Directions: Welcome to the Open Physics topic test! The following conventions are to be used:

- *The frame of refrence for any problem is assumed to be inertial*
- *Positive work is defined as work done on a system by the environment*
- *Current is conventional: the direction positive charge would drift*
- *All circuit components are ideal unless otherwise stated*
- *All thermal expansion effects are negligible*

And, most importantly, ***for the duration of this test, let*** $g = 10m/s^2$. *Good luck!*

1. Andy throws a ball upwards with an initial velocity of 10 m/s. How long will Andy have to wait for the ball to reach his hand again?

   A. 1 s $\boxed{\text{B. 2 s}}$ C. 4 s D. 8 s E. NOTA

   Solution: The quickest way to do this is to notice the symmetry; it takes 1 second for gravity to reduce the velocity to 0, and thus 2 seconds to return the same distance down as well.

2. A block of mass 1 kg is initially at rest on a 30-degree inclined plane with coefficient of kinetic friction of $\frac{\sqrt{3}}{5}$. How far will the block travel in 5 seconds? Assume the coefficient of static friction is 0.

   A. $\frac{125}{2}$ m B. $\frac{75}{2}$ m C. 50 m $\boxed{\text{D. 25 m}}$ E. NOTA

   Solution: The force down the plane is $10 \sin 30 = 5$. The normal force is $10 \cos(30) = 5\sqrt{3}$, so the frictional force up the plane is $5\sqrt{3} * \frac{\sqrt{3}}{5} = 3$. The net force is $5 - 3 = 2$, so the acceleration is $\frac{2}{1} = 2\,\text{m/s}^2$, which will travel $\frac{1}{2} * 2 * 5^2 = 25\,\text{m}$ in five seconds.

3. A lump of clay of mass 3 kg moving at 2 m/s to the left collides perfectly inelastically with a ball of mass 7 kg moving to the right at 8 m/s. After the collision, what is the velocity of the resulting object, assuming right is positive?

   $\boxed{\text{A. 5 m/s}}$ B. 6.2 m/s C. −5 m/s D. −6.2 m/s E. NOTA

   Solution: There is $-3 * 2 + 7 * 8 = 50$ momentum to the right, contained in an object of mass $7 + 3 = 10$, for a velocity of positive 5.

4. An ideal spring has its equilibrium point at x = 0. If it takes 6 J of work to stretch this spring from x = 1 to x = 2, how much work does it take to stretch it from x = 2 to x = 3?

   A. 6 J B. 9 J $\boxed{\text{C. 10 J}}$ D. 12 J E. NOTA

   Solution: It takes $\frac{1}{2}kx^2$ Joules to stretch a spring from equilibrium. Thus, we set $6 = \frac{1}{2}k(2)^2 - \frac{1}{2}k(1)^2$ to find $k = 4$, and solve $\frac{1}{2}(4)(3)^2 - \frac{1}{2}(4)(2)^2 = 10\,\text{J}$ to stretch the rest of the way.

5. Consider an ideal gas with pressure and volume given by (V, P) in m^3 and atm for the following three states: A (1, 1), B (1, 4), C (3, 3). How much work is done on the gas by the environment in the cycle ABCA?

A. 3 J B. 6 J $\boxed{\text{C. } -3\,\text{J}}$ D. −6 J E. NOTA

Solution: The area of the triangle is 3, and the work from B to C is greatest, when the volume is increasing and thus work is being done on the environment, meaning the total work is negative.

6. A circuit consists of n resistors wired in series with a 5 V battery. Find the current in each resistor, if R_i is the resistance of the i-th resistor in the series, and the following holds true:

$$\sum_{i=1}^{n} R_i = 10\,\Omega$$

A. .02 A $\boxed{\text{B. .5 A}}$ C. 2 A D. 50 A E. NOTA

Solution: The total resistance is literally given as $10\,\Omega$. The current is .5 A.

7. A circuit consists of a 9 V battery and 9 resistors of $9\,\Omega$ each wired in parallel. Find the current through the battery.

A. $\frac{1}{3}$ A $\boxed{\text{B. 1 A}}$ C. 3 A D. 9 A E. NOTA

Solution: The resistors total $1/\sum_1^9 \frac{1}{9} = 1\,\Omega$. Ohm's law then gives $9 = 9I$, or $I = 1$ A.

