1) How many positive integers less than 1000 have the property that the sum of their positive factors is odd?

A) 31 B) 32 C) 53 D) 54 E) NOTA

2) How many positive divisors does 2021 have?

A) 2 B) 4 C) 6 D) 8 E) NOTA

3) When written in base 12, how many 0's does 2021! end with?

A) 1005 B) 1006 C) 1007 D) 1008 E) NOTA

4) A Gaussian integer is a number of the form $a + bi$, $a, b \in \mathbb{Z}$. A Gaussian integer is considered a Gaussian prime if either of the following properties hold: (1) a or b are 0 and the magnitude of the non-zero part is a prime that is 3 mod 4, or (2) $a, b \neq 0$ and $a^2 + b^2$ is a prime number. Given this, let $p + qi$ be the sum of the 3 Gaussian primes with the lowest magnitude and argument (for instance, $1 + 2i$ would be chosen over $-1 - 2i$ as the former has a smaller complex argument). Calculate $p + q$.

A) 6 B) 7 C) 8 D) 9 E) NOTA

5) Compute the sum of the digits of the largest prime factor of $4^7 - 1$.

A) 8 B) 10 C) 11 D) 12 E) NOTA

6) How many of the first 500000 terms of the Fibonacci sequence (starting with $F_1 = F_2 = 1$) are divisible by 13?

A) 71427 B) 71428 C) 71429 D) 71430 E) NOTA

7) In what base is the following equation true: $123 \times 321 = 39383$?

A) 10 B) 11 C) 12 D) 13 E) NOTA

8) Let $n!! = (n)(n-2)(n-4)\cdots$, stopping when $n-k=1$ or $n-k=2$. Calculate the largest prime factor of 2020!!.

A) 1009 B) 2017 C) 1003 D) 997 E) NOTA

9) Which of the following sets does NOT have the same cardinality as $\mathbb{Z}$?

A) $\mathbb{Q}$ B)$\mathbb{N}$ C) $\mathbb{C}$ D) $\mathbb{P}$ (the set of prime numbers) E) NOTA

10) What is the sum of the last 2 digits of $7^{5296}+3$?

A) 1 B) 4 C) 7 D) 10 E) NOTA

11) Which of these pairs of numbers will take the most steps when using the Euclidean Algorithm to calculate the GCD?

A) 610,377 B) 610,379 C) 612,377 D) 612,379 E) NOTA

12) Which of these numbers is perfect?

A) 26 B) 27 C) 28 D) 29 E) NOTA

13) Evaluate $31^{16} \mod 23$.

A) 13 B) 14 C) 15 D) 16 E) NOTA

14) Evaluate $31^{25} \mod 23$.

A) 3 B) 4 C) 5 D) 6 E) NOTA

15) How many right triangles with integer side lengths have equal perimeter and area?

A) 0 B) 1 C) 2 D) 3 E) NOTA

16) Which of the following numbers has a continued fraction with an odd period?

A) $\sqrt{37}$ B) $\sqrt{38}$ C) $\sqrt{39}$ D) $\sqrt{40}$ E) NOTA

17) Compute the product of the positive factors of 576.

A) 24^{20} B) 24^{21} C) 24^{22} D) 24^{23} E) NOTA

18) Given that x is an integer and $x^6 - 6x = 148036027$, what is x?

A) 21 B) 23 C) 25 D) 27 E) NOTA

19) $\dfrac{x!}{p} = (y!)^2$. Given that none of p, x, and y are greater than 15 or less than 3, and that p is prime, find $p + x + y$.

A) 20 B) 21 C) 22 D) 23 E) NOTA

20) Binary Coded Decimal (BCD) is a number format that computers sometimes use. In BCD, base 10 digits are encoded in groups of 4 binary digits ($0_{10} = 0000_2$, $1_{10} = 0001_2$,... $9_{10} = 1001_2$). For example, if I were to write the base 10 number 47 in BCD, I would write it as 0100 0111. Given this, let a be the sum of the digits of 24867 when written in BCD, and let b be the sum of the digits of 24867 when written in binary. Calculate $a + b$.

A) 12 B) 13 C) 14 D) 15 E) NOTA

21) Let $A = \{\binom{49}{k}, 0 \leq k \leq 49\}$, and $B = \{\binom{50}{k}, 0 \leq k \leq 50\}$. Let j be the number of $a \in A$ such that $a \equiv 0 \mod 7$ and let k be the number of $b \in B$ such that $b \equiv 0 \mod 7$. Compute $j + k$

A) 48 B) 49 C) 95 D) 96 E) NOTA

22) Given that $\sum_{n=0}^{\infty} \dfrac{x^n}{n!} = e^x$ and that $0!! = 1$, calculate $\sum_{n=0}^{\infty} \dfrac{1}{(2n)!!}$

A) 1 B) $\sqrt{e}$ C) e D) $e\sqrt{e}$ E) NOTA

23) Let a be a positive integer. $a \equiv 3 \mod 5$, $a \equiv 5 \mod 7$, and $a \equiv 6 \mod 11$. Given this, which of the following could a be?

A) 5913 B) 5948 C) 5968 D) 6003 E) NOTA

24) How many positive integers $x < 100$ satisfy the following equation: $(x-1)! \equiv -1 \mod x$?

A) 23 B) 24 C) 25 D) 26 E) NOTA

25) Which of these numbers is prime?

A) $2^{14}-1$ B) $2^{15}-1$ C) $2^{16}-1$ D) $2^{17}-1$ E) NOTA

26) Evaluate $\dfrac{\phi(300)}{\phi(15)}$, where $\phi(x)$ is Euler's totient function.

A) $\phi(20)$ B) $5\phi(20)$ C) $\dfrac{\phi(20)}{4}$ D) $\dfrac{5\phi(20)}{4}$ E) NOTA

27) What is the sum of the digits of the smallest positive integer to have exactly 64 factors?

A) 18 B) 19 C) 20 D) 21 E) NOTA

28) Given that the following magic square (when filled in) uses all the numbers from 1 to 16, what is $A+B$?

	11	14	1
A		7	12
3			6
	5	B	

A) 15 B) 16 C) 17 D) 18 E) NOTA

29) With the addition of Min-Min into Super Smash Bros. Ultimate, there are now a total of 80 characters in the game. Assume that the players are identical (so Player 1 picking Mario and Player 2 picking Luigi is the same as Player 2 picking Mario and Player 1 picking Luigi), and that any character may be picked by multiple players. Let P be the number of possible character selections for an 8-player game of Smash. Find the remainder when P is divided by 9.

A) 0 B) 1 C) 2 D) 3 E) NOTA

30) Which of the following triplets satisfies the following equation: $(x+y)^3 = z^3+3xy(x+y)$?

A) $(136, 384, 432)$ B) $(348, 734, 766)$ C) $(456, 865, 897)$
D) $(6348, 47597, 48743)$ E) NOTA