1. Simplify: $|-3+4i|$

   a. $\sqrt{7}i$ b. $5i$ c. 1 d. 5 e. NOTA

2. Simplify: $e^{\frac{2i-\pi}{2i}}$

   a. $e-i$ b. $e+i$ c. $-ei$ d. ei e. NOTA

3. What is the product of the non-real roots of $f(x)=x^6-1$?

   a. -1 b. 1 c. $\text{cis}\frac{\pi}{10}$

   d. There are no non-real roots of $f(x)$ e. NOTA

4. Let z be a complex number such that $z^2=-12+16i$ and $|Re(z)|=2$. What is $z\cdot\bar{z}$, where $\bar{z}$ is the complex conjugate of z?

   a. 4 b. 12 c. 16 d. 20 e. NOTA

5. If $f(i^3z)=z^2-e^z+\cosh(z)$, where $\cosh(x)=\frac{e^x+e^{-x}}{2}$, what is $f(z)$? (Hint: $\sin(x)=\frac{e^{ix}-e^{-ix}}{2i}$ and $\cos(x)=\frac{e^{ix}+e^{-ix}}{2}$)

   a. z^2-e^z b. $-z^2-\cos(iz)$ c. $-z^2-i\sin(z)$

   d. $-z^2+i\sin(z)$ e. NOTA

6. Simplify: $\frac{1}{2+i}+\frac{1}{2-i}$

   a. $-\frac{2i}{5}$ b. $-\frac{2i}{3}$ c. $\frac{4}{3}$ d. $\frac{4}{5}$ e. NOTA

7. Which of the following is true?

   a. The product of two purely imaginary numbers is a non-positive, real number.

   b. The sum of two complex numbers is not necessarily complex.

   c. The difference of two complex numbers is never an integer.

   d. The absolute value of a complex number is a positive, real number.

   e. NOTA

8. Simplify: $(1+i)^{11}$

   a. -32 b. $-32+32i$ c. $32-32i$ d. $32i$ e. NOTA

9. Given that $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$, what is the value of $\cosh^2(x) - \sinh^2(x)$?

a. 1 b. $\cosh 2x$ c. $2\cosh^2 x - 1$ d. $1 - 2\sinh^2 x$ e. NOTA

10. Simplify: $\left|e^{\ln 5 + \frac{16}{\pi i}}\right|$

a. 5 b. $\frac{80}{\pi}$ c. $\sqrt{25 - \frac{256}{\pi^2}}$ d. $\sqrt{25 + \frac{256}{\pi^2}}$ e. NOTA

11. Let $f(z) = |z|$. Which of the following is true for all complex numbers z?

i) $f(z)$ is a complex number ii) $f(-z) = -f(z)$

iii) $f(\bar{z}) = \overline{f(z)}$ iv) $f(iz) = i\,f(z)$

a. i b. iii c. i, iii, iv d. ii, iii, iv e. NOTA

12. Simplify: $\prod_{n=1}^{2015} i^{n!}$

a. $-i$ b. i c. -1 d. 1 e. NOTA

13. Katie and Zach are playing a game. They both pick a complex number on the unit circle in the Argand plane. If the distance between the numbers in the plane is greater than 1, Katie wins. If the distance is less than 1, Zach wins. If it is equal to 1, it is a draw. Katie picks $\text{cis}\left(\frac{\pi}{4}\right)$, and Zach picks $\text{cis}\left(-\frac{\pi}{4}\right)$. Who wins?

a. Katie b. Zach c. It's a draw

d. They did not both choose valid numbers e. NOTA

14. Which of the following is equal to π^i?

a. -1 b. πi c. i^π d. $\text{cis}(\ln \pi)$ e. NOTA

15. Find the determinant: $\begin{vmatrix} \text{cis}\left(\frac{5\pi}{32}\right) & i^{16777218} \\ e^{i\pi} & \text{cis}\left(\frac{27\pi}{32}\right) \end{vmatrix}$

a. -2 b. 0 c. $\cos\frac{135\pi}{1024} - 1$ d. $\cos\frac{135\pi}{1024} + 1$ e. NOTA

16. Consider the circle in the Argand plane that goes through the points $-1+i, 1-i$, and $\sqrt{2}$. What is the area enclosed by the circle?

a. No such circle exists b. 2π c. $2\pi\sqrt{2}$

d. 4π e. NOTA

17. Consider the n-gon created by connecting the nth roots of unity, i.e. the n complex roots of $x^n - 1$, in counter-clockwise order in the Argand plane. Let P_n be the perimeter of this polygon. What is P_4?

a. $\frac{1}{2}$ b. 1 c. 2 d. 4 e. NOTA

18. Simplify: $i^{2^{2015}}$

a. $-i$ b. i c. -1 d. 1 e. NOTA

For problems 19-21, let φ and θ be functions such that $\varphi(x+yi) = \begin{bmatrix} x & y \\ -y & x \end{bmatrix}$, $\theta\left(\begin{bmatrix} x & y \\ -y & x \end{bmatrix}\right) = x+yi$, where x and y are real numbers. Additionally, let $a, b, c,$ and d be real numbers.

19. Which of the following is equivalent to $\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} c & d \\ -d & c \end{bmatrix}\right)$?

a. $\begin{bmatrix} a+c & b+d \\ -b-d & a+c \end{bmatrix}$ b. $\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right) + \theta\left(\begin{bmatrix} c & d \\ -d & c \end{bmatrix}\right)$

c. $\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right)\theta\left(\begin{bmatrix} c & d \\ -d & c \end{bmatrix}\right)$ d. $(ac-bd)+(ad+bc)i$

e. NOTA

20. Which of the following is equivalent to $\varphi(a+bi)\varphi(c+di)$?

a. $\begin{bmatrix} a+c & b+d \\ -b-d & a+c \end{bmatrix}$ b. $\varphi(a+bi)+\varphi(c+di)$

c. $\varphi\big((a+bi)(c+di)\big)$ d. $(ac-bd)+(ad+bc)i$

e. NOTA

21. Which of the following is true?

i) $\varphi\left(\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right)\right) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ ii) $\theta\big(\varphi(a+bi)\big) = a+bi$

iii) If $\varphi(a+bi) = \varphi(c+di)$, then $a=c$ and $b=d$.

a. i b. ii c. iii d. i, ii e. NOTA

22. Simplify: $\prod_{i=1}^{2015}\left(\operatorname{cis}\left(\frac{\pi}{2015}\right)\right)^i$

a. -1 b. $-\frac{1}{2}$ c. 0 d. 1 e. NOTA

23. If $\log_2(\log_3(x)) = 2$, what is i^x?

a. $-i$ b. i c. -1 d. 1 e. NOTA

24. What is the sum of the series $e^{\frac{i\pi}{3}} + e^{-\ln 2+\frac{i\pi}{3}} + e^{-2\ln 2+\frac{i\pi}{3}} + e^{-3\ln 2+\frac{i\pi}{3}} + \cdots$?

a. $1+\sqrt{3}i$ b. $\sqrt{3}+i$ c. $e^{\frac{\pi}{3}}$

d. The sum does not converge e. NOTA

25. What is the area inside the graph of the equation $|z-30+40i| = 10$ in the Argand plane?

a. 0 b. 100π c. 400 d. 2500π e. NOTA

26. Which of the following numbers is equal to i^i?

a. $-\operatorname{cis} i$ b. $\operatorname{cis} 1$ c. $e^{-\frac{\pi}{2}}$ d. $e^{\frac{\pi}{2}}$ e. NOTA

27. Let $\varphi(a,b) = a+bi\sqrt{5}$, where a and b are real numbers. Which of the following is true for any real numbers $a, b, c,$ and d?

a. $\varphi(a,b)\varphi(c,d) < \varphi(ac-5bd, ad+bc)$

b. $\varphi(a,b)\varphi(c,d) = \varphi(ac-5bd, ad+bc)$

c. $\varphi(a,b)\varphi(c,d) > \varphi(ac-5bd, ad+bc)$

d. $\varphi(a,b)+\varphi(c,d) = \varphi(ac-5bd, ad+bc)$

e. NOTA

28. Which of the following is a sixth root of unity?

a. $-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ b. $\frac{1}{2}-\frac{\sqrt{3}}{2}i$ c. $-i$

d. $-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$ e. NOTA

29. The following sequence is what type of sequence?

$$\log_2(3^i), \log_4(3^i), \log_8(3^i), \log_{16}(3^i), \ldots$$

a. Arithmetic b. Geometric c. Harmonic

d. Oscillating e. NOTA

30. What is the imaginary part of $i^{i^{2014}}+e^{\frac{i\pi}{2}}+\left|e^{83i}+12\pi i-ei\right|$?

a. -2 b. 2 c. $i\sqrt{\pi}$ d. $\ln\pi$ e. NOTA