8. Bryan finds himself on a ladder parallel to the y-axis, floating in space. As Bryan begins to climb the ladder in the positive y direction, which of the following describes the y-coordinate of the center of mass of the Bryan-ladder system?

A. Increasing $\boxed{\text{B. Constant}}$ C. Decreasing D. Can't Tell E. NOTA

Solution: Without an outside force, the center of mass of the system cannot move.

9. Two blocks of mass 2 kg and 4 kg respectively are tied together by a string, and the string is then draped over an ideal pulley such that when one block is pulled down, the other rises, and vice versa, forming an Atwood machine. Find the tension in the string.

A. 20 N B. 60 N $\boxed{\text{C. } \frac{80}{3}\,\text{N}}$ D. $\frac{40}{3}$ N E. NOTA

Solution: Let F_T be the tension force. The net force on the big block is $40 - F_T$, and on the small block is $F_T - 20$. We can equate the accelerations to receive $\frac{40-F_T}{4} = \frac{F_T-20}{2}$, or $F_T = \frac{80}{3}$ N.

10. An oscillating spring with an angular frequency of $\omega = 2$ Hz has a block attached to it. Given the block goes a maximum of 5 m and no further from the spring's equilibrium in each oscillation, find the maximum speed of the block during any given oscillation.

A. .1 m/s B. .4 m/s C. 2.5 m/s $\boxed{\text{D. 10 m/s}}$ E. NOTA

Solution: The energy in the system is entirely stored in the spring when stretched completely, and entirely in the kinetic movement of the block when the block is moving its fastest (when the spring is at equilibrium). Thus, we can equate these energies as $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Solving for v gives $v = x\sqrt{\frac{k}{m}}$, but we know that $\omega = \sqrt{\frac{k}{m}}$ for a spring. Thus, we find $v = (5)(2) = 10$ m/s.

11. DZ is staring down at a loop of wire on his desk. Using his magnetic abilities, he begins to increase the magnetic field downwards into the loop at a linear rate. From DZ's perspective, which of the following are correct?

    I. The induced current is clockwise

    II. The magnetic field generated by the induced current is in a direction opposite to the field exerted by DZ

    A. I and II　B. I only　$\boxed{\text{C. II only}}$　D. Neither　E. NOTA

    Solution: The induced current must generate an opposing flux to DZ's. Right hand rule tells us this must be counterclockwise, so I is false. II is trivially true.

12. A hoop with radius 1m and mass 10 kg is spinning about its central axis at 2 rad/s. If a tangential force of 5 N is applied to the edge of the disc to slow it down, for long does the disc remain spinning?

    A. 2 s　$\boxed{\text{B. 4 s}}$　C. 5 s　D. 8 s　E. NOTA

    Solution: The inertia is $I = 10(1)^2 = 10$. We can use torque to find the angular acceleration as $\tau = Fr = (5)(1) = I\alpha = 10(\alpha)$, so $\alpha = .5\,\text{rad/s}^2$. Thus, it will take $\frac{2}{.5} = 4\,\text{s}$ to slow the hoop to a stop.

13. A balloon is filled with air and submerged underwater to depth $d > 1\,\text{m}$ below the surface such that it is neutrally buoyant. A small eddy current causes the balloon to move slightly deeper. Which of the following describes the depth of the balloon right after this downwards movement (Increasing depth would be moving downwards)?

    $\boxed{\text{A. Increasing}}$　B. Constant　C. Decreasing　D. Can't Tell　E. NOTA

    Solution: When the balloon is pushed downwards, the volume will decrease slightly due to the increased pressure from the water. This will cause the upwards buoyant force to decrease, which makes it less than gravity now, giving the balloon a positive downwards acceleration.

14. A 100 m long rope with a linear mass density of 2 kg/m is hanging from a building. During a workout routine, Konwoo grabs the rope by its middle and pulls it straight up to be even with the top of the rope. If Konwoo's biceps grow by 1 mm for every kilo-joule of work he does, by how much did Konwoo's biceps grow?

