ANSWERS

1) C
2) A
3) B
4) E (all valid)
5) B
6) A
7) C
8) C
9) D
10) A
11) B
12) D
13) C
14) B
15) A
16) D
17) B
18) C
19) B
20) D
21) C
22) A
23) D
24) B
25) D
26) D
27) B
28) C
29) A
30) C

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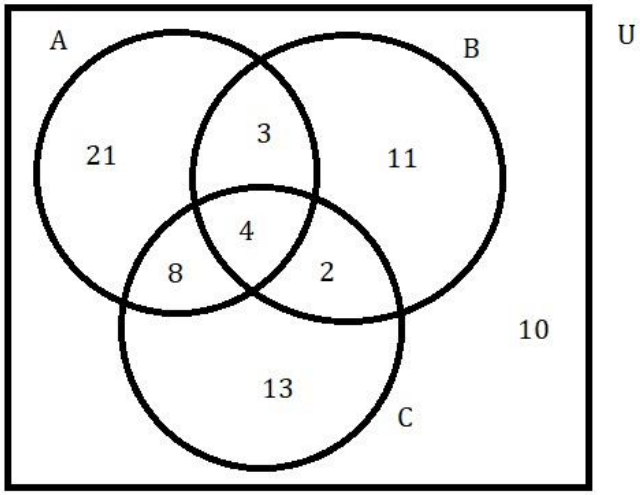
SOLUTIONS

1) Try dividing 178 by 4 to get $178 = 44 \cdot 4 + 2$. This indicates that if we pack each house with 4 people until we can't fit anymore we could only fill the first 46 houses. If we rewrite it as $178 = 43 \cdot 4 + 6$, this could have people in $43 + 6 = 49$ of the 50 houses. Therefore only 42 of the 50 houses can hold 4 people if all are to be occupied. **C**

2) Here is the Venn Diagram. 10 **A**



3) $32768 \div 15 = 2184.5\overline{3}$ = 2185, which is option **B**

4) We will examine each option. For option A, if the right combination of people older than 34 moved away, they would no longer be part of the sample so this one is valid. For option B, introducing babies would lower the average, potentially back to exactly 34, so this is a valid explanation. For option C, if the sample changes, the average may change; perhaps the new city limits include a neighborhood of very young people, lowering the average. Option D is also valid, while it is illegal to intentionally misrepresent demographic information to the Census, mistakes do occur. The only answer choice left is **E**

5) Below is the chart with another row added showing the ratio of registered voters to EC votes. The lowest ratio is in Wyoming, at 90,000. $90{,}000 \div 360 = 250 \rightarrow 2 + 5 + 0 = 7$ **B**

State	TX	CA	AL	WA	WY	NE
# Registered	16,200,008	22,040,040	3,700,008	4,860,000	270,000	1,250,000
Electoral Votes	38	55	9	12	3	5
Ratio	426,316	400,728	411,112	405,000	90,000	250,000

6) First, we see that his earnings are larger than $2 \cdot 40 \cdot 19 = 1520$, so we know he worked at least some overtime hours. Let the number of overtime hours worked be x. Then we have $1520 + 28.5x = 2118.5 \rightarrow 28.5x = 598.5$. Solving this for x gives $x = 21$ **A**

7) $3021 = 3 \cdot 19 \cdot 53 \rightarrow a = 3 + 19 + 53 = 75$. $3091 = 11 \cdot 281 \rightarrow b = 11 + 281 = 292$. Then $b - a = 292 - 75 = 217$ **C**

8) Below is the chart with an extra row added showing the area:perimeter ratio. A way to cut down on time for this is to check only the states that are close to squares, as that has a high A:P ratio. It is clear that CO wins here **C**

State	GA	SC	CO	WY	WV
Area	57,513 mi^2	30,061 mi^2	103,642 mi^2	97,093 mi^2	24,038 mi^2
Perimeter	3,288 mi	3,437 mi	1,302 mi	1,257 mi	996 mi
Ratio	17.49	8.75	79.6	77.24	24.13

9) $t = \frac{598+808+830+2260}{3} = 1498.\bar{6}$ which rounds up to 1499 hours. $\frac{1499}{24} = 62\ r\ 11$, meaning it would take 62 days and 11 hours **D**

10) Let the areas of each state be the first letter of the state name. We have the system of equations
$K + M = 47826$
$M + P = 52543$. If we add all three equations together, then divide by 6, we'll have $\frac{K+M+P}{3} = 30766.33$ option **A**
$K + P = 84229$

11) If the cities were perfect squares, then $s^2 = 2870$ and $t^2 = 778$. Then $\frac{s}{t} = \sqrt{\frac{s^2}{t^2}} = \sqrt{\frac{2870}{778}} \approx \sqrt{3.57}$. The square root of $\sqrt{3.57}$ is about 1.889. So, the interval determined by choice B is the answer. **B**

12) First, find the number of households in America: $\frac{330,000,000}{3} = 110,000,000$. One percent of those make above $500,000, meaning 1,100,000 earn above, leaving $110,000,00 - 1,100,000 = 108,900,000$ earning below **D**

