2022 MAΘ National Convention
Theta Individual Answers and Solutions

Answers:

1. D
2. A
3. B
4. B
5. C
6. C
7. B
8. C
9. C
10. C
11. A
12. B
13. D
14. D
15. D
16. A
17. B
18. D
19. D
20. B
21. A
22. D
23. B
24. C
25. D
26. A
27. B
28. B
29. C
30. B

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Solutions:

1. $20(.5)+20(1)+2x=60\rightarrow x=15\rightarrow 55\rightarrow 10$ D

2. Draw a picture and you see triangle RLZ is isosceles with angle LRZ making a measure of 36 degrees. That leaves 72 degrees in angle R. Angle WRL bisects a 36-degree angle so the answer is 18 degrees A

3. If you started listing them you would see that 3,6,9,2,5 and 8 would be counted twice and 1,4 and 7 would be counted once. Add them up and you get 78 B

4.Simplify the top piece to: $\frac{x^2+x+1}{x+1}$ plug in -1 and you get -1.5 B

5. $|x+5|=-3|x+5|+6\rightarrow |x+5|=\frac{3}{2}$. If you graph this you see that the region is a kit with diagonals of 6 and 3. And the area is ½ the product of the diagonals, so 9 C

6.Easier to figure out how many don't contain a 1 and then subtract from 9999. Break into 4 cases of 1,2,3, and 4-digit numbers respectively.
$8+8\bullet 9+8\bullet 9\bullet 9+8\bullet 9\bullet 9\bullet 9=8+72+648+5832=6560\rightarrow 9999-6560=3439$ C

7.Write out the first few terms: $a_1=0, a_2=i, a_3=-1+i, a_4=-i, a_5=-1+i$
All the odd numbered terms are going to be -1+i. So $\sqrt{1^2+1^2}=\sqrt{2}$ B

8.Call the points (P,0), (0,I), and (4,3) use your slope formula to get:
$\frac{0-3}{P-4}=\frac{I-3}{0-4}\rightarrow 12=(P-4)(I-3)\rightarrow P-4=1\rightarrow P=5$ These are the only 2 cases where 12 can
$P-4=3\rightarrow P=7$
be factored and produce the given requirement. C

9. $\frac{3}{4}\bullet\frac{4}{3}\pi r^3=\frac{1}{3}\pi r^2 h\rightarrow r=\frac{h}{3}\rightarrow\frac{r}{h}=\frac{1}{3}$ C

10.Solve for the equations of the 2 lines $\begin{array}{l} y=-2x+12 \\ y=\frac{-1}{2}x+9 \end{array}$ set y=x and solve and you get intersection points (6,6) and (4,4). The distance is $2\sqrt{2}$ C

11.Daw a picture and call the angles that are congruent x. call angle WLU y and angle ZUL w. So, x+y-w =42 and x=w+y solve the system and you get 2y=42 so y=21 A

12. Let x= side of larger square and y= side of smaller square

$$4x-4y=10\to x-y=\frac{5}{2}\to x^2-y^2=(x-y)(x+y)=100$$

$$\frac{5}{2}(x+y)=100\to x+y=40\to x=\frac{85}{4},y=\frac{75}{4}$$ B

$$4y=75\to 7+5=12$$

13.

$$\frac{n}{2}\left[2a_1+(n-1)(d)\right]=180(n-2)\to n\left[2\bullet 172+(n-1)(-4)\right]=360n-720$$

$$344n-4n^2+4n=360n-720\to 4n^2+12n-720=0\to n^2+3n-240=0$$ D

$$(n+15)(n-12)=0\to n=12$$

14.Draw a picture. Triangles LFW and WJU are isosceles triangles. Call the congruent angles in JWU x and the congruent angles in triangle LWF y. you then get

$$180-2y+180-2x+32=180\to 212=2x+2y$$

$$x+y=106$$

Our angle is 180-106=74 D

15. Draw a picture and use power of the point. Let LW=x and UF=y. That makes MF=40-y and RW=38-x. MF +RW=MR by power of the point. You then get 40-y+38-x=36 x+y=42 D