    A. 12.5 mm　B. 50 mm　$\boxed{\text{C. 62.5 mm}}$　D. 100 mm　E. NOTA

    Solution: Imagining the rope as split into fourths of 25 m and 50 kg each, we see that Konwoo has raised the second part 25 m, and the third and fourth part both by 50 m. This is a total energy difference of $50 * 10 * 25 + (2)50 * 10 * 50 = 62.5\,\text{KJ}$. Konwoo's biceps thus grow by 62.5 mm. Wow!

15. Yared, a sphere with radius r, angular velocity ω, and linear velocity v (all positive) is rolling and slipping on a surface. Given that ω is decreasing, which of the following is true?

    $\boxed{\text{A. } v < r\omega}$　B. $v = r\omega$　C. $v > r\omega$　D. Can't tell　E. NOTA

    Solution: Friction will cause Yared's angular velocity to approach an equilibrium at which $v = r\omega$. If ω is decreasing, $r\omega$ is too large, and so $v < r\omega$.

16. A cup of ice water sits outside on a hot summer day. As the ice melts, which of the following describes the height of the water level in the cup?

A. Increasing $\boxed{\text{B. Constant}}$ C. Decreasing D. Can't Tell E. NOTA

Solution: Let the density of liquid water be ρ_w and of solid water be ρ_i, and let there initially be V_w liters of liquid water in the cup and V_i liters of solid water. Consider the moment in which the fraction of the volume of the ice melted is X, where $0 \leq X \leq 1$. The water level is in direct proportion to the volume of liquid water plus the volume of submerged ice. The liquid water has volume $\rho_w V_w + \frac{\rho_i}{\rho_w}(XV_i)$. The submerged volume V_s of ice can be found by equating the buoyant force and gravity: $\rho_w(V_s)g = \rho_i((1-X)V_i)g$, which gives $V_s = \frac{\rho_i}{\rho_w}(1-X)V_i = \frac{\rho_i}{\rho_w}V_i - \frac{\rho_i}{\rho_w}XV_i$. Adding this submerged volume to the liquid volume gives a total of $\rho_w V_w + \frac{\rho_i}{\rho_w}(XV_i) + \frac{\rho_i}{\rho_w}V_i - \frac{\rho_i}{\rho_w}XV_i = \rho_w V_w + \frac{\rho_i}{\rho_w}V_i$ liters of volume that contribute to the water height. Thus, we see that this does not depend on or change due to how much ice melts, and therefore the water level stays constant. This conclusion can also be reached intuitively based on the fact that the mass of water in the cup remains constant.

17. A ramp of mass 4 kg is sitting on a table, and a sphere of mass 2 kg and radius 1 m is sitting with its center 4 m above the table on the ramp. *All* surfaces are frictionless. The system is initially at rest, but the ball and ramp will start to move due to gravity. The ramp is curved such that once the ball reaches the table, it is only moving parallel to the table (no vertical velocity). How fast is the ball moving at that time?

A. $\sqrt{10}$ m/s $\boxed{\text{B. } 2\sqrt{10}\text{ m/s}}$ C. $3\sqrt{10}$ m/s D. $4\sqrt{10}$ m/s E. NOTA

Solution: The energy from gravity that goes into the movement of the ramp and ball is $(2)(10)(4-1) = 60$ J. Once moving, we know that $4v_r = 2v_b$ due to conservation of momentum, and that $\frac{1}{2}(4)(v_r)^2 + \frac{1}{2}(2)(v_b)^2 = 60$ due to conservation of energy. Solving this system yields $v_b = 2\sqrt{10}$ m/s

18. A ball of mass 10kg has an initial velocity vector of $(8, 0)$. After a 1 second collision with a rigid wall, its velocity becomes (0, 6). What is the average magnitude of force the ball exerts on the wall during the collision?

A. 10 N B. 20 N $\boxed{\text{C. 100 N}}$ D. 200 N E. NOTA

Solution: The impulse is the hypotenuse of a 6-8-10 triangle (realize that momentum is a vector) times the mass of the ball, or $(10)(10) = 100$. Thus, the average force was $100/1 = 100$ N.

