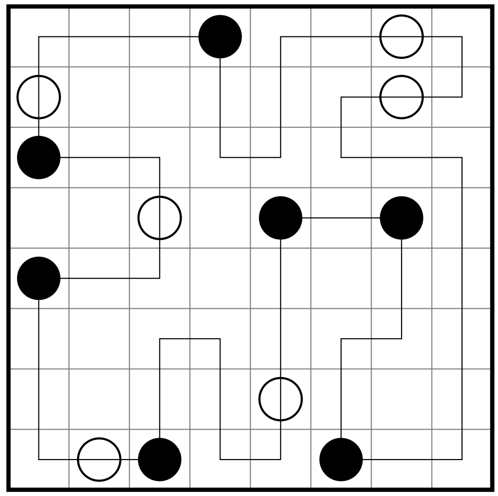
29) Rows 0 through 5 of the Rule 30 automaton are shown here.



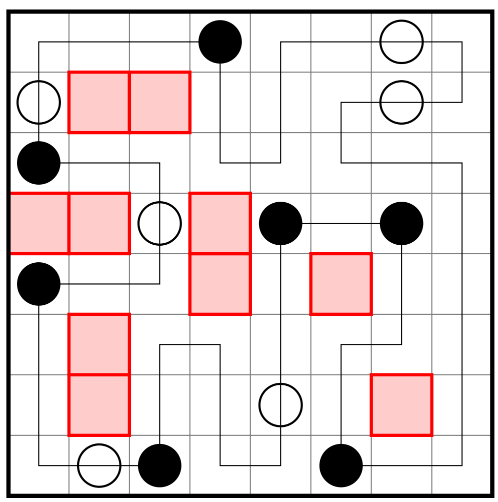
Row 0
Row 1
Row 2
Row 3
Row 4
Row 5

Row 5 has 9 black cells. $\boxed{\text{C}}$

30) The unique solution to the Masyu is shown here, with unused cells highlighted in red.



Unused cells are highlighted in red.



There are 10 unused cells. E