1. What is the smallest prime that divides 703?

a) 13 b) 17 c) 29 d) 37 e) NOTA

2. Evaluate: $(1+i)^{12}$

a) $-64i$ b) $64i$ c) -64 d) 64 e) NOTA

3. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, and let X be a matrix such that $AX = B$. What is X?

a) $\begin{bmatrix} 2 & 1 \\ -5 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 4 & -5 \\ -1 & 3 \end{bmatrix}$ d) No such X exists e) NOTA

4. A Freedy Frisbee is thrown in the Argand plane at time $t = 0$. It's position in the Argand plane at a time t is described by the equation $z(t) = \left(\text{cis}\frac{3\pi}{5}\right)^t$. Where is the Freedy Frisbee at time $t = \frac{5}{4}$?

a) 1 b) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ c) $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ d) $\frac{\sqrt{3}}{2} - \frac{1}{2}i$ e) NOTA

5. A Caesar cipher is an ancient technique used to encode and decode messages. It does so by shifting all of the letters and assigning them to new ones. To see how this might be used by a computer, consider the function f that assigns to the i^{th} letter of the alphabet the number i. In other words, let $f(a) = 1, f(b) = 2, f(c) = 3$, and so on. Additionally, let f^{-1} be its inverse, i.e. f^{-1} is defined by $f^{-1}(1) = a, f^{-1}(2) = b, f^{-1}(3) = c$, and so on. To decode the message "lt gzhx" where a shift of 5 characters was used to encode it, we can find the value of $f^{-1}(f(l)-5)f^{-1}(f(t)-5) \qquad f^{-1}(f(g)-5)f^{-1}(f(z)-5)f^{-1}(f(h)-5)f^{-1}(f(x)-5)$. What is this value?

a) qy lemc b) go math c) hi zach d) ay bucs e) NOTA

6. What is the greatest common divisor of 483 and 989?

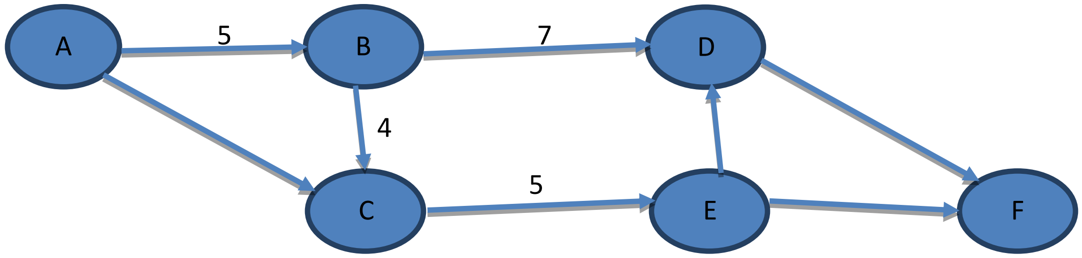
a) 7 b) 17 c) 23 d) 63 e) NOTA

7. Find a solution to the following set of equations:

$$\begin{cases} x - z = 2 \\ x + 2y - 3z = 0 \\ -x - 10y + 11z = -12 \end{cases}$$

a) No solution exists b) $x = 1, y = -3, z = 9$ c) $x = -1, y = 3, z = 9$ d) Infinitely many solutions exist e) NOTA

8. Consider the following graph, where the numbers given on the edges represent the "weight" of that edge:



We consider the weight of a path along the graph to be the sum of the numbers on the edges used in the path. For example, if I were to travel from node B to node C to node E to node D, then the weight of this path (from B to C to E to D) would be 4+5+1 = 10. Note that this is different than the direct path from B to D, which simply has weight 7. Suppose you are starting at node A. What is the least possible weight of a path that will get you to node F if you are only allowed to travel along the edges to other nodes in the direction of the arrow?

a) 10 b) 11 c) 13 d) 14 e) NOTA

9. Katie wishes to use Cramer's rule to solve for the variable x in the following system of equations:

$$\begin{cases} x + y + z = 0 \\ -10x + y - 15z = 13 \\ 22x - 10y = 15 \end{cases}$$

