For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Simplify $(1 + i)^{20} - (1 - i)^{18}$.

A) $-1024 + 512i$ B) $-1024 - 512i$ C) $1024 - 512i$ D) $1024 + 512i$ E) NOTA

2. If $a = 1 + i\sqrt{3}$ and $b = \cos 15° - i \sin 15°$, find $\frac{a}{b}$.

A) $2\text{cis}(75°)$ B) $\text{cis}(75°)$ C) $2\text{cis}(45°)$ D) $\text{cis}(45°)$ E) NOTA

3. Define a sequence for each natural number n by $a_{n+1} = a_n^2 + i$ where $a_1 = 0$. What is $|a_{2016}|$?

A) $\sqrt{2}$ B) 1 C) $12\sqrt{14}$ D) $\sqrt{2015}$ E) NOTA

4. Let x be a complex number and let m be a positive integer. If $x + \frac{1}{x} = 2\cos\theta$ where $0 < \theta < \pi$, find $x^m + \frac{1}{x^m}$.

A) $2^m \cos\theta$ B) $2(\cos\theta)^m$ C) $2^m(\cos\theta)^m$ D) $2\cos(m\theta)$ E) NOTA

5. Let $A = 1 + i\sqrt{3}$ and $B = -1 - i\sqrt{3}$. Which of the following values of C would make $\triangle ABC$ a right triangle in the Argand plane?

A) $-1.2 - 1.6i$ B) $1.25 - 1.75i$ C) $-1 + i$ D) $1.5 + 1.5i$ E) NOTA

6. If $c = 7 - 12i$, what is $\text{Re}(c) + \text{Im}(c) - \bar{c} + |c|^2$?

A) $188 - 12i$ B) $181 + 12i$ C) $181 - 12i$ D) $205 + 12i$ E) NOTA

7. What is the sum of the non-real solutions to the equation $x^{2016} - 1 = 0$?

A) 1 B) $-i$ C) i D) 0 E) NOTA

8. Write the following expression in $a + bi$ form:
$$ie^{i\frac{\pi}{6}}$$

A) $-\frac{\sqrt{3}}{2} + \frac{1}{2}i$ B) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ C) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ D) $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$ E) NOTA

9. What is the coefficient of y in the expansion of $(y + 1 + i)^{11}$?

A) $11i$ B) -11264 C) $32 + 320i$ D) $352i$ E) NOTA

10. Let x, y, and h denote the side lengths of a right triangle with a hypotenuse of length h. If $z = ax + ayi$, where $a > 0$, what is $z \cdot \bar{z}$ in terms of a and h?

A) ah^2 B) a^2h C) ah D) a^2h^2 E) NOTA

11. Find the magnitude of $\sqrt{-\frac{1}{10} - i}$.

A) $\frac{10-\sqrt{10}}{10}$ B) $\frac{11-2\sqrt{10}}{10}$ C) $\frac{9}{10}$ D) $\frac{\sqrt{10}-10}{10}$ E) NOTA

12. Evaluate the following expression over the complex numbers, where $\ln(-a) = \ln a + \pi i$ for positive a:

$$\sum_{n=0}^{101} \ln[(-e)^n]$$

A) 5151 B) $101! + \pi i$ C) $5151(\pi i + 1)$ D) $101i$ E) NOTA

13. Let x and y be non-real, complex numbers such that $x^2 = y$, $y^2 = x$, and $x \neq y$. What is $x + y$?

A) i B) $-\frac{1}{2}$ C) 1 D) -1 E) NOTA

14. Find the determinant of the following matrix:
$$\begin{bmatrix} 316 - 122i & |316 - 122i| \\ |316 - 122i| & 316 + 122i \end{bmatrix}$$

A) 0 B) $316 - 122i$ C) $(316 - 122i)^2$ D) 1 E) NOTA

15. Simplify $(-1 - i^{-1})^{-1} \cdot i^{-1}$.

A) $-\frac{1}{2} - \frac{1}{2}i$ B) $\frac{1}{2} - \frac{1}{2}i$ C) $-1 + i$ D) $-\frac{1}{2} + \frac{1}{2}i$ E) NOTA

16. Let $f(z) = \frac{z^2}{|z|^2}$. Which of the following is equivalent to $f(121 - 144i)$?

A) $11 - 12i$ B) $\frac{121+144i}{121-144i}$ C) $\frac{121-144i}{121+144i}$ D) $\frac{11+12i}{11-12i}$ E) NOTA

17. Randy is throwing darts at his imaginary dartboard on the Argand plane. If all of his darts hit at random points within the region $\pi|z| - 5\sqrt{\pi} \le 0$, what is the probability of a dart hitting in the region defined by $|Re(z)| \le 1$ and $|Im(z)| \le 1$?