For the next three questions, consider the scenario in which Jeffrey is driving his Honda Civic in a circle of radius 5 m, where the coefficients of static and kinetic friction between his wheels and the ground are both .5. Jeffrey drives as fast as he can while still maintaining the 5 m radius circle, and never turns the steering wheel from this path.

19. How fast is Jeffrey going?

A. 2 m/s $\boxed{\text{B. 5 m/s}}$ C. 10 m/s D. $10\sqrt{2}$ m/s E. NOTA

Solution: Jeffrey's uniform circular motion must follow $F_c = \frac{mv^2}{r}$. The centripetal force is $F_c = mg\mu = 5m$, so $v = 5$ m/s.

20. If Jeffrey were to encounter an oil slick, the coefficients of friction would be reduced by 5%. Which of the following would describe Jeffrey's distance from the center of his original circular path right after contacting the oil slick?

$\boxed{\text{A. Increasing}}$ B. Constant C. Decreasing D. Can't Tell E. NOTA

Solution: The lower static friction causes a decreased centripetal force, so the circle must expand to continue at the same speed.

21. Finally, if Jeffrey were to encounter a patch of ice, the coefficient of kinetic friction would be reduced by 5%, while the coefficient of static friction would remain the same. Which of the following would describe Jeffrey's distance from the center of his original circular path right after contacting the patch of ice?

A. Increasing $\boxed{\text{B. Constant}}$ C. Decreasing D. Can't Tell E. NOTA

Solution: Because the wheels are not moving relative to the ground as they roll (they are not slipping), the coefficient of static friction is what is important; the kinetic coefficient is irrelevant. The static coefficient is unchanged, and thus so is the circle.

22. A firework of mass 6 kg is launched from the point $(-6, 0, 0)$ such that at time t for $0 < t < 6$ its position is the point $(-6 + 4t, 0, 9t - t^2)$. At t = 6, the firework explodes into three pieces of equal mass that land in the x-y plane. If one piece lands at $(20, 5, 0)$ and another lands at $(25, -10, 0)$, at what point does the third piece land?

A. $(-55, 5, 0)$ $\boxed{\text{B. } (45, 5, 0)}$ C. $(30, 0, 0)$ D. $(-55, -5, 0)$ E. NOTA

Solution: The center of mass of the pieces must follow the original path of the rocket. The rocket would have landed at $t = 9$ at (30, 0, 0), and so the center of the three equally massive pieces must be here. Thus, the final piece is at (45, 5, 0).

23. A .5 m thick metal disc with a radius of 5 m has a hole in the middle with radius 1 m. The mass of this object is 12 kg and the metal the disc is made of has a specific heat of 2 J/kg*K. An axle is fit through the hole in the middle and the disc is given an initial angular velocity of 6 rad/s. Due to friction with the axle, the disc eventually slows to a stop as its rotational kinetic energy is lost as heat. Assuming all of this heat is transferred uniformly to the disc and the disc is initially at 300 K, at what temperature is the disc when it stops?

A. 415 K B. 416 K $\boxed{\text{C. 417 K}}$ D. 418 K E. NOTA

Solution: We must find the energy stored in the rotating disc. The Inertia can be found as $\frac{1}{2}m(r^2 + R^2) = 6(1 + 25) = 156$ (or some clever addition and subtraction if this formula is unknown). Thus, the rotational kinetic energy is $\frac{1}{2}m\omega^2 = \frac{1}{2}(156)(6^2) = 2808\,\text{J}$. This is completely transferred to heating the disc. It takes $2 * 12 = 24$ Joules to heat the disc by one degree Kelvin, so the disc heats by $\frac{2808}{24} = 117\,\text{K}$, bringing the temperature up to $300 + 117 = 417\,\text{K}$.

For the next two questions, consider an ideal massless spring with length 2 m and spring constant $k = 1$, which has 1 kg masses attached to each end and is floating in space. The spring is stretched and given an angular velocity of 1 rad/sec such that during rotation, the spring remains a constant length.

24. What is the angular momentum of the rotating spring-mass system?

A. $1\,\mathrm{kg\,m^2/s}$ B. $2\,\mathrm{kg\,m^2/s}$ C. $4\,\mathrm{kg\,m^2/s}$ $\boxed{\text{D. } 8\,\mathrm{kg\,m^2/s}}$ E. NOTA

Solution: We need to find out how much the string is stretched. To do this, we can equate the restorative force of the spring to the centripetal force of the mass; $kx = mr\omega^2$. Let the string be extended by $2X$, so that the radius of motion is $1+X$. We then have that $(1)(2X) = (1)(1+X)(1)^2$, or $X = 1\,\mathrm{m}$. The inertia is thus $2(1)(1+1)^2 = 8$, so the angular momentum is $(8)(1) = 8$.

25. The rotating spring-mass system is magically and instantly transported to a non-vacuum atmosphere, where the masses now face a non-negligible air resistance (and no other additional forces). Right after this happens, which of the following describes the length of the spring?

A. Increasing B. Constant $\boxed{\text{C. Decreasing}}$ D. Can't Tell E. NOTA

Solution: The new drag force will cause ω to decrease. The spring will not have to pull as hard on the masses, and so will decrease in length. $F = mr\omega^2$ and $F = 2r - 2 = r\omega^2 : 2 = r(2 - \omega^2)$

26. What is the rotational inertia about an axis through the center and normal to a square of mass m and side length L? (Hint: The inertia about one of its sides is $\frac{1}{3}mL^2$)

A. $\frac{1}{24}mL^2$ B. $\frac{1}{12}mL^2$ $\boxed{\text{C. } \frac{1}{6}mL^2}$ D. $\frac{1}{3}mL^2$ E. NOTA

Solution: First, we find the inertia about the center parallel to an edge using parallel axis theorem: $\frac{1}{3}mL^2 = I_{com} + m(\frac{L}{2})^2$ yields $I_{com} = \frac{1}{12}mL^2$. Then, we can use perpendicular axis theorem with two of these axes to get the final inertia as $\frac{1}{12}mL^2 + \frac{1}{12}mL^2 = \frac{1}{6}mL^2$.

27. For an insulating cube with uniform charge density ρ and side length s, the electric potential at the center of the cube is given by $V = c\rho s^2$ for some constant c. Find the electric potential at one of the corners of the same cube.

$\boxed{\text{A. } \frac{1}{2}c\rho s^2}$ B. $\frac{1}{3}c\rho s^2$ C. $\frac{1}{4}c\rho s^2$ D. $\frac{1}{6}c\rho s^2$ E. NOTA

Solution: We can envision the voltage from the corner of the cube as one eighth of the voltage at the center of a cube with twice the side length, by splitting the larger cube into octants. Thus, the voltage is $\frac{1}{8}cp(2s)^2 = \frac{1}{2}cps^2$.

When playing professional billiards, the spin imparted to the cue ball when struck is of utmost importance. For the next two questions, consider a situation in which Marc is trying to hit the cue ball into the 8-ball to beat Alex, but wants to show off, so he tries to also pocket the cue ball after the 8-ball. The center of the cue ball is 1.7 m from the center of the 8-ball, and the center of the 8-ball is .8 m from the edge of the pocket. The shot is perfectly straight. The pool balls all have radius .1 m and mass 1 kg with uniform density, and are perfectly spherical ($I = \frac{2}{5}mr^2$). The pool table has static and kinetic coefficients of friction of .1. A ball will go in the pocket once its center passes the edge. Marc will hit the cue ball such that its linear velocity immediately after impact is 1 m/s towards the 8-ball. Assume that once the cue ball hits the 8-ball, its linear momentum is completely and instantly transferred to the 8-ball, and its angular momentum is unaffected.

28. The amount of topspin (i.e. angular velocity) ω that Marc must impart on the cue ball while hitting it to sink it after the 8-ball in a finite amount of time follows $\omega > \omega_{min}$. What is the value of ω_{min}?

