

**2022 Mu Alpha Theta National Convention
Theta Individual**

Choose the letter of the correct answer. In all cases, E. NOTA means “none of these answers”.

1. Lillian wants to drink 60 ml of MULU juice to help her concentration. The first 20 glasses each have 0.5 ml, the next 20 glasses are each 1 ml and, thereafter, each glass is 2 ml. Find the number of glasses that Lillian drinks altogether. What is the sum of the digits of this number?

- A. 5 B. 6 C. 9 D. 10 E. NOTA**

2. In regular pentagon MRZLU, W is the midpoint of segment LU. What is the degree measure of angle WRL?

- A. 18 B. 24 C. 36 D. 54 E. NOTA**

3. What is the sum of the unit’s digits of all the multiples of 3 between 0 and 50?

- A. 8 B. 78 C. 80 D. 84 E. NOTA**

4. Given the piece-wise function : $\left\{ \begin{array}{ll} \frac{x^3 + 1}{x^2 - 1} & x \neq 1, -1 \\ K & x = -1 \end{array} \right\}$. What value of K will make the function continuous at $x = -1$?

- A. -2 B. -1.5 C. 0 D. 1.5 E. NOTA**

5. Find the area of the quadrilateral bounded by: $y = |x + 5|$ and $y = -3|x + 5| + 6$.

- A. 3 B. 6 C. 9 D. 18 E. NOTA**

6. How many positive integers less than 10,000 contain the digit 1 at least once?

- A. 3016 B. 3430 C. 3439 D. 3447 E. NOTA**

7. Define a sequence of complex numbers by $a_1 = 0$, $a_{n+1} = a_n^2 + i$ for $n \geq 1$. In the complex plane, how far from the origin is a_{2021} ?

- A. 1 B. $\sqrt{2}$ C. $\sqrt{110}$ D. $\sqrt{2^{55}}$ E. NOTA**

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8. How many lines whose x-intercept is a positive prime number and whose y-intercept is a positive integer pass through the point (4, 3)?

- A. 0** **B. 1** **C. 2** **D. 3** **E. NOTA**

9. An ice cream cone consists of a sphere of ice cream and a right circular waffle cone. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies three-fourths of the volume of the frozen ice cream. What is the ratio of the cone's radius to its height if the cone and the sphere have the same diameter?

- A. 3:16** **B. 1:4** **C. 1:3** **D. 1:2** **E. NOTA**

10. Let $X = (0,9)$ and $Y = (0,12)$. Points W and Z are on the line $y = x$, and \overline{XW} and \overline{YZ} intersect at $K = (2,8)$. What is the length of \overline{WZ} ?

- A. 2** **B. 3** **C. $2\sqrt{2}$** **D. $3\sqrt{2}$** **E. NOTA**

11. Point W is on side \overline{ZU} of triangle ZLU such that $\angle ZWL = \angle ZLW$, and $\angle ZLU - \angle ZUL = 42^\circ$. Find the degree measure of $\angle ULW$.

- A. 21** **B. 42** **C. 48** **D. 84** **E. NOTA**

12. The perimeters of 2 squares differ by 10 units and their areas differ by 100 square units. If the perimeter of the smaller square is L , what is the sum of the digits of L ?

- A. 3** **B. 12** **C. 13** **D. 15** **E. NOTA**

13. The interior angles of a convex polygon form an arithmetic sequence with a common difference of 4 degrees. If the largest interior angle measures 172 degrees, what is the number of sides of the polygon?

- A. 7** **B. 8** **C. 9** **D. 12** **E. NOTA**

14. Let the incircle of triangle ZLU be tangent to sides \overline{LU} , \overline{ZU} , and \overline{ZL} at points W , J , F , respectively. Given that $\angle Z = 32^\circ$, find the degree measure of angle JWF .

