1. How many ways can Vickie, Caroline, and five of their friends sit in a circle, if Vickie and Caroline must always sit next to each other?

   a. 240 b. 120 c. 60 d. 280 e. NOTA

2. You have twelve points arranged in a circle. How many different pentagons whose vertices are from these twelve points can you make?

   a. 1282 b. 574 c. 792 d. 800 e. NOTA

3. A flying reindeer is attached by a rope to the upper corner of a cube-shaped house. The house has a side length of 40 feet, and the rope is 18 feet long. How much volume (in cubic yards) of air can the reindeer fly in?

   a. $\frac{28\pi}{3}$ b. $\frac{224\pi}{3}$ c. $2{,}400\pi$ d. 252π e. NOTA

4. Find the area of the triangle spanned by the two vectors <-1,0,5> and <0,-2,3>.

   a. $\frac{\sqrt{113}}{2}$ b. $\sqrt{49}$ c. 5.5 d. $\frac{\sqrt{129}}{2}$ e. NOTA

5. Two particles start at the same location and then begin to move away from each other at an angle of 60°. If one particle is moving at 4 m/s and the other at 7 m/s, how far apart are they after 3 seconds?

   a. 19 b. $\sqrt{837}$ c. $\sqrt{333}$ d. 23 e. NOTA

6. You kayak upstream for four miles and then go back downstream for three miles. The entire trip takes you three hours. If the upstream part of the trip takes four times as long as the downstream part of the trip, how fast is the current in mph?

   a. 2 mph b. 5/3 mph c. 2/3 mph d. 3 mph e. NOTA

7. You have a twenty liter tub. Water is flowing in at a rate of 6*t liters/minute (where t is the time in minutes), and out at a rate of 2 liters per minute. How long does it take the tub to fill up, to the nearest minute?

   a. 1 b. 2 c. 3 d. 4 e. NOTA

8. How many ways can a group of 10 people be split up into two equal teams?

   a. 252 b. 248 c. 504 d. 126 e. NOTA

9. If it is externally driven by a periodic force with a frequency that matches its natural frequency, a bridge can undergo resonance, possibly leading to its collapse. If the natural frequency of a bridge is $\frac{1}{2\pi}$, which external force would not lead to resonance?

   a. F(t) = sin(3t)+cos(4t) b. F(t) = sin(2t)+cos(4t)

   c. F(t) = sin(5t)+cos(3t) d. F(t) = sin(2t)+cos(t) e. NOTA

10. $\sum_1^{\infty} \frac{3^n}{4^{n+2}} =$

a. $\frac{3}{4}$ b. $\frac{3}{16}$ c. $\frac{1}{4}$ d. $\frac{3}{2}$ e. NOTA

11. You have a square piece of paper with side length 4. What is the volume of the largest box that you can make by cutting out squares of side length x from each corner and folding to form a box?

a. 66 b. 68 c. 70 d. 65 e. NOTA

12. Katie and Zach create a phone tree to distribute messages about how awesome they are. Assume that each phone call takes ten minutes, and people begin making calls as soon as the call notifying them ends, and that each person continues to make calls until everyone has been notified. If you start with two people making calls, and have a total of five hundred people to notify, how long will the process take?

a. 60 minutes b. 70 minutes c. 50 minutes d. 80 minutes e. NOTA

13. Sound vibrations within the ear are conducted through the basilar membrane. The basilar membrane varies in thickness, causing different parts of the membrane to vibrate at different frequencies. The frequency of vibration varies *exponentially* along the membrane (i.e. if the frequency at the middle is equal to the frequency at the bottom multiplied by 10, the frequency at the top will be equal to the frequency at the bottom multiplied by 100). If the frequency of the base is 100 hertz, and the frequency at top is 62.5 kilo-hertz, what is the frequency at a point that is 1/4 the length of the membrane from the top?

a. 32,000 Hz b. 16,500 Hz c. 625 Hz d. 12,500 Hz e. NOTA

14. What is the volume enclosed by the figure formed when the curve $(y-4)^2+(x-3)^2=1$ is rotated about the origin?

a. $20\pi^2$ b. $15\pi^2$ c. 6π d. 12π e. NOTA

15. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}^5 =$

a. $\begin{bmatrix} 32 & 0 \\ 0 & 243 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 32 \\ 243 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 120 & 0 \\ 0 & 64 \end{bmatrix}$ d. $\begin{bmatrix} 12 & 32 \\ 81 & 12 \end{bmatrix}$ e. NOTA

16. What is the volume of the largest sphere that can be inscribed in a cylinder of height 6 and base radius 3?

a. 72π b. 48π c. 36π d. 16π e. NOTA

17. Medication A will lead to side effects if it is taken when there is more than 30 mg (1 tablet) of Medication B in the body, but will only be effective if the ratio of medication B to Medication A (in mg) is at least 1:3. If Medication B has a half-life of 45 minutes and Medication A has a half-life of 1 hour, what is the greatest amount (in mg) of Medication A that can be taken at the same time as 1 tablet of Medication B so that Medication A will not lead to side effects but also be effective for at least 3 hours?