13) $\underbrace{0.1 \times 9875}_{first\ bracket} + \underbrace{0.12 \times 30250}_{second\ bracket} + \underbrace{0.22 \times (75000 - 40126)}_{third\ bracket} = 987.5 + 3630 + 7672.5 = 12,290$ **C**

14) Steve's donations total $\sum_{n=0}^{35} 2^n = 2^{36} - 1$ cents. This number is upwards of 68 billion pennies, which is well above the $12,400 standard deduction, so Steve should chose the Charitable Deduction **B**

15) The Adjusted Gross Income is Total Income – Standard Deduction: $40 \cdot 50 \cdot 19.5 - 12,400 = 26,600$ The amount Owed is calculated using the table and the AGI: $0.1 \times 9875 + 0.12 \times (26,600 - 9875) = 2994.5$. The amount paid is based on the Total Income: $0.2 \times 39,000 = 7,800$. Then the amount the employee receives is the difference between Paid and Owed: $7800 - 2994.5 = 4805.50$ **A**

16) One might try shoelacing this if you're feeling like indulging in a little pain. However, the coordinates pair up so that there are 2 pairs with matching x coordinates and 2 pair with matching y coordinates, implying this is a rectangle. When we subtract the x's and y's we have a rectangle which is $311 \times 335 = 104,185$ square units **D**

17) Let the number of problems on the test be p. We need $\frac{42}{p} \geq \frac{4}{5} \rightarrow 210 \geq 4p \rightarrow p \leq 52.5$. Since p must be an integer, the possibilities for p are 48, 49, 50, 51, and 52, a total of valid p out of $75 - 48 + 1 = 28$ possible values of p. Therefore the probability is $\frac{5}{28}$ **B**

18) This quantity is $\frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{4 \cdot 3 \cdot 2 \cdot 4 \cdot 3 \cdot 2 \cdot 2} = 34{,}650$ option **C**

19) We will use Heron's formula. The semiperimeter is $s = \frac{82+128+110}{2} = 160$. Then the Area $A = \sqrt{160(160-82)(160-128)(160-110)} = \sqrt{160(78)(32)(50)} = 320\sqrt{195}$ option **B**

20) Given two choices in 32 spots, we have $\overbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}^{32\ times} = 2^{32}$. **D**

21) There are 2^{256} possible 256-bit strings. The number of guesses all the humans an make in one second is equal to $2 \cdot 2^{33} = 2^{34}$. We then divide the two $\frac{2^{256}}{2^{34}} = 2^{256-34} = 2^{222}$ **C**

22) The amount of bytes in a gigabyte is $\underbrace{2^{10}}_{kilo} \cdot \underbrace{2^{10}}_{mega} \cdot \underbrace{2^{10}}_{giga} = 1024^3$, and the number of grams in a gigagram is $\underbrace{10^3}_{kilo} \cdot \underbrace{10^3}_{mega} \cdot \underbrace{10^3}_{giga} = 1000^3$. The desired quantity is then $\frac{1024^3 - 1000^3}{24} = \frac{(1024-1000)(1024^2+1024 \cdot 1000+1000^2)}{24}$. The denominator and the first term cancel, leaving a big quadratic expression. This can be simplified by substituting $k = 1000$ to get $((k+24)^2 + k(k+24) + k^2) = 3k^2 + 72k + 576 = 3{,}072{,}576$ option **A**

23) The curve given is a logistical curve. We want to find out what happens as x gets larger. Plugging in a large $x = 1000$, we have $y = 343 + \frac{-101}{1+\frac{1}{e^{1000}}}$. The denominator of the fraction is essentially 1, and only gets closer to 1 as x grows arbitrarily large, so the limit is $y = 343 + \frac{-101}{1} = 242$ **D**

24) The minimum average would be if each of the 13 states had 2,000,000 residents. Then the population of the remaining states is $330{,}000{,}000 - 13 \times 2{,}000{,}000 = 304{,}000{,}000$. When we divide this by the 39 states in consideration we have $304{,}000{,}000 \div 39 = 7{,}794{,}872$ rounded up **B**

25) There are 9 true positives and 89 false positives meaning 98 total positive tests. Since we are only concerned with people who tested positive, this will be our total sample space. The probability is then $\frac{9}{9+89} = \frac{9}{98}$ **D**

26) This is a fun trivia question. The Lone Star State is, of course, Texas. **D**

27) First factor $330{,}000{,}000 = 2^7 3^1 5^7 11^1$. Then $330{,}000{,}000^2 = 2^{14} 3^2 5^{14} 11^2$. This number of factors of this gargantuan quantity is $(14+1)(2+1)(14+1)(2+1) = 45^2 = 2025$ **B**

28) -26.7 - 0.12 = -2.5log x where x = $\frac{b_2}{b_1}$

-26.82 = -2.5logx
10.728 = logx
1010.728 = x
5.346 x 1010 **C**

29) Plugging in the known information, we get $(0 - h)^2 + (0 - (-4))^2 = 25 \rightarrow h^2 + 16 = 25 \rightarrow$ $h^2 = 9 \rightarrow h = \pm 3$. The y-coordinate of the center must be -4 since the center lies on the line y = -4. If we allow h = 3, we get $(x - 3)^2 + (y + 4)^2 = 25$. **A**

30) A, B - are speakers at this year's convention; D - um, not female!; E - you were told this was not the answer.

C