| Question | Seat 1 A | Seat 2 B | Seat 3 C |
|---|---|---|---|
| 1 | 3 | 45 | $\frac{7}{15}$ |
| 2 | -1 | $\frac{\sqrt{2}}{2}$ | $\frac{4}{9}$ |
| 3 | 12 | 13 | $24\sqrt{3}$ |
| 4 | 9 | 4 | 4 |
| 5 | 21 | 42 | 74 |
| 6 | 40 | 14 | $\frac{4\sqrt{2}}{3}$ |
| 7 | $\frac{10}{3}$ | 10 | $648\pi$ |
| 8 | 13 | 12 | $\frac{24}{5}$ |
| 9 | 3 | $\frac{1}{2}$ | 12 |

0.Everything is zero!!

1. $$x^4-3x^3+5x^2-27x-36=(x-Ai)(x+Ai)\left(x^2+bx+c\right)=\left(x^2+A^2\right)\left(x^2+bx+c\right)$$
$$bx^3=-3x^3\to b=-3\to -3A^2=-27\to A=3$$

$$(ZU)^2=144+(ZL)^2-24(ZL)\cos 30\to (ZL)^2-12\sqrt{3}(ZL)+144-(ZU)^2=0$$
$$D>0\to 432-4\left(144-(ZU)^2\right)>0\to (ZU)^2<144$$
$$\text{Prod}>0\to (ZU)^2-36>0\to 6<ZU<12\to 7+8+9+10+11=45=B$$

$$t=\frac{d}{r}=\frac{\sqrt{81+x^2}}{27}+\frac{9-x}{45}\to dt=\frac{1}{2}\frac{\left(81+x^2\right)^{\frac{-1}{2}}(2x)}{27}-\frac{1}{45}=0\to \frac{x}{27\sqrt{81+x^2}}-\frac{1}{45}=0$$
$$C=\frac{7}{15}$$
$$5x=3\sqrt{81+x^2}\to 25x^2=729+9x^2\to x=\frac{27}{4}\to t=\frac{\sqrt{81+\frac{729}{16}}}{27}+\frac{\frac{9}{4}}{45}=\frac{5}{12}+\frac{1}{20}=\frac{7}{15}$$

2. $|z+3|=|z-1|=|z-i| \to |a-1+bi|=|a+3+bi| \to (a-1)^2+b^2=(a+3)^3+b^2 \to -2a+1=6a+9$
$a=-1 \to b=-1 \to z=-1-i \to A=-1$

$$\lim_{x\to\infty}\left(\sqrt{2(x^2+\frac{1}{2}x+\frac{1}{16})+2020-\frac{1}{8}}-\sqrt{2(x^2-\frac{1}{2}x+\frac{1}{16})-\frac{1}{8}}\right)$$

$$\lim_{x\to\infty}\left(\sqrt{2(x+\frac{1}{4})^2+2020-\frac{1}{8}}-\sqrt{2(x-\frac{1}{4})^2}\right)=\sqrt{2}\left(x+\frac{1}{4}\right)-\sqrt{2}\left(x-\frac{1}{4}\right)=\frac{\sqrt{2}}{2}=B$$

$$\int_C^1 \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx=\frac{1}{5} \to u=1+\sqrt{x} \to du=\frac{dx}{2\sqrt{x}} \to 2du=\frac{dx}{\sqrt{x}} \to \int_C^1 \frac{2}{u^2}du=\frac{-2}{u}$$

$$\left.\frac{-2}{1+\sqrt{x}}\right|_C^1=\frac{1}{5} \to -1+\frac{2}{1+\sqrt{C}} \to \frac{2}{1+\sqrt{C}}=\frac{6}{5} \to 1+\sqrt{C}=\frac{5}{3} \to C=\frac{4}{9}$$

3.
$$X^2-8X+16+Y^2+4Y+4=25 \to (X-4)^2+(Y+2)^2=25$$
$$X-4=\pm5, Y+2=0 \to X-4=\pm3, Y+2=\pm4 \to X-4=\pm4, Y+2=\pm3 \to X-4=0, Y+2=\pm5$$

