1. D – $|-3+4i| = \sqrt{(-3)^2+(4)^2} = 5$
2. D – $e^{\frac{2i-\pi}{2i}} = e^{\frac{2i}{2i}-\frac{\pi}{2i}} = e^{1-\frac{\pi i^3}{2}} = e \cdot e^{\frac{\pi i}{2}} = e \cdot \text{cis}\frac{\pi}{2} = e \cdot i$
3. B – The product of all of the roots of $f(x)$ is $(-1)^6\left(\frac{-1}{1}\right) = -1$. However, note that $x^6-1 = (x^3-1)(x^3+1) = (x-1)(x^2+x+1)(x+1)(x^2-x+1)$. Since x^2+x+1 and x^2-x+1 do not have any real roots (since the discriminant of both polynomials is -3), ± 1 are the only real roots of $f(x)$. Therefore, the product of the real roots is -1, so the product of the complex roots is $\frac{-1}{-1} = 1$.
4. D – Let $z = a+bi$ for real numbers a and b. Then $-12+16i = z^2 = (a+bi)^2 = a^2+2abi+b^2i^2 = (a^2-b^2)+2abi$, so $a^2-b^2 = -12 \Longrightarrow b^2 = a^2 + 12$. Additionally, $|a| = |Re(z)| = 2$, so $a^2 = |a|^2 = 4$. Therefore, $b^2 = a^2+12 = 4+12 = 16$. As a result, we see that $z \cdot \bar{z} = (a+bi)(a-bi) = a^2+b^2 = 4+16 = 20$.
5. C – Note that $f(z) = f(i^4 z)$

$$\begin{aligned} &= f\left(i^3(iz)\right) \\ &= (iz)^2 - e^{iz} + \cosh(iz) \\ &= i^2z^2 - \cos(z) - i\sin(z) + \frac{e^{iz}+e^{-iz}}{2} \\ &= -z^2 - \cos(z) - i\sin(z) + \cos(z) \\ &= -z^2 - i\sin(z) \end{aligned}$$

6. D – $\frac{1}{2+i} + \frac{1}{2-i} = \frac{2-i}{(2+i)(2-i)} + \frac{2+i}{(2-i)(2+i)} = \frac{(2-i)+(2+i)}{(2-i)(2+i)} = \frac{4}{2^2-i^2} = \frac{4}{5}$
7. E – a) $-i$ and i are purely imaginary numbers, but $(-i)(i) = -i^2 = 1$. Therefore, (a) is false.

   b) For any complex numbers $a+bi$ and $c+di$, $(a+bi)+(c+di) = (a+c)+(b+d)i$ is a complex number (note that even if $b+d=0$, $(a+c)+(b+d)i$ is still complex). Therefore, (b) is false.

   c) i is a complex number and $i - i = 0$ is an integer. Therefore, (c) is false.

   d) 0 is a complex number, and $|0| = 0$. Therefore, (d) is false.
8. B – Since $(1+i)^2 = 1+2i+i^2 = 2i$, $(1+i)^{11} = [(1+i)^2]^5(1+i) = (2i)^5(1+i) = 32i^5(1+i) = 32i^5+32i^6 = 32i-32$
9. A – $\cosh^2(x) - \sinh^2(x) = \left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$

$$= \frac{e^{2x}+2+e^{-2x}-(e^{2x}-2+e^{-2x})}{4}$$

$= 1$

10. A – First, note that for any real number z,

$\left|e^{iz}\right| = |\cos z + i\sin z| = \sqrt{(\cos z)^2 + (\sin z)^2} = 1$. Therefore, $\left|e^{\ln 5 + \frac{16}{\pi i}}\right| =$

$\left|e^{\ln 5} e^{\frac{16 i^3}{\pi}}\right| = \left|5e^{-\frac{16}{\pi}i}\right| = 5\left|e^{-\frac{16}{\pi}i}\right| = 5.$

11. E – i) The absolute value of a complex number is a real number, and all real numbers are complex. Therefore, (i) is true.

ii) $f(-1) = 1 \neq -1 = -f(1)$, so (ii) is false.

iii) Let $z = a + bi$. Then $f(\bar{z}) = f(a - bi) = |a - bi| = \sqrt{a^2 + b^2} = f(z)$. Since $f(z)$ is a real number, $f(z) = \overline{f(z)}$, so $f(\bar{z}) = \overline{f(z)}$. Therefore, (iii) is true.

iv) $f(i) = 1 \neq i = if(1)$, so (iv) is false.

12. B – For $n \geq 4$, 4 divides $n!$, so $i^{n!} = 1$. Therefore, $\prod_{n=1}^{2015} i^{n!} = \prod_{n=1}^{3} i^{n!} \prod_{n=4}^{2015} i^{n!} = \prod_{n=1}^{3} i^{n!} \prod_{n=4}^{2015} 1 = ii^2 i^6 = i^9 = i.$

13. A – Note that Katie's number is $\operatorname{cis}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ and Zach's number is $\operatorname{cis}\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$. Graphing this in the Argand plane, it is easy to see that the distance is $2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2} > 1$. Alternatively, the distance between two complex numbers $a + bi$ and $c + di$ is $|(a - c) + (b - d)i|$, so the distance between Zach and Katie's numbers is $\left|\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)i\right| = \left|i\sqrt{2}\right| = \sqrt{2}.$

14. D – $\pi^i = \left(e^{\ln \pi}\right)^i = e^{i \ln \pi} = \operatorname{cis}(\ln \pi)$

15. A – First, note that $\left(\operatorname{cis}\frac{5\pi}{32}\right)\left(\operatorname{cis}\frac{27\pi}{32}\right) = \operatorname{cis}\left(\frac{5\pi}{32} + \frac{27\pi}{32}\right) = \operatorname{cis}\pi = -1$. Also, $e^{i\pi} = -1$ and $i^{16777218} = i^2 = -1$ since 16777218 has remainder 2 when divided by 4.

Therefore, $\begin{vmatrix} \operatorname{cis}\left(\frac{5\pi}{32}\right) & i^{16777218} \\ e^{i\pi} & \operatorname{cis}\left(\frac{27\pi}{32}\right) \end{vmatrix} = \left(\operatorname{cis}\frac{5\pi}{32}\right)\left(\operatorname{cis}\frac{27\pi}{32}\right) - e^{i\pi}\, i^{16777218} = -1 - 1 = -2.$

16. B – Note that $|-1 + i|^2 = |1 - i|^2 = \left|\sqrt{2}\right|^2 = 2$. Therefore, the points all lie on the circle $|z|^2 = 2$, which has area 2π.

17. E – The roots of $x^4 - 1$ are $\pm i$ and ± 1. Viewing this in the Argand plane, we see that it is a square with side length $\sqrt{2}$ (the Argand plane is constructed with the real line as the x-axis and the purely imaginary numbers listed on the y-axis).

18. D – Since $2^{2015} = 4 \cdot 2^{2013}$, $i^{2^{2015}} = i^{4 \cdot 2^{2013}} = (i^4)^{2^{2013}} = 1^{2^{2013}} = 1.$

19. B – By the definition of θ,

$$\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} c & d \\ -d & c \end{bmatrix}\right) = \theta\left(\begin{bmatrix} a+c & b+d \\ -b-d & a+c \end{bmatrix}\right)$$
$$= \theta\left(\begin{bmatrix} a+c & b+d \\ -(b+d) & a+c \end{bmatrix}\right)$$
$$= (a+c) + (b+d)i$$
$$= (a+bi) + (c+di)$$
$$= \theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right) + \theta\left(\begin{bmatrix} c & d \\ -d & c \end{bmatrix}\right)$$

20. C – By the definition of φ,

$$\varphi(a+bi)\varphi(c+di) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} c & d \\ -d & c \end{bmatrix}$$
$$= \begin{bmatrix} ac-bd & ad+bc \\ -bc-ad & -bd+ac \end{bmatrix}$$
$$= \begin{bmatrix} ac-bd & ad+bc \\ -(ad+bc) & ac-bd \end{bmatrix}$$
$$= \varphi\big((ac-bd) + (ad+bc)i\big)$$
$$= \varphi\big((a+bi)(c+di)\big)$$

21. E – i) By the definitions of φ and θ,

$\varphi\left(\theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right)\right) = \varphi(a+bi) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. Therefore, (i) is true.

ii) By the definitions of φ and θ,

$\theta\big(\varphi(a+bi)\big) = \theta\left(\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\right) = a+bi$. Therefore, (ii) is true.

iii) Suppose $\varphi(a+bi) = \varphi(c+di)$. By the definition of φ, this means that $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} = \begin{bmatrix} c & d \\ -d & c \end{bmatrix}$. Therefore, setting the elements of the matrix equal, we see that $a = c$ and $b = d$. Therefore, (iii) is true.