A) 16%   B) 4%   C) 80%   D) 25%   E) NOTA

18. Simplify the following expression:

$$\left(i + \cfrac{1}{i + \cfrac{1}{i + \cfrac{1}{i + \cdots}}} \right)^{3}$$

A) $-i$ B) 1   C) i D) -1 E) NOTA

19. What does the graph of the following look like on the Argand plane?

"All complex numbers with magnitude less than or equal to 2π having imaginary part 3."

A) vertical line segment   B) annulus   C) arc of a circle   D) horizontal line segment   E) NOTA

20. Let $P(z)$ be a polynomial with real coefficients such that $P(z_1) = 3 - 4i$, and suppose that z_1 and $\bar{z_1}$ are roots of $Q(z) = P(z) - (3 - 4i)$. Find the value of $P(z_1)P(\bar{z_1})$.

A) -7 B) 0   C) $25 - 24i$ D) $-7 - 24i$ E) NOTA

21. Let $z = \frac{1}{2} - \frac{1}{2}i$ and $w = -\frac{1}{2} - \frac{1}{2}i$. Let $K = \{k : z^k + w^k = 0\}$. Find $\min|K|$, where $|K| = \{|k| : k \in K\}$.

A) 2   B) $\frac{10}{3}$ C) 1   D) 0   E) NOTA

22. Evaluate the following expression:

$$\prod_{n=1}^{360} 2^{n(-1)^n}(\cos(n°) + i\sin(n°))$$

(Note: $n°$ denotes degrees, not an exponent of 0).

A) 2^{180} B) -2^{180} C) 0   D) -1 E) NOTA

23. Given z is a complex number, which of the following statements is false?

A) $\overline{(z^n)} = (\bar{z})^n$ for any integer n B) $|z^2| = |z|^2$ C) $e^{\pi i}$ is a complex number
D) i^i is an imaginary number   E) NOTA

24. Given the following system of equations with x and y as complex numbers,

$$\begin{cases} 2x + xi + 3y - 2yi = 21 - 6i \\ -4x - 2xi + 2y + 6yi = 4 + 32i \end{cases}$$

find $|x + y|$.

A) $\sqrt{53}$ B) 53   C) 9   D) $\sqrt{45}$ E) NOTA

25. Which quadrant of the Argand plane does the following expression lie in?

$$\left[\frac{\sqrt{6} + \sqrt{2}}{4} + i\left(\frac{\sqrt{6} - \sqrt{2}}{4} \right) \right]^{40}$$

A) IV B) III C) II D) I E) NOTA

26. Evaluate $i^{-2016} + i^{-2015} + \ldots + i^{n-2017} + \ldots + i^{2016}$.

A) 0   B) -1 C) 1   D) $-i$ E) NOTA

27. Simplify the following expression:

$$\sum_{n=1}^{102} ni^n$$

A) 0   B) $-51 + 52i$ C) $52 - 51i$ D) $-52 + 51i$ E) NOTA

28. Let P be a set of points, z, in the Argand plane such that $(1 + i)z$ is a real number. Which of the following best describes the graph of P?

A) circle   B) line   C) hyperbola   D) parabola   E) NOTA

29. Define $a \odot b = a^b - b^a$. What is $e^{i\frac{\pi}{2}} \odot e^{i\pi}$?

A) $-\frac{1}{e^{\pi}} + i$ B) $-1 - i$ C) $-e^{\pi} - i$ D) undefined   E) NOTA

30. Jim decides to label the faces of a fair 6-sided die with each of the sixth roots of unity. He rolls it twice and multiplies the two results to obtain Q. What is the probability Q lies wholly within in the third quadrant of the Argand plane?

A) $\frac{1}{4}$ B) $\frac{5}{36}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) NOTA