Instructions : Choose the letter of the correct answer. In all cases, E. NOTA means none of these answers.

1. If a, b, c, d, and p are distinct real numbers such that $\left(a^2+b^2+c^2\right)p^2-2(ab+bc+cd)p+\left(b^2+c^2+d^2\right)\le 0$, then a, b, c, and d are in:

   A. arithmetic progression B. geometric progression C. harmonic progression D. the Beatles E. NOTA

2. Find the equation of the line perpendicular to $y=4$ that passes through (–20, 21).

   A. $x=-20$ B. $y=-20$ C. $x=21$ D. $y=21$ E. NOTA

3. Which of the following values does **not** make $\begin{bmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{bmatrix}$ a singular matrix?

   A. –3 B. 0 C. 1 D. 2 E. NOTA

4. For what value of x does $8y+7=2y^2+3x$ have a unique solution for y?

   A. 0 B. 1 C. 3 D. 5 E. NOTA

5. If a, b, c, and d are different positive numbers in harmonic progression, which of the following inequalities must be true?

   A. $a+b>c+d$ B. $a+c>b+d$ C. $a+d>b+c$ D. $a-b>cd$ E. NOTA

6. If $z=x^y$ and $x=y^z$, then z^x is always equal to which of the following expressions?

   A. x^{y^z} B. y^{x^z} C. y^{xyz} D. x^{xyz} E. NOTA

7. How many ordered pairs (x, y) of positive integers satisfy $2x+3y=73$?

   A. 12 B. 13 C. 14 D. 15 E. NOTA

8. If $M!=(3!)(5!)(7!)$, find the value of M.

   A. 10 B. 12 C. 14 D. 16 E. NOTA

9. Four distinct random integers a, b, c, and d are chosen between 1 and 2021 inclusive. What is the probability that $a>c$ and $b>d$, given that $a>b$ and $c>d$?

   A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. NOTA

10. How many ordered triples (a, b, c) of positive integers satisfy $a+b+c=12$?

    A. 44 B. 55 C. 63 D. 72 E. NOTA

11. Find the least value of $x+y$ when $\log_2 x+\log_2 y\geq 6$.

A. 4 B. 8 C. 12 D. 16 E. NOTA

12. Find the coordinates of the orthocenter of the triangle formed by the lines $xy=0$ and $x+y=1$.

A. $\left(\frac{1}{3},\frac{1}{3}\right)$ B. $\left(\frac{1}{3},\frac{2}{3}\right)$ C. $\left(\frac{2}{3},\frac{1}{3}\right)$ D. $\left(\frac{2}{3},\frac{2}{3}\right)$ E. NOTA

13. Find the area of the triangle formed by the lines $xy=0$ and $x+y=1$.

A. 0.5 B. 1 C. 2 D. there is no such triangle E. NOTA

14. Which of the following inequalities has the same solution set as $2x>15-x^2$?

A. $|x+3|>7$ B. $|x+6|>1$ C. $|x+1|>4$ D. $|x-1|>5$ E. NOTA

15. If $2\left({}_nP_3\right)=3\left({}_{n-1}P_3\right)$, what is the value of $\frac{(n!)^2}{(n+1)!(n-1)!}$?

A. $\frac{9}{10}$ B. $\frac{10}{9}$ C. 9 D. 81 E. NOTA

16. The equation $x^2+6x-6\sqrt{x^2+6x-2}+3=0$ has four solutions: a, b, and $c\pm d\sqrt{3}$, where a, b, c, and d are all integers. Find the value of $a+b+c+d$.

A. −12 B. −10 C. −8 D. −7 E. NOTA

17. The solution set of $\begin{cases}x^2+y<0\\x^2>4-y^2\end{cases}$ lies entirely in quadrants:

A. I and II B. II and III C. I and IV D. I and III E. NOTA

18. If $\log_{3-x}\left(3x^3+3x^2-31x+27\right)=3$, find ${}_{2x+9}C_{7-2x}$.

A. 1 or 36 B. 36 or 286 C. 1 or 286 D. 1 or 36 or 286 E. NOTA

19. Real numbers x, y, and z satisfy $2^{x+y}=10$, $2^{y+z}=20$, and $2^{x+z}=30$. Find the value of 2^x.

A. $\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{5}}{2}$ C. $\frac{\sqrt{6}}{2}$ D. $\sqrt{15}$ E. NOTA

20. The sum of three non-zero real numbers is 8 times the first number, 3 times the second number, and *k* times the third number. Find the value of *k*.

A. $\frac{13}{24}$ B. $\frac{24}{35}$ C. $\frac{35}{24}$ D. $\frac{24}{13}$ E. NOTA

21. If $z = x + yi$ and $|z-1|+|z+3| \le 8$, what is the range of values of $|z-4|$?

A. $[0, 8]$ B. $[1, 9]$ C. $[3, 7]$ D. $[5, 9]$ E. NOTA

22. Which of the following equations generates the locus of points that are equidistant from (6, 0) and (0, 8)?

A. $(x-6)^2+(y-8)^2=100$ B. $(x+6)^2+(y+8)^2=100$
C. $4x+3y=24$ D. $3x-4y=-7$ E. NOTA

23. The solution set (of real numbers) for which $x^2-|x-2|+x>0$ is $(-\infty, a)\cup(b, \infty)$. Find $|ab|$.

A. $\frac{1}{2}$ B. 1 C. 2 D. 4 E. NOTA

24. Find the sum of the entries of matrix *X* if $X\begin{bmatrix}4 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}6 & 9\\ 7 & 2\end{bmatrix}$.

A. $\frac{1}{5}$ B. $\frac{12}{5}$ C. $\frac{24}{5}$ D. 48 E. NOTA

25. Find the value of *n* if $\left(10^{12}+25\right)^2-\left(10^{12}-25\right)^2=10^n$.

A. 12 B. 13 C. 14 D. 15 E. NOTA

26. There are 2021 solutions to $x^{2021}=2020x+2019$. If the solutions are $a_1, a_2, a_3, ..., a_{2021}$, find the value of $a_1^{2021}+a_2^{2021}+a_3^{2021}+...+a_{2021}^{2021}$.

A. 4,076,361 B. 4,078,389 C. 4,080,399 D. 4,084,441 E. NOTA

27. Find the area of the region bounded by $|x|\le 2$, $|y|\le 2$, and $||x|-|y||\le 1$.

A. 8 B. 10 C. 12 D. 14 E. NOTA

28. If $x^2-x=10$, then $(x-4)(x+1)(x+2)$ is:

A. rational, integral B. rational, non-integral C. irrational, positive D. irrational, negative E. NOTA

29. If $P(x)=x^3-6x^2+\ldots$ has real coefficients and a zero of $1+i\sqrt{5}$, then $P(x)=$

A. $x^3-6x^2-14x-24$ B. $x^3-6x^2+14x-24$
C. $x^3-6x^2+12x-24$ D. $x^3-6x^2+13x-24$ E. NOTA

30. Suppose *a*, *b* and *c* are in arithmetic progression and a^2, b^2, and c^2 are in geometric progression. If $a<b<c$ and $a+b+c=1.5$, what is the value of *a*?

A. $-\frac{\sqrt{2}}{4}$ B. $\frac{1}{2}-\frac{\sqrt{2}}{2}$ C. $\frac{\sqrt{2}}{4}$ D. $\frac{1}{2}+\frac{\sqrt{2}}{2}$ E. NOTA