- A. 32** **B. 58** **C. 64** **D. 74** **E. NOTA**

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15. You are given quadrilateral MRLU with $\overline{MR}=36$, $\overline{RL}=38$, $\overline{LU}=39$, and $\overline{MU}=40$. A circle inside the quadrilateral is tangent to sides \overline{MR} , \overline{RL} , and \overline{MU} . Points W and F are the tangent points on sides \overline{RL} and \overline{MU} respectively. What does $LW+UF=?$

- A. 39** **B. 40** **C. 41** **D. 42** **E. NOTA**

16. Find the area of the ellipse with a horizontal major axis, one focus at (4,1) and an endpoint of the minor axis at (6, 5).

- A. $8\pi\sqrt{5}$** **B. 12π** **C. $16\pi\sqrt{5}$** **D. 24π** **E. NOTA**

17. Find the number of solutions to the equation $|x - |2x + 1|| = 3$.

- A. 1** **B. 2** **C. 3** **D. 4** **E. NOTA**

18. If $k^3 = 1$ (k cubed equals 1) and $k \neq 1$, find the value of $(1 - k + k^2)(1 + k - k^2)$.

- A. -2** **B. 1** **C. 2** **D. 4** **E. NOTA**

19. How many quadrants does the graph of $y = \log x^2$ pass through?

- A. 1** **B. 2** **C. 3** **D. 4** **E. NOTA**

20. The zeros of the polynomial $x^4 + ax^2 + bx + 225$ are distinct real numbers that form an arithmetic sequence. What is the absolute value of the common difference in the sequence?

- A. 4** **B. $2\sqrt{5}$** **C. $\frac{\sqrt{30}}{2}$** **D. 6** **E. NOTA**

21. In Mr. Wiggle's most recent triathlon, he ran 15 miles at 6 mph, he biked 6 miles in 15 minutes, and he swam for 10 minutes at 1.5 mph. His average speed (in mph) can be represented as $\frac{J}{W}$, where J and W are relatively prime. What does $J+W=?$

- A. 58** **B. 64** **C. 117** **D. 127** **E. NOTA**

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22. If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, then what does $x + y =$?

- A. -5** **B. -3** **C. 3** **D. 5** **E. NOTA**

23. How many integral solutions to the inequality $\frac{2k-1}{k+1} \leq 1$ are there?

- A. 2** **B. 3** **C. 4** **D. infinitely many** **E. NOTA**

24. A line N has a slope of -2 and passes through the point (5, -3). A second line, M, is perpendicular to N at (L, U) and passes through the point (6, 5). What does L equal?

- A. -6** **B. 1.4** **C. 2** **D. 3** **E. NOTA**

25. An arithmetic sequence, with all integral terms has a first term of 81, a last term of 256 and another term of 144. If the common difference is not 1, how many terms does the sequence have?

- A. 12** **B. 24** **C. 25** **D. 26** **E. NOTA**

26. Simplify the expression: $\frac{1 + \sqrt{2k-1}}{\sqrt{k + \sqrt{2k-1}}}$

- A. $\sqrt{2}$** **B. 2** **C. $2\sqrt{2}$** **D. 4** **E. NOTA**

27. If Mr. Lu and J Wigs each toss 3 fair coins, what is the probability that they get the same number of tails?

- A. $\frac{1}{8}$** **B. $\frac{5}{16}$** **C. $\frac{21}{64}$** **D. $\frac{3}{8}$** **E. NOTA**

28. MuLu Juice says it will give you 20% more juice than Snow Slush for a total price that is 10% less than Snow Slush's price. What is the ratio of the unit price of MuLu Juice to the unit price of Snow Slush?

- A. $\frac{3}{5}$** **B. $\frac{3}{4}$** **C. $\frac{4}{3}$** **D. $\frac{5}{3}$** **E. NOTA**

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29. If 2 is the second entry on the second row of Pascal's triangle, find the sum of the entries in the 12th row of Pascal's triangle if you were to skip the 8th entry.

A. 2892

B. 3298

C. 3304

D. 3564

E. NOTA

30. Find the value of $Z+L+U$ if: $\begin{pmatrix} 1 & -2 & -1 \\ 2 & 3 & 3 \\ 1 & -1 & -1 \end{pmatrix} \cdot \begin{pmatrix} Z \\ L \\ U \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

A. -0.6

B. 1

C. 1.8

D. 6

E. NOTA