For all questions, answer E) NOTA means "None Of These Answers". Any question mentioning "die/dice", "cards", or "coins" assumes that those objects are fair and standard, unless otherwise stated.

The following information is used for Questions 1-2: Mark is trying to get dressed, but the light in his room went out. He has a drawer of socks and must get a matching pair. There are 6 red socks, 6 black socks, and 6 purple socks, and he can only draw out 1 sock at a time.

1. What is the probability that Mark only needs 2 draws to get a matching pair of socks?

   A) $\frac{5}{17}$ 　　　 B) $\frac{5}{18}$ 　　　 C) $\frac{6}{17}$ 　　　 D) $\frac{6}{18}$ 　　　 E) NOTA

2. What is the expected number of draws for Mark to take to get a matching pair of socks?

   A) $\frac{99}{34}$ 　　　 B) $\frac{101}{34}$ 　　　 C) $\frac{103}{34}$ 　　　 D) $\frac{105}{34}$ 　　　 E) NOTA

3. How many ways can you get to the point (4,7) if you start at the origin and can only move one unit up or right at a time?

   A) 210 　　　 B) 252 　　　 C) 330 　　　 D) 462 　　　 E) NOTA

4. How many ways can you get to the point (4,7) if you start at the origin and can only move one unit up or right at a time and you must move through the point (1,3)?

   A) 60 　　　 B) 80 　　　 C) 140 　　　 D) 224 　　　 E) NOTA

5. How many rectangles can be made from a 4x6 grid of unit squares?

   A) 50 　　　 B) 120 　　　 C) 210 　　　 D) 240 　　　 E) NOTA

6. Alex is about to play his favorite dice-based role-playing game, Prisons and Poltergeists! He decides to make a barbarian character and needs to choose his weapons. He can wield either a 2-handed weapon, 2 1-handed weapons, or a 1-handed weapon and a shield. Given there are 4 types of 2-handed weapons, 6 types of one-handed weapons, and 3 types of shields, how many combinations does Alex have to choose from?

    A) 43   **B) 50**   **C) 58**   **D) 65**   **E) NOTA**

7. Alex ended up selecting a greataxe for his barbarian to use. Now that his character is done, he immediately gets into a fight. To hit with his greataxe, Alex needs to roll at least a 10 on his 20-sided die. However, being a barbarian, he can perform a crazy attack, which lets him roll twice and take the higher result. If the greataxe deals 9 damage on a successful attack and no damage on a failed attack, what is Alex's expected damage, to the nearest whole number?

    A) 4   **B) 5**   **C) 6**   **D) 7**   **E) NOTA**

8. A positive integer n between 1 and 12 inclusive is chosen at random. It is then squared and taken mod 13. This process repeats until there is a repeated number. What is the expected number of times the process is performed?

    A) $\frac{35}{12}$ **B) 3**   **C)** $\frac{37}{12}$ **D)** $\frac{19}{6}$ **E) NOTA**

9. Steven is playing a game, and originally he only has a 50% chance to win the game. However, every time he plays the game, his chance of losing is halved (so it goes 50%, 25%, 12.5%, . . .). What is the chance that Steven loses every one of his first 10 attempts?

    A) 2^{-10}   **B) 2^{-45}**   **C) 2^{-55}**   **D)2^{-66}**   **E) NOTA**

10. How many nonnegative integers exist with no digit being greater than or equal to the digit to its left?

    A) 510   **B) 511**   **C) 1022**   **D) 1023**   **E) NOTA**

11. How many of the integers generated in Problem 10 are divisible by 4?

    A) 255   **B) 256**   **C) 270**   **D) 288**   **E) NOTA**

12. Given that 2 people must be either friends or strangers, what is the minimum number of people required to guarantee that there is a group of 3 people who are either all mutual friends or mutual strangers?

    A) 5   **B) 6**   **C) 7**   **D) 8**   **E) NOTA**

13. You have a deck of 5 cards labeled 1 to 5. How many ways are there to rearrange the deck so that no card is in its correct spot?

    A) 42   **B) 43**   **C) 44**   **D) 45**   **E) NOTA**

14. If you roll a 30-sided die twice, what is the expected value of the higher roll to the nearest whole number?

    A) 15   **B) 17**   **C) 18**   **D) 20**   **E) NOTA**

15. 7 people are sitting at a circular table, including Jake and Arthur. However, Jake and Arthur don't want to sit next to each other. How many distinct arrangements of the 7 people don't have Jake and Arthur next to each other? 2 arrangements are considered distinct if one is not a rotation of the other.

    A) 240   **B) 480**   **C) 720**   **D) 960**   **E) NOTA**

16. Find the constant term of $(3x^2 + 2x + 4 + 1/x)^4$.

    A) 782   B) 808   C) 952   D) 1192   E) NOTA

17. Alice and Bob are playing a game of table tennis. They are playing to 7, and after 20 points, they are tied 10-10. If Bob never had more points than Alice, how many possible orders could the points have been won in?

    A) 120   B) 124   C) 128   D) 132   E) NOTA

18. What is the maximum number of distinct intersection points between 15 lines?

    A) 60   B) 75   C) 90   D) 105   E) NOTA

19. Evaluate

$$\sum_{i=0}^{9} \binom{i+3}{i}$$

    A) 286   B) 715   C) 1001   D) 1287   E) NOTA

20. Consider the set $S = \{1,2,3,4,5,6,7,8,9\}$. Let S' be a subset of S with the property that if $s \in S$ is also in S', then each multiple of s that was in S is also in S'. How many such S' exist?

    A) 48   B) 60   C) 72   D) 84   E) NOTA

21. The government is attempting to choose members for a committee. If the committee will have 5 members, and the pool from which members are drawn has 8 males and 12 females, what is the number of ways there will be more males than females on the committee?

    A) 4592   B) 4608   C) 4624   D) 4640   E) NOTA

22. If a coin is flipped 7 times, what are the chances that the first string of 2 heads in a row occurs at positions 6 and 7?

    A) $\frac{1}{16}$ B) $\frac{1}{8}$ C) $\frac{3}{16}$ D) $\frac{1}{4}$ E) NOTA

23. Lucas and Arod are taking turns throwing free throws. Lucas starts, and the first person to miss a throw loses. If Arod has a 60% chance of scoring on his free throw, and they have equal chances of winning, what are Lucas's chances of scoring on his free throw?

    A) $\frac{1}{4}$ B) $\frac{2}{5}$ C) $\frac{3}{8}$ D) $\frac{3}{5}$ E) NOTA

24. Gus is walking on the number line. If he starts at the origin, and can walk one unit right (positive) or left (negative) every second, what is the probability he is at 1 after 5 turns?

    A) $\frac{1}{8}$ B) $\frac{3}{16}$ C) $\frac{1}{4}$ D) $\frac{5}{16}$ E) NOTA

25. Which of the following pairs or events are mutually exclusive when rolling three fair 6-sided dice?

    A) Sum of positive pairwise differences of 9, at least one 3

    B) At least one pair, mean of 4

    C) Sum is less than 11, product is more than 25

    D) All 3 dice different, product of 18

    E) NOTA

26. What is the expected number of digits less than 2 in a random 6-digit number?

    A) 1   B) $\frac{10}{9}$ C) $\frac{6}{5}$ D) $\frac{5}{4}$ E) NOTA

27. A certain game has a probability p that you win $5 and probability 1-p that you lose $5. If your expected profit after 30 games is $20, what is p, assuming the different games are independent of each other?

    A) 0.55   **B) 0.6**   **C) 0.65**   **D) 0.7**   **E) NOTA**

28. My dog Luna likes to dig holes and bury bones in them. If Luna digs 5 holes and wants to put 11 bones inside them with each hole having at least one bone, how many ways can she distribute the bones?

    A) 210   **B) 280**   **C) 330**   **D) 350**   **E) NOTA**

29. My other dog Lexi isn't as picky and doesn't care if there is a bone in every hole. If she is trying to put the 11 bones in the 5 holes, how many ways can she do it?

    A) 462   **B) 1001**   **C) 1365**   **D) 4368**   **E) NOTA**

30. Carson's favorite number is 8. Jason chooses a number between $n \in \left[1, 10^{10^{420}}\right]$. Jason asks Carson what the nth digit of π is, and Carson always guesses 8. What is the probability that he is correct on any given guess?

    A) 0.1   **B) 0.2**   **C) 0.05**   **D) 0.8**   **E) NOTA**