A. $10\,\mathrm{rad/s}$ B. $20\,\mathrm{rad/s}$ $\boxed{\text{C. } 25\,\mathrm{rad/s}}$ D. $50\,\mathrm{rad/s}$ E. NOTA

Solution: As long as there is any angular momentum left in the cueball after it strikes the 8, it will eventually make it to the pocket. The cue ball must travel 1.5 m to hit the 8 (remembering to subtract the radii). To find out how long this takes, we can calculate the acceleration due to friction. With topspin, the frictional force will be towards the 8-ball because the cue ball is spinning too quickly. The frictional force is $mg\mu = (1)(10)(.1) = 1\,\text{N}$, so the linear acceleration is $\frac{1}{1} = 1\,\text{m/s}^2$. Thus, we can set up the distance equation: $1.5 = 1t + \frac{1}{2}(1)t^2$ to find that it takes $t = 1$ second to hit the 8, at which point it must still be spinning. We need to find out how much angular velocity is lost in this time. The inertia of the cue ball is $I = \frac{2}{5}(1)(.1)^2 = \frac{1}{250}$. We then calculate the angular acceleration using torque as $\tau = Fr = (1)(.1) = I\alpha = \frac{1}{250}\alpha$, so $\alpha = 25\,\text{rad/s}^2$. Thus, the angular velocity will decrease by 25 rad/s in the one second it takes to hit the 8-ball, so Marc must put more than 25 rad/s of topspin on the cue ball when he hits it.

29. The amount of topspin (i.e. angular velocity) ω that Marc must impart on the cue ball while hitting it to sink it after the 8-ball, *within* 2.5 *seconds of hitting the cue ball*, follows $\omega > \omega_{min}$. What is the value of ω_{min}?

A. 40 rad/s $\quad$ $\boxed{\text{B. 60 rad/s}}$ $\quad$ C. 95 rad/s $\quad$ D. 130 rad/s $\quad$ E. NOTA

Solution: Now we have to consider how much spin is left after the cue ball hits the 8-ball. From before, we know that the cue ball loses 25 rad/s in the 1 s it takes to hit the 8. Then we have 1.5 s left to make it to the pocket, which is .8+.2 = 1 m away. Let $W = \omega - 25$ be the angular velocity of the cue ball after hitting the 8. There are two distinct motions in the ensuing time: when the ball is slipping, and when it is not. We first must find what time this occurs. The angular acceleration is still $25\,\text{rad/s}^2$, and the linear $1\,\text{m/s}^2$. The cue ball stops completely after hitting the 8, so we see the time T at which the slipping stops; at which v is equal to $r\omega$. We set the equation $(1)T = (.1)(W - 25T)$. This yields $T = \frac{W}{35}$. So, for the first $\frac{W}{35}$ seconds after hitting the 8, the cue ball accelerates at $1\,\text{m/s}^2$, and then after this moves with constant velocity $at = (1)(\frac{W}{35})$. Therefore the distance it travels in 1.5 seconds is equal to $\frac{1}{2}(1)(\frac{W}{35})^2 + \frac{W}{35}(1.5 - \frac{W}{35})$ (Realize we assume here that $T < 1.5$. This is because we are seeking the minimum angular velocity. A quick check reveals that 1.5 s of constant acceleration at $1/,\text{m/s}^2$ can go further than 1 m, so we assume it stops spinning beforehand to find the actual minimum). If we set this distance equal to 1 m, we get the quadratic in W : $\frac{1}{2}(1)(\frac{W}{35})^2 + \frac{W}{35}(1.5 - \frac{W}{35}) = 1$. At this point, either by inspection, factoring, or answer-choice checking, we see that $W = 35$ is a solution, and then that the other is 70. The minimum is thus 35, but this is after losing the initial 25 rad/s; remember $W = \omega - 25$. Thus, the final answer is $35 + 25 = 60$ rad/s.

30. Which of the following is equivalent to 1 V?

A. $1\,\frac{\text{m}}{\text{s}^3\text{C}^2}$ $\quad$ B. $1\,\frac{\text{m}^2\text{kg}}{\text{sC}}$ $\quad$ C. $1\,\frac{\text{m}^2\text{kg}}{\text{s}^3\text{C}}$ $\quad$ $\boxed{\text{D. } 1\,\frac{\text{m}^2\text{kg}}{\text{s}^2\text{C}}}$ $\quad$ E. NOTA

Solution: 1 volt is one joule per coulomb. One joule is one newton-meter. One newton is one kilogram-meter per second squared, so the final answer is one kilogram-meter squared per coulomb-second squared.