16.You can quickly see that the center of the ellipse is (6,1) this makes c=2 and b=4. The area is πab $a^2=2^2+4^2=20\to a=2\sqrt{5}\to 8\pi\sqrt{5}$ A

17.

$$x\pm 3=\left|2x+1\right|\to 2x+1=x+3\to x=2$$

$$2x+1=x-3\to x=-4$$

$$2x+1=-x+3\to x=\frac{2}{3}$$

We produced 4 solutions but 2 were extraneous so

$$2x+1=-x-3\to x=\frac{-4}{3}$$

$$2,\frac{-4}{3}$$

B

18. $(k-1)(k^2+k+1)=0 \to (k^2+k+1)=0 \to 1+k^2=-k \to 1+k=-k^2$
$(1-k+k^2)(1+k-k^2)=(-2k)(-2k^2)=4k^3=4$ D

19. All 4!! The graph of $y=\log x$ goes through quadrants I and IV. $y=\log x^2$ is a reflection across the y-axis D

20. Call the roots $r-\frac{3}{2}d, r-\frac{1}{2}d, r+\frac{1}{2}d, r+\frac{3}{2}d$ so that we can exploit some symmetry. The sum of roots 4r=0 by Vieta's formula, so r=0 that makes the roots $-\frac{3}{2}d, -\frac{1}{2}d, \frac{1}{2}d, \frac{3}{2}d$. Convert roots to factors. $\left(x-\frac{3}{2}d\right)\left(x+\frac{3}{2}d\right)\left(x-\frac{1}{2}d\right)\left(x+\frac{1}{2}d\right)=\left(x^2-\frac{9}{4}d^2\right)\left(x^2-\frac{1}{4}d^2\right)=x^4-\frac{5}{2}x^2+\frac{9}{16}d^4$ So by product of roots $\frac{9}{16}d^4=225 \to d=\pm 2\sqrt{5}$. B

21.Total distance divided by total time: $\dfrac{21+\frac{1}{4}}{\frac{5}{2}+\frac{1}{4}+\frac{1}{6}}=\dfrac{21\bullet 12+3}{30+3+2}=\dfrac{255}{35}=\dfrac{51}{7}\to 58$ A

22. $\frac{4^x}{2^{x+y}}=8 \to 2^{x-y}=2^3 \to x-y=3$
$\frac{9^{x+y}}{3^{5y}}=243 \to 3^{2x-3y}=3^5 \to 2x-3y=5$ D
$x=4, y=1 \to 4+1=5$

23. $\frac{2k-1}{k+1}-\frac{k+1}{k+1}\le 0 \to \frac{k-2}{k+1}\le 0 \to (-1,2] \to$ so 0,1, and 2 work B

24. $y=-2x+b \to y=-2x+7$
$y=\frac{1}{2}x+b \to y=\frac{1}{2}x+2 \to \frac{1}{2}x+2=-2x+7 \to x+4=-4x+14 \to 5x=10 \to x=2$
C

25. $81+(n_1-1)d=256\to(n_1-1)d=175$
$81+(n_2-1)d=256\to(n_2-1)d=63$ D
$\frac{(n_1-1)}{(n_2-1)}=\frac{175}{63}=\frac{25}{9}\to n_1=26$

26. $\frac{1+\sqrt{2k-1}}{\sqrt{k+\sqrt{2k-1}}}=x\to x^2=\frac{1+2k-1+2\sqrt{2k-1}}{k+\sqrt{2k-1}}=2\to=\sqrt{2}$ A

27.This means that they both get 0,1,2 or 3 each
$\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)+\left(\frac{3}{8}\right)\left(\frac{3}{8}\right)+\left(\frac{3}{8}\right)\left(\frac{3}{8}\right)+\left(\frac{1}{8}\right)\left(\frac{1}{8}\right)=\frac{20}{64}=\frac{5}{16}$ B

28. $\frac{9}{10}SP_s=\frac{6}{5}SP_L\to\frac{45}{60}=\frac{P_L}{P_S}=\frac{3}{4}$ B

29. $2^{12}-{}_{12}C_7=4096-792=3304$ C

30. $Z-2L-U=2$
$2Z+3L+3U=1$
$Z-L-U=3$ B
$(2,1,-2)\to1$