0. 0 — $\cos 90°$ is part of the product, making the entire product 0.

1. $[-5,-1] \cup [5,9]$ — The graph of $\left||x-2|-5\right|$ is a W with tips at $(-3,0), (2,5)$, and $(7,0)$. The sections below the line $y=2$ are $[-5,-1] \cup [5,9]$.

2. $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ — The equation can be rewritten as

$$M\begin{bmatrix} 5 & 11 \\ 3 & 3 \end{bmatrix} + M\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$

$$M\begin{bmatrix} 7 & 11 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$

Then simply multiply both sides by the inverse of $\begin{bmatrix} 7 & 11 \\ 3 & 5 \end{bmatrix}$ on the right, to get $M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} \frac{5}{2} & -\frac{11}{2} \\ -\frac{3}{2} & \frac{7}{2} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

3. $2+\sqrt{3}$ — The 6th roots of any complex number have arguments $\frac{\pi}{3}$ apart. The 6th root next to $3+i$ is $(3+i)\left(cis\left(\frac{\pi}{3}\right)\right) = (3+i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = \left(\frac{3}{2}-\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}+\frac{3\sqrt{3}}{2}\right)i$. So $a+b = 2+\sqrt{3}$.

4. 80 — Clearly, $B=2$. Let $\theta = \arcsin\left(\frac{C}{D}\right)$. Then $A\cos(2x+\theta) = A(\cos 2x\cos\theta - \sin 2x \sin\theta)$, which means $A\cos\theta = 12$ and $A\sin\theta = 5$.
Squaring both equations and add them together, we have $A^2 = 169$, or $A = 13$. Furthermore, $\cos\theta = \frac{12}{13}$ and $\sin\theta = \frac{5}{13}$, so $\theta = \arcsin\left(\frac{5}{13}\right)$. So $A+B+CD = 13+2+(5)(13) = 80$.

5. $8\pi\sqrt{3}$ — The graph is an ellipse with eccentricity of $\frac{1}{2}$. (In general, $r = \frac{d}{1+e\cos\theta}$ is a conic section with eccentricity e.)
We can find a by looking for the points on the x-axis by plugging in 0 and π for θ, which give us the points $(2,0)$ and $(6,\pi)$. Thus $a=4, c=2, b=2\sqrt{3}$, and the area of the ellipse is $8\pi\sqrt{3}$.

6. $6\sqrt{5}$ — Consider rays extending from point A towards circle O. Each ray intersects the circle twice, except the two tangent lines from point A to the circle. Call a point of tangency B, and a different ray intersecting the circle at points C and D. We have $AB^2 = AC \cdot AD$. Thus, for any ray, the segment(s) formed have a geometric mean of the length of AB, which is also the geometric mean of all segments.
The easiest segments to consider are formed by the ray that goes through point O. The two segments have lengths 10 and 18, which has a geometric mean of $\sqrt{180} = 6\sqrt{5}$.

7. 23

Start the problem by inspecting powers of 7 mod 40. It can be seen that $7^4 \equiv 1\ (mod\ 40)$, so powers of 7 go in cycles of 4 mod 40. It remains to find the exponent (tower of 6 7s) mod 4.
7^7 leaves a remainder of 3 when divided by 4. So inspecting part of the tower, $7^{7^7} \equiv 7^3 \equiv 3\ (mod\ 4)$. The same pattern continues down the exponential tower, and the desired remainder is 7^3 mod 40, which is 23.

8. 6043

$S_n = \{1, 4, 8, 15, 26, \ldots\}$. By inspecting these first few terms of S_n, it can be seen that $S_n = A_{n+2} - 3$. Thus, the desired expression is

$$A_1 + A_2 + \sum_{k=1}^{2013} (A_{k+2} - S_k) = 1 + 3 + 2013(3) = 6043$$

9. e^3

$4^{\ln x} = \left(2^{\ln x}\right)^2$ and $x^{\ln 2} = 2^{\ln x}$, so the equation can be factored as

$$\left(2^{\ln x} - 4\right)\left(2^{\ln x} - 2\right) = 0$$

So $2^{\ln x} = 4{,}2$, or $\ln x = 2, 1$, and $x = e^2, e$. The product of the roots is e^3.

10. $\frac{5}{6}$

The probability of getting a fair coin and 3 tails in a row is $\frac{4}{5} \cdot \frac{1}{8} = \frac{1}{10}$.
The probability of getting the unfair coin and 3 tails in a row is $\frac{1}{5} \cdot 1 = \frac{1}{5}$.
So given that the coin came up 3 tails in a row, there is a $\frac{1}{3}$ probability that it is a fair coin, and $\frac{2}{3}$ probability that it is the unfair coin. Thus the probability of getting tails on the fourth flip is $\frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot 1 = \frac{5}{6}$.

11. $-\frac{5}{11}$

$a + 2, b + 2$, and $c + 2$ are the three roots of $f(x-2)$, so the expression desired is the sum of the reciprocals of the roots of $f(x-2)$.
$f(x-2) = x^3 - 6x^2 + 5x + 11$, so the desired sum is $-\frac{5}{11}$.

12. $\left(-\infty, -\sqrt{2}\right] \cup [0,0] \cup \left[\sqrt{2}, \infty\right)$

For f to be real-valued, $x^2 - 1 \neq 0 \Rightarrow x \neq \pm 1$, and additionally, $\ln\left(\left|x^2 - 1\right|\right) \geq 0 \Rightarrow \left|x^2 - 1\right| \geq 1 \Rightarrow x^2 - 1 \geq 1$ or $x^2 - 1 \leq -1 \Rightarrow x^2 \geq 2$ or $x^2 \leq 0$. The solution to the first inequality is $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$ while 0 is the only solution to the second inequality. The domain, therefore, is $\left(-\infty, -\sqrt{2}\right] \cup [0,0] \cup \left[\sqrt{2}, \infty\right)$.