12 cases: A=12

Must have passed out 12 or 10 correctly. Can't have 11 so ${}_{12}C_2+{}_{12}C_0=66+1=67 \to B=13$

$$4x^2+16-4B+By^2=4y^2 \to \frac{x^2}{B-4}+\frac{y^2}{4}=1 \to y=\frac{\sqrt{36-4x^2}}{3}=S$$

$$A=\frac{3S^2\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}\int_{-3}^{3}\left(\frac{\sqrt{36-4x^2}}{3}\right)^2dx=3\sqrt{3}\int_0^3 4-\frac{4x^2}{9}dx=3\sqrt{3}\left(4\bullet3-\frac{4(3)^3}{27}\right)=24\sqrt{3}=C$$

4. $$A=2\left[\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(3\sin\theta)^2\,d\theta-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(2-\sin\theta)^2\,d\theta\right]=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(-4\cos 2\theta+4\sin\theta)d\theta$$ A=9

$$-2\sin 2\theta-4\cos\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=3\sqrt{3}$$

$$\frac{(x_2)^2-(x_1)^2}{(x_2)-(x_1)}=9\rightarrow x_2+x_1=9\rightarrow x_2>x_1\rightarrow 8,7,6,5\rightarrow 4=B$$

$$\log_4 k+\log_{k^2}\frac{1}{8}=1\rightarrow\frac{1}{2}\log_2 k-\frac{1}{6}\log_k 2=1\rightarrow 3\log_2 k-\frac{1}{\log_2 k}=6$$

$$3(\log_2 k)^2-6\log_2 k-1=0\rightarrow\frac{6\pm4\sqrt{3}}{6}\rightarrow k=2^{\frac{6\pm4\sqrt{3}}{6}}\rightarrow 2^2=4=C$$

5. Notice the function has a zero when x = 1. Simplifying quickly leads to the other two roots, x = -2 and x = 2.5. For the problem, the only root which is of concern is x = 1. The solution is:

$$\int_{-1}^{2}\left|2x^3-3x^2-9x+10\right|dx=\int_{-1}^{1}(2x^3-3x^2-9x+10)dx+\int_{1}^{2}(-2x^3+3x^2+9x-10)dx=$$

$$\left(\frac{x^4}{2}-x^3-\frac{9x^2}{2}+10x\Big|_{-1}^{1}\right)+\left(-\frac{x^4}{2}+x^3+\frac{9x^2}{2}-10x\Big|_{1}^{2}\right)=(5-(-13))+(-2-(-5))=18+3=21.$$

Use your power of a point skills. The tangent segments are congruent. Call LW=x so WJ=x. Call UF=y then JF=y. WZ=21-x and ZF=21-y. The perimeter is 21-x+x+y+21-y=42

Draw a picture. The incenter is where the angle bisectors meet. Use alternate interior knowledge and you create some small isosceles triangles you can exploit. Call YJ=a, JX=32-a, PZ=b, and XP=42-b. The perimeter is: 32-a+a+b+42-b=74

6.
$$g\left(f(x)^2\right)=g\left(g^{-1}(80x+2020)\right)=80x+2020$$
$$2f(x)f'(x)g'\left(f(x)^2\right)=80\rightarrow f(x)f'(x)g'\left(f(x)^2\right)=40=A$$

$$a_1=2,a_2=2+2(1),a_3=2+2(1)+2(2),a_{40}=2+2(1)+2(2)+...2(39)$$
$$2+2(1+2+3+...39)=2+39(40)=1562\rightarrow 14=B$$

$$A=\frac{1}{2}(8)(14)\left(\frac{1}{3}\right)=\frac{56}{3}\rightarrow \sin 2x=2\sin x\cos x=2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)=\frac{4\sqrt{2}}{9}$$
$$A=\frac{1}{2}(8)(14)\left(\frac{4\sqrt{2}}{9}\right)=\frac{224\sqrt{2}}{9}\rightarrow \frac{\frac{224\sqrt{2}}{9}}{\frac{56}{3}}=\frac{4\sqrt{2}}{3}=C$$

