1) Let $f(2x-1)=15x$. What is the value of $f(15)$?

A) 15 B) 120 C) 225 D) 435 E) NOTA

2) Let $f(x,y)=x^2+y^2$. What is the value of $f(\sqrt{5}-1,\sqrt{5}+1)$?

A) $2\sqrt{5}$ B) 6 C) $4\sqrt{5}$ D) 12 E) NOTA

3) The function $f(x)$ achieves its unique maximum value at (a,b), for a and b real numbers. At what point does the function $f(3x)$ achieve its unique maximum value?

A) $(\frac{a}{3},\frac{b}{3})$ B) $(\frac{a}{3},b)$ C) $(a,\frac{b}{3})$ D) (a,b) E) NOTA

4) When the function $f(x)=3x^2-12x+1$ is translated m units up and n units to the left on the Cartesian plane, it becomes the function $g(x)=3x^2$. What is the value of $m+n$?

A) -9 B) -3 C) 9 D) 13 E) NOTA

5) The quadratic function $f(x)=5x^2+nx+13$ (n a real number) can be factored as a product of two linear factors

$$f(x)=(Ax+B)(Cx+D)$$

for positive integers A,B,C,D. What is the minimum possible value of $A+B+C+D$?

A) 7 B) 15 C) 18 D) 20 E) NOTA

6) What is minimum value that the function $f(x)=4^x-2^{x+3}$ reaches on its domain (all reals)?

A) -16 B) -14 C) -12 D) -7 E) NOTA

7) The distance between the Cartesian points $(6,m)$ and $(m+5,2)$ for some real number m is 5. What is sum of all possible values of m?

A) 2 B) 3 C) 4 D) 5 E) NOTA

8) The region R in the Cartesian Plane is bounded above by $f(x) = x - 4$ and below by the arc in the fourth quadrant of $x^2 + y^2 = 16$. What is the area of R?

A) $16 - 4\pi$ B) $4\pi - 8$ C) 2π D) 8 E) NOTA

9) Let $f(x) = \dfrac{1}{x^2+1}$. For how many real values of a does the horizontal line $y = a$ intersect $f(x)$ exactly once?

A) 0 B) 1 C) 2 D) infinitely many E) NOTA

10) Let I_1 be the range of real values that k_1 can take on so that the function $f(x) = x^2 - k_1x + 1$ has no real roots and let I_2 be the range of real values that k_2 can take on so that the function $g(x) = x^2 - k_2x - 4$ has no real roots. What is $I_1 \cup I_2$?

A) $(-\infty, \infty)$ B) $(-2, 2)$ C) $(-2, -1) \cup (1, 2)$ D) $\emptyset$ E) NOTA

11) Let $r(s)$ be a function that gives the inradius of an equilateral triangle with side length s. Compute the value of $\dfrac{r(15)}{r(5)}$.

A) $\sqrt{3}$ B) 3 C) 5 D) 9 E) NOTA

12) A linear function is a function $f(x)$ satisfying

$$f(ax + by) = af(x) + bf(y)$$

for all real numbers a, b, x, y. Which of the following is an example of a linear function?

A) $f(x) = 12x + 3$
B) $f(x) = x\log(x)$
C) $f(x) = x^3$
D) $f(x) = x^2 - 4x + 4$
E) NOTA

13) Let $S = \{1, 2, 3, 4\}$. Define f by $f : S \to S$ such that f is one-to-one and maps even integers in S to odd integers in S. How many functions f exist?

A) 4 B) 6 C) 12 D) 24 E) NOTA

For questions $14 - 16$, let $f(x) = 4x^3 - 2x^2 - 1$.

14) How many real roots does $f(x)$ have?

A) 0 B) 1 C) 2 D) 3 E) NOTA

15) Most cubic functions have no inverse function, and $f(x)$ is no exception. Which of the following is a necessary and sufficient condition for a function, say $g(x)$, to have an inverse function $g^{-1}(x)$?

A) Monotonic Increasing
B) No real roots
C) One-to-One
D) Surjective
E) NOTA

16) If $f(x)$ were to have an inverse $f^{-1}(x)$, $f^{-1}(x)$ would be the reflection of $f(x)$ across the line $y = x$. What is the sum of the x coordinates of the intersection between $f(x)$ and its reflection across $y = x$?

A) -2 B) -1 C) 0 D) 1 E) NOTA

17) If the line $f(x) = 3x - 2$ intersects the ellipse $3x^2 + 2y^2 = 12$ at point (a, b) in the first quadrant, then what is the value of $12a - 4b$?

A) 4 B) $\frac{4\sqrt{57}}{7}$ C) 8 D) $\frac{8\sqrt{57}}{7}$ E) NOTA

18) Let a_k be the kth element of a sequence with first term a_1. If

$$S(n) = n^2 2^n$$

gives the sum of the first n elements of this sequence, what is the value of a_4?

A) 56 B) 100 C) 184 D) 256 E) NOTA

19) Let $f(x)$ be a cubic polynomial satisfying

$$f(-x) = -f(x)$$

for all real inputs x. Given that $f(x)$ has a root at $x = 3$, what is the maximum possible value of the sum of the remaining two (possibly non-distinct) roots of $f(x)$?

A) -6 B) -3 C) 0 D) 3 E) NOTA

20) Let $f(n) = 2n - 3$ and $g(n) = \sum_{i=0}^{n} f(i)$. If $g(n)$ is defined for only non-negative integers n, which of the following is an expression for $g(n)$ written as a polynomial in n?

A) $n^2 - 3n - 3$ B) $n^2 - 3n$ C) $n^2 - 2n - 3$ D) $n^2 - 2n$ E) NOTA

21) Let $f(x) = x^2 - 4x + 6$ and let $g(x)$ be a quadratic function in x that intersects $f(x)$ at $x = 2$. If $g(x)$ has a coefficient of -1 on its quadratic term, what is the minimum possible value of $g(3)$?

A) -3 B) -1 C) 1 D) 3 E) NOTA

22) If, for a real number a,

$$\frac{a}{a^2+1} = \frac{1}{3}$$

then compute the sum of all distinct possible values for

$$\frac{a^3}{a^6+1}$$

A) $\frac{1}{18}$ B) $\frac{1}{15}$ C) $\frac{1}{12}$ D) $\frac{1}{9}$ E) NOTA

23) If $f(x) = x^x$, then which of the following is least in value?

A) $f(\frac{1}{4})$ B) $f(\frac{1}{3})$ C) $f(\frac{1}{2})$ D) $f(1)$ E) NOTA

24) The equation

$$(x-6)(x-2)(x+1)(x+5)=60$$

has four distinct real solutions. If the greatest of these solutions is m and the least of these solutions is n, then the value $m-n=\sqrt{k}$ for some integer k. What is the value of k?

A) 129 B) 131 C) 133 D) 135 E) NOTA

25) Let

$$f(x)=\prod_{i=0}^{10}\sum_{j=0}^{i}x^i$$

Compute the coefficient of the x^{54} term of $f(x)$ when expanded completely.

A) 1 B) 5 C) 10 D) 45 E) NOTA

26) Let $f(x)=(x+3)^6$. What is the remainder when $f(49)$ is divided by 100?

A) 29 B) 43 C) 48 D) 64 E) NOTA

27) Let $f(8x-3)=16x^3-5x-3$. If

$$f(x)=ax^3+bx^2+cx+d$$

for real values a,b,c,d, then compute the value of $(a+c)(b+d)$.

A) $-\frac{16}{15}$ B) $-\frac{15}{16}$ C) $\frac{15}{16}$ D) $\frac{16}{15}$ E) NOTA

28) Let $f(x)=x^3-4x-1$. If the 3 real roots of f are a,b, and c, then compute the value of

$$\frac{a}{a^2-4}+\frac{b}{b^2-4}+\frac{c}{c^2-4}$$

A) 0 B) 4 C) 8 D) 12 E) NOTA

For questions 29 – 30, let $f(n)$ be a function defined on positive integers n such that $f(1) = 0, f(p) = 1$ for all prime numbers p, and

$$f(mn) = nf(m) + mf(n)$$

for all positive integers m and n.

29) Compute the value of $f(2021)$.

A) 70 B) 80 C) 90 D) 100 E) NOTA

30) Let

$$n = 88942644 = 2^2 3^3 7^7$$

Compute the value of $f(n)$ in terms of n.

A) $3n$ B) $12n$ C) $21n$ D) $42n$ E) NOTA