22. D – $\prod_{i=1}^{2015}\left(\text{cis}\left(\frac{\pi}{2015}\right)\right)^i = \left(\text{cis}\left(\frac{\pi}{2015}\right)\right)^{\sum_{i=1}^{2015} i}$

$$= \left(\text{cis}\left(\frac{\pi}{2015}\right)\right)^{\frac{2015(2016)}{2}}$$
$$= \left(\text{cis}\left(\frac{2015(2016)}{2}\cdot\frac{\pi}{2015}\right)\right)$$
$$= \text{cis}(2008\pi)$$
$$= 1$$

23. B – Since $\log_2(\log_3(x)) = 2$, $\log_3 x = 2^2 = 4$, so $x = 3^4 = 81$. Therefore, $i^{81} = i^{4(20)+1} = i$.

24. A – First, note that we can the sum is equivalent to $e^{\frac{i\pi}{3}} \cdot \left(1 + e^{-\ln 2} + e^{-2\ln 2} + \cdots\right)$. Therefore, we will first find the sum $1 + e^{-\ln 2} + e^{-2\ln 2} + \cdots$. Note that for any integer j, $e^{-j\ln 2} = \left(e^{\ln 2}\right)^{-j} = 2^{-j}$. Therefore, $1 + e^{-\ln 2} + e^{-2\ln 2} + \cdots = 1 + 2^{-1} + 2^{-2} + \cdots = \frac{1}{1-\frac{1}{2}} = 2$. Therefore, the original sum equals $2e^{\frac{i\pi}{3}} = 2\operatorname{cis}\frac{\pi}{3} = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 1 + \sqrt{3}i$.
25. B – Note that $|z - 30 + 40i| = 10$ is the graph of a circle with radius 10 in the Argand plane. To see this, let $z = x + yi$, where x and y are real numbers. Then

$$|z - 30 + 40i| = 10$$
$$\Leftrightarrow |x + yi - 30 + 40i| = 10$$
$$\Leftrightarrow |(x - 30) + (y + 40)i| = 10$$
$$\Leftrightarrow \sqrt{(x - 30)^2 + (y + 40)^2} = 10$$
$$\Leftrightarrow (x - 30)^2 + (y + 40)^2 = 100$$

Therefore, the area inside the circle is 100π.
26. C – Since $e^{\frac{i\pi}{2}} = \operatorname{cis}\frac{\pi}{2} = i$, $i^i = \left(e^{\frac{i\pi}{2}}\right)^i = e^{\frac{i^2\pi}{2}} = e^{-\frac{\pi}{2}}$.
27. B – $\varphi(a,b)\varphi(c,d) = \left(a + bi\sqrt{5}\right)\left(c + di\sqrt{5}\right)$

$$= ac + bci\sqrt{5} + adi\sqrt{5} + bdi^2\left(\sqrt{5}\right)^2$$
$$= (ac - 5bd) + (bc + ad)i\sqrt{5}$$
$$= \varphi(ac - 5bd, bc + ad)$$

28. B – A sixth root of unity is a root of $x^6 - 1$. Note that $\frac{1}{2} - \frac{\sqrt{3}}{2}i = \operatorname{cis}\frac{5\pi}{3}$, so $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^6 - 1 = \left(\operatorname{cis}\frac{5\pi}{3}\right)^6 - 1 = \operatorname{cis}\left(6 \cdot \frac{5\pi}{3}\right) - 1 = \operatorname{cis} 10\pi - 1 = 1 - 1 = 0$. Therefore, $\frac{1}{2} - \frac{\sqrt{3}}{2}i$ is a sixth root of unity.
29. C – The n^{th} term of the sequence is $\log_{2^n} 3^i = \frac{1}{n}\log_2 3^i$, which is a harmonic sequence. Specifically, it is the harmonic sequence $\frac{1}{n}$, multiplied by the constant $\log_2 3^i$.
30. E – Since the absolute value of a complex number is a real number, we only need to worry about finding the imaginary part of $i^{i^{2014}} + e^{\frac{i\pi}{2}}$. Since $i^{2014} = -1$, $i^{i^{2014}} = i^{-1} = i^3 = -i$. Also, $e^{\frac{i\pi}{2}} = i$, so $i^{i^{2014}} + e^{\frac{i\pi}{2}} = -i + i = 0$. Therefore, the imaginary part is 0.