7.
$$9(x^2+6x+9)+5(y^2+8y+16)=-116\rightarrow \frac{(x+3)^2}{5}+\frac{(y+4)^2}{9}=1$$
$$A=\frac{1}{2}\bullet 2c\bullet\frac{b^2}{a}=\frac{cb^2}{a}=\frac{2\bullet 5}{3}=\frac{10}{3}=A$$

$$\frac{d(y-x)}{dt}=\frac{dy}{dt}-\frac{dx}{dt}=\frac{10}{3}\rightarrow\frac{dx}{dt}=B-5\rightarrow\frac{B+5}{6}=\frac{y}{y-x}$$
$$y(B+5)-6y=(B+5)x\rightarrow y=\frac{(B+5)x}{B-1}\rightarrow\frac{dy}{dt}=\frac{(B+5)}{B-1}\frac{dx}{dt}$$
$$\frac{10}{3}=\frac{(B+5)(B-5)}{B-1}-(B-5)\rightarrow B=10$$

Draw a 2-dimensional picture with a triangle inscribed in a circle. From the center of the sphere draw 2 radii to 2 vertices with side lengths 12. This creates a 30-30-120 triangle and a 30-60-90 triangle with a hypotenuse of 12. The radius is $6\sqrt{3}$ and the height is 12+6=18. $V=\frac{1}{3}\pi\left(6\sqrt{3}\right)^2\bullet 18=648\pi$

8. Total score for both teams: 1+2+3+…+10=55. Lowest possible score is: 1+2+3+4+5=15. Winning score ranges from 15 to 27 for a total of 13 numbers possible. A=13

Shift back the graph 10 units horizontally to make the calculations easier.

$$-6x = 6x^2 - 18x \rightarrow x = 0, 2 \rightarrow \int_1^2 (-6x^2 + 12x)dx - \int_2^3 (-6x^2 + 12x)dx$$

$$-2(2)^3 + 6(2)^2 - 4 - \left[-2(3)^3 + 6(3)^2 - 8\right] = 8 - 4 + 8 = 12 = B$$

Draw a picture and set up similar triangles $\frac{6}{8} = \frac{\frac{x}{2}}{8-x} \rightarrow 48 - 6x = 4x \rightarrow x = \frac{24}{5}$ =C

9.
$$\cos^2 L + 2\cos L \cos U + \cos^2 U = 1$$

$$\sin^2 L + 2\sin L \sin U + \sin^2 U = \frac{5}{3}$$

$$2\sin L \sin U + 2\cos L \cos U = \frac{8}{3} - 2 \rightarrow \sin L \sin U + \cos L \cos U = \frac{1}{3}$$

$$\cos(L - U) = \frac{1}{3} \rightarrow \sec(L - U) = 3 = A$$

$$\int_0^B \sqrt{\frac{1}{1-t^2} + \left(\frac{1}{2} \bullet \frac{-2t}{1-t^2}\right)^2}\, dt = \int_0^B \frac{dt}{1-t^2} = \frac{1}{2}\int_0^B \left(\frac{1}{1-t} + \frac{1}{1+t}\right)dt = \ln\sqrt{3}$$

$$-\ln(1-t) + \ln(1+t) = \ln 3 \rightarrow \frac{1+t}{1-t} = 3 \rightarrow t = \frac{1}{2} = B$$

$$x^2 - 4y^2 + 10x + 24y + 25 = 0 \rightarrow x^2 + 10x + 25 - 4(y^2 - 6y + 9) = -36$$

$$(x+5)^2 - 4(y-3)^2 = -36 \rightarrow \frac{(y-3)^2}{9} - \frac{(x+5)^2}{36} = 1 \rightarrow b = 6 \rightarrow 2b = 12 = C$$