Which of the following correctly solves for x?

a) $\dfrac{\begin{vmatrix} 0 & 1 & 1 \\ 13 & 1 & -15 \\ 15 & -10 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ -10 & 1 & -15 \\ 22 & -10 & 0 \end{vmatrix}}$ b) $\dfrac{\begin{vmatrix} 0 & 1 & 1 \\ -10 & 13 & -15 \\ 22 & -10 & 15 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ -10 & 1 & -15 \\ 22 & -10 & 0 \end{vmatrix}}$ c) $\dfrac{\begin{vmatrix} 1 & 1 & 1 \\ -10 & 1 & -15 \\ 22 & -10 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & 1 \\ 13 & 1 & -15 \\ 15 & -10 & 0 \end{vmatrix}}$

d) $\dfrac{\begin{vmatrix} 1 & 1 & 1 \\ -10 & 1 & -15 \\ 22 & -10 & 0 \end{vmatrix}}{\begin{vmatrix} 0 & 1 & 1 \\ -10 & 13 & -15 \\ 22 & -10 & 15 \end{vmatrix}}$ e) NOTA

10. How many real solutions are there to the equation $0 = 10x^6 + 4x^4 - 13x^3 - 10x - 5$?

a) 0 b) 1 c) 2 d) 6 e) NOTA

11. How many complex solutions are there to the equation $0 = 10x^6 + 4x^4 - 13x^3 - 10x - 5$?

a) 0 b) 4 c) 5 d) 6 e) NOTA

12. For what values of θ in the interval $[0,2\pi)$ does the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ not have two unique eigenvalues? In other words, for what values of θ in this interval does the equation $\det\left(\lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\right) = 0$ not have two unique solutions for λ (in terms of θ), where det is the determinant of a matrix?

a) $0, \pi$ b) $\frac{\pi}{2}, \frac{3\pi}{2}$ c) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ d) No such values exist e) NOTA

13. Let $a_1 = 1$, and define $a_n = \frac{na_{n-1}}{n-1}$ for all $n > 1$. Which of the following is an explicit formula for a_n in terms of n?

a) 1 b) n c) $\frac{n(n+1)}{2}$ d) $(n-1)!$ e) NOTA

14. A binary search algorithm is an effective algorithm often used to find a certain element in an ordered list of numbers. To understand how this algorithm works, suppose we are searching for the number n in a list of numbers ordered from least to greatest. The algorithm starts by picking the number in the middle of the list (which we will call x) and checking to see if $x = n$. If it is, then the algorithm terminates since n has been found. Otherwise, if $x > n$ it will run the algorithm again on the list of all elements to the left of x. Since the list is ordered from least to greatest and $x > n$, this means that n must come before x, i.e. it is to the left of x. Similarly, if $x < n$, it will run the procedure again on all of the elements to the right of x. It continues this procedure until it finds n.

Suppose we are trying to use the algorithm to find the number 7 in the list below. How many numbers will we have to compare to 7 by the time 7 is found? In other words, how many steps of the algorithm will it take to find 7?

-10, -8, -7, -2, 0, 4, 7, 11, 13, 19, 20, 29, 52, 75, 128

a) 1 b) 2 c) 3 d) 4 e) NOTA

For the next two problems, suppose Zach is given the following list of 7 numbers:

7, 13, 18, 20, 25, 31, 42

Additionally, Zach is trying to find the number x, where x is in the above list.

15. If Zach chooses three arbitrary elements of the list, what is the probability that x is one of the three numbers that he has chosen?

a) $\frac{1}{7}$ b) $\frac{1}{5}$ c) $\frac{3}{7}$ d) 1 e) NOTA

16. Instead, suppose Zach applies the binary search algorithm. What is the probability that after three iterations of the binary search algorithm that he has found his number x?

a) $\frac{1}{7}$ b) $\frac{1}{5}$ c) $\frac{3}{7}$ d) 1 e) NOTA

17. If 1,1, and 2 are first three Fibonacci numbers, what is the 11th Fibonacci number?

a) 55 b) 89 c) 144 d) 233 e) NOTA

18. Find the determinant: $\begin{vmatrix} 3 & -4 & 2 \\ 0 & -1 & -2 \\ 1 & -1 & 3 \end{vmatrix}$

a) -21 b) -7 c) -5 d) 0 e) NOTA

19. Consider the following binary number: 10010100_2. What is its equivalent value in hexadecimal?

a) 74_{16} b) 94_{16} c) 98_{16} d) $B8_{16}$ e) NOTA

20. Which of the following is the partial fraction decomposition of $\frac{-1}{x^2(x-1)}$?

a) $\frac{1}{x^2} - \frac{1}{x-1}$ b) $-\frac{1}{x^2} - \frac{1}{x-1}$ c) $\frac{1}{x^2} - \frac{1}{x} - \frac{1}{x-1}$

d) $-\frac{1}{x^2} - \frac{1}{x} + \frac{1}{x-1}$ e) NOTA

21. If p is true and q is false, what is the value of $((\neg p \vee q) \wedge (p \vee \neg q)) \Rightarrow q$?

a) true b) false c) $p \Rightarrow q$ d) undefined e) NOTA

22. Define a binary operation $*$ by $a * b = a + b^{ia}$, where $i = \sqrt{-1}$. What is the value of $\pi * e$?

a) $\pi - 1$ b) $\pi + 1$ c) $e + \pi^{ie}$ d) $e - 1$ e) NOTA

23. Consider the following chart for the odd function f and the even function g:

	0	1	4	5
$f(x)$	1	4	10	12
$g(x)$	-1	0	-5	8

What is the value of $f\left(g\left(f\left(g(g(1))\right)\right)\right)$?

a) -12 b) -8 c) 8 d) 10 e) NOTA

24. Let $s(n)$ denote the sum of the digits in base 10 of the number n. For example, $s(167) = 1 + 6 + 7 = 14$. Additionally, let $s^2(n) = s\big(s(n)\big), s^3(n) = s\Big(s\big(s(n)\big)\Big)$, etc. What is the value of $s^{2015}(65536)$?
    a) 0 b) 1 c) 4 d) 25 e) NOTA

25. Evaluate: $|3 - 4i|$
    a) -5 b) -1 c) 1 d) 5 e) NOTA

26. For what value of x are the vectors $\langle 1,3,-1\rangle$ and $\langle x,2,2\rangle$ perpendicular?
    a) -6 b) -4 c) 8 d) No such value exists e) NOTA

27. What are the fewest number of coins that can be used to make $0.87 (where you are allowed to use pennies, nickels, dimes, and quarters only)?
    a) 5 b) 6 c) 7 d) 8 e) NOTA

For questions 28-30, consider the following:

Julia starts at the point (1,0) in the Cartesian plane and then moves to the point (-2,-4). After she is at this latter point, she is given instructions to find where she should travel next. Specifically, she is told that if she is currently at the point (x_1, y_1) and she was at the point (x_2, y_2) just before that, then she should travel to the point $\left(x_1 + x_2, -\sin\left(\frac{\pi(y_1 - y_2)}{4}\right)\right)$ next.

28. In how many steps will Julia reach the origin?
    a) 4 b) 6 c) 10
    d) She never reaches the origin e) NOTA

29. Evan is given the same set of instructions. However, he starts at the point (0,1) and moves to the point (0,-1) before he is given the instructions. On what point does he eventually stop? In other words, at what point do his instructions tell him for the rest of the time to return to the same point on which he is currently standing?
    a) 4 b) 6 c) 10
    d) No such point exists e) NOTA

30. Will begins at the same two points as Evan does, just in the reverse order. However, he is instead given the instructions to move to the point $\left(x_1 + x_2, \left|e^{i(y_2+y_1)}\right|\right)$. Including when he visits the points (0,-1) and (0,1), how many points does Will visit?

a) 2 b) 3 c) 6 d) Infinitely many e) NOTA