This is the Theta 3D Geometry test. E. NOTA specifies None of the Above. The questions are arranged roughly in increasing difficulty. That being said, you may wish to take a look at number 30 even if you don't get there. Good luck!

1. Find the volume of a cube with side length 2.

   A. $\sqrt[3]{2}$ B. 4 C. 8 D. 16 E. NOTA

2. Find the volume of a sphere inscribed in a cube with side length 2.

   A. $\frac{4\pi}{3}$ B. $\frac{32\pi}{3}$ C. $\frac{4\pi}{9}$ D. $\frac{32\pi}{9}$ E. NOTA

3. Find the volume of a cube inscribed in a sphere inscribed in a cube with side length 2.

   A. $\frac{8}{27}$ B. $\frac{8\sqrt{3}}{27}$ C. $\frac{8}{9}$ D. $\frac{8\sqrt{3}}{9}$ E. NOTA

4. Cone P has one-third the radius and three times the height of cone Q. Find the ratio of the volume of cone Q to cone P.

   A. 1 : 27 B. 1 : 3 C. 3 : 1 D. 27 : 1 E. NOTA

5. The centers of three spheres of radius $\sqrt{3}$ form an equilateral triangle with side length 6. Find the radius of the smallest sphere which can enclose these three spheres.

   A. $6 + \sqrt{3}$ B. $2\sqrt{3}$ C. $3\sqrt{3}$ D. $6\sqrt{3} - 3$ E. NOTA

6. DZ the fly is inside a cube of side length 4, and tied to a corner of the cube by a rope of length 3. How much volume does DZ have to fly around in?

   A. $\frac{9\pi}{2}$ B. 36π C. $\frac{63\pi}{2}$ D. $\frac{27\pi}{2}$ E. NOTA

7. Two spheres with centers A and B have radius 1. If the distance form A to B is also 1, find the area of the circle whose circumference is formed by the intersections of the two spheres.

   A. π B. $\frac{3\pi}{4}$ C. $\frac{\pi}{2}$ D. $\frac{\pi}{4}$ E. NOTA

8. A cube of side length 6 has all of its vertices truncated (sliced off) by planes such that the shortest distance from the center of the cube to each face created by the truncation is 3. Let V_n represent the polyhedron formed by n iterations of this process. Find

   $$\lim_{n\to\infty} V_n$$

   A. 27 B. 72 C. 27π D. 32π E. NOTA

9. Find the volume of a regular tetrahedron of side length 6.

   A. $18\sqrt{2}$ B. $18\sqrt{3}$ C. $24\sqrt{2}$ D. $24\sqrt{3}$ E. NOTA

10. Find the radius of a sphere inscribed in a right circular cone with radius 3 and height 4.

A. 1 B. $\frac{3}{2}$ C. $\frac{2}{3}$ D. $\frac{4}{3}$ E. NOTA

11. Find the number of edges in a convex polyhedron with 11 faces and 33 vertices.

A. 20 B. 46 C. 42 D. 24 E. NOTA

12. What is the intersection of the graphs $z^2 = x^2 + y^2$ and $x = 2$?

A. Circle B. Ellipse C. Parabola D. Hyperbola E. NOTA

13. What is the maximum number of spheres that can be put into a configuration such that each sphere is externally tangent to every other sphere?

A. 3 B. 4 C. 5 D. 6 E. NOTA

For the next three questions, consider cube $ANDYA'N'D'Y'$ which has base $ANDY$, A adjacent to A', N adjacent to N', D adjacent to D', Y adjacent to Y', and $AN = 2$.

14. Find the area of triangle ADY'.

A. $\sqrt{2}$ B. $\sqrt{3}$ C. $2\sqrt{2}$ D. $2\sqrt{3}$ E. NOTA

15. Let P be the midpoint of AA' and Q be the midpoint of DD'. Find the area of quadrilateral $PNQY'$.

A. $\sqrt{3}$ B. $\sqrt{6}$ C. $2\sqrt{3}$ D. $2\sqrt{6}$ E. NOTA

16. The plane that bisects and is perpendicular to diagonal AD' intersects the edges of the cube at six different points. Find the area of the convex polygon formed by these six points.

A. $2\sqrt{3}$ B. $2\sqrt{6}$ C. $3\sqrt{3}$ D. $3\sqrt{6}$ E. NOTA

A right cylinder has radius 2 and height 100. Plane P makes a 30° angle with the plane containing one of the cylinder's faces, and intersects the center axis of the cylinder at a point 6 units from that base, cutting the cylinder into two sections. For the next two questions, consider the smaller of those sections.