a. 45 mg b. 40 mg c. 35 mg d. 30 mg e. NOTA

18. A decagon is inscribed in a circle. How many of its diagonals do not go through the center of the circle?
    a. 35 b. 30 c. 25 d. 40 e. NOTA

19. If we list all the distinguishable permutations of PRECALC in alphabetical order, what is the 360th one?
    a. APRCCLE b. ARPLECC c. CACPREL d. CCARPEL e. NOTA

20. Catherine is playing miniature golf, and trying to get the ball through a windmill obstacle. In order to get it through the obstacle, two things must happen: 1) they must hit the ball at the right place, and 2) the ball must go through the opening of the obstacle when the blade of the windmill is not in front of it. If there is a 1/6 chance of the ball missing all of the windmill blades, and if there is a 20% probability that she will aim properly on any given shot, what is the probability that she will get the ball through the obstacle in one of the first two shots? (You can assume that the hole in the obstacle is on the ground, and that the size of the hole and the ball are negligible)
    a. $\frac{59}{900}$ b. $\frac{1}{30}$ c. $\frac{1}{900}$ d. $\frac{49}{900}$ e. NOTA

21. How many different paths are there from (0,0) to (7,10), if you can only go up and right one lattice point at a time, and you must go through (3,5)?
    a. 3620 b. 5264 c. 8204 d. 7056 e. NOTA

22. Solve this system. What is y?

$$\frac{1}{x}+\frac{2}{y}=5$$
$$\frac{3}{x}+\frac{1}{y}=-2$$

    a. 4/9 b. 2/9 c. 1/5 d. 9/2 e. NOTA

Complex numbers are very useful for analyzing circuits in physics. Let's start by having some fun with complex numbers, then we'll see how they can be useful in circuit analysis.

23. What is $\left|e^{\frac{3i\pi}{2}}\right|$?
    a. 1 b. -1 c. 0 d. ½ e. NOTA

24. What is the area contained in the graph of $|z-1|=2$ in the Argand plane?
    a. 5 b. 4π c. 9π d. 2π e. NOTA

25. Evaluate, if possible, the geometric series: $1-\frac{i}{2}-\frac{1}{4}+\frac{i}{8}+\frac{1}{16}\ldots$
    a. $\frac{4-2i}{5}$ b. $\frac{1-2i}{3}$ c. $\frac{4-i}{5}$ d. $\frac{2-2i}{3}$ e. NOTA

26. How far is the point 5-12i from the center of the Argand plane?

a. 15 b. 10 c. 13 d. 12 e. NOTA

Time for circuits! We'll start by looking at two types of circuit elements: resistors and inductors. The *impedance* of a circuit element is defined as the amount of resistance that circuit elements has to current flow. The impedance of a resistor is R (the resistance), and the impedance of an inductor is $i\omega L$, where $i = \sqrt{-1}, \omega$ =the frequency of the voltage source, and L = the inductance. When you have circuit elements in series, the sum of the individual impedances is the total impedance. One other thing you will need is a variation on Ohm's Law: the total complex current through a circuit is $I_c = \frac{max\ voltage}{total\ impedance}$.

27. You have a circuit with one resistor (resistance = 1) and one inductor ($L = \frac{\sqrt{3}}{2}$) in series, and a current source given by $voltage = 10\sin(4\pi t)$. What is the total unsimplified complex current? (don't worry about units)

a. $\frac{5}{4+i}$ b. $\frac{5}{\sqrt{3}+2i}$ c. $\frac{10}{1+2i}$ d. $\frac{10}{1+i\sqrt{3}}$ e. NOTA

28. What is the total complex current $I_C = I_0 cis\Theta$?

a. $\frac{1}{2}e^{\frac{i\pi}{6}}$ b. $5e^{\frac{3i\pi}{2}}$ c. $\frac{3}{2}e^{\frac{i\pi}{4}}$ d. $5e^{\frac{5i\pi}{3}}$ e. NOTA

29. The complex current you just found can be expressed as $I_T(t) = I_C * e^{i\omega t}$. This will be in the form $I_T(t) = I_0 e^{i(\omega t+\varphi)}$. What is φ?

a. $\frac{\pi}{6}$ b. $\frac{5\pi}{3}$ c. $\frac{\pi}{2}$ d. $\frac{7\pi}{6}$ e. NOTA

30. Obviously, current can't be imaginary! The actual current through the circuit is given by $I(t) = Re(I_T(t))$. What is $I(t)$?

a. $I(t) = 5\cos\left(\frac{\pi}{3} + 2t\right)$ b. $I(t) = 5\cos\left(\frac{5\pi}{3} + 2t\right)$

c. $I(t) = \frac{5}{2}cos(\frac{5i\pi}{2} + 4t)$ d. $I(t) = 5cos\left(\frac{5\pi}{6} + 4t\right)$

e. NOTA