17. Find the volume of this figure.

A. 12π B. 24π C. 36π D. 48π E. NOTA

18. Find the surface area of this figure.

A. $\frac{4\pi\sqrt{3}}{3} + 24\pi$ B. $\frac{4\pi\sqrt{3}}{3} + 28\pi$ C. $\frac{8\pi\sqrt{3}}{3} + 28\pi$ D. $\frac{8\pi\sqrt{3}}{3} + 24\pi$ E. NOTA

A cube has side length 3. On each open face of this cube, a cube with $\frac{1}{3}$ the side length is attached to the center. This process is repeated for each new cube, and so on forever. (For reference, there are 7, 37, then 187 total cubes after the first 1, 2 and 3 iterations). For the next two questions, consider this fractal.

19. Find the volume of this figure.

A. $\frac{729}{22}$ B. $\frac{378}{11}$ C. $\frac{243}{7}$ D. $\frac{81}{2}$ E. NOTA

20. Find the surface area of this figure.

A. 99 B. 108 C. $\frac{441}{4}$ D. $\frac{243}{2}$ E. NOTA

For the next two questions, consider the plane which passes through the points $(3, 0, 0)$, $(0, 4, 0)$, and $(0, 0, 4)$.

21. Find the volume of the tetrahedral region bounded the first octant (positive x, y, z) and the plane.

A. 6 B. 8 C. 10 D. 12 E. NOTA

22. Find the distance from the origin to the plane.

A. $\frac{5\sqrt{34}}{17}$ B. $\frac{6\sqrt{34}}{17}$ C. $\frac{8\sqrt{34}}{17}$ D. $\frac{12\sqrt{34}}{17}$ E. NOTA

23. Find the volume of the figure formed when the region bound in the first quadrant by the equation $y = 4 - |x - 4|$ is revolved around the y-axis.

A. $\frac{256\pi}{3}$ B. 96π C. $\frac{1024\pi}{9}$ D. 128π E. NOTA

24. Three mutually tangent spheres of radius 1 are placed on a table. A sphere of radius 2 is placed on top of them so that it is tangent to each smaller sphere. Find the minimum distance between the table and the largest sphere.

A. $\frac{3}{2}$ B. $\frac{5\sqrt{3}}{3} - 1$ C. $\sqrt{5} - 1$ D. $\frac{\sqrt{69}}{3} - 1$ E. NOTA

25. A cube has two adjacent vertices at $(4, 3, 25)$ and $(4, 3, 27)$. If each of these cube's vertices are transformed by being multiplied by the matrix

$$\begin{bmatrix} 2 & 1 & 3 \\ -2 & -8 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

then what is the volume of the resulting figure?

A. 0 B. 8 C. 16 D. 64 E. NOTA

26. Tetrahedron $ABCD$ has side length 6. Sphere X has diameter AB, and sphere Y has diameter CD. Find the area of the circle whose circumference is formed by the intersections of the two spheres.

A. $\frac{27\pi}{4}$ B. $\frac{27\pi}{2}$ C. $\frac{9\pi}{4}$ D. $\frac{9\pi}{2}$ E. NOTA

27. DZ the fly has escaped his cubic prison, but finds himself tied down yet again. His new leash is of length 6, and he is tied to the center of a thin (negligible height) disc of radius 3. Assuming the disc is fixed in place, how much volume does DZ have to fly around?

A. $72\pi + 27\pi^2$ B. $72\pi + 54\pi^2$ C. $144\pi + 27\pi^2$ D. $144\pi + 54\pi^2$ E. NOTA

28. A sphere with radius 3 has diameter AB. Two planes which contain AB and intersect at an angle of 40° cut the sphere into two small and two large wedges. Find the outer surface area of one of the smaller wedges (not including the two semi-circular faces).

A. 4π B. 8π C. 32π D. 64π E. NOTA

29. A sphere with radius 3 has points A, B, and C on its surface. A bug walks along the surface of the sphere directly from A to B, then B to C, then from C back to A. The path the bug traces out is known as a spherical triangle. Given that the angles of this spherical triangle are 60° at A, 150° at B, and 90° at C, find the area of this spherical triangle. (Hint: Think about the previous problem. What is the total area of the wedges the spherical triangle would make? What is overcounted?)

A. 3π B. $3\pi\sqrt{3}$ C. 6π D. 9π E. NOTA

30. A cube is filled with water such that when its base is parallel to the ground, the water fills $\frac{1}{3}$ of the cube. The cube is then tilted such that its space diagonal is normal to the ground. How much of the cube does the water fill now?

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA