For all questions, answer choice "E. NOTA" means none of the above answers is correct.

1. Which Greek mathematician do most historians credit with the discovery of conic sections as a solution to solving the Delian problem, also known as "doubling the cube"?

A) Eratosthenes B) Menaechmus C) Pythagoras D) Apollonius E) NOTA

2. How many points in general linear position are required to uniquely determine a conic section?

A) 3 B) 4 C) 5 D) 6 E) NOTA

3. What is the maximum number of intersections of two distinct non-degenerate conic sections?

A) 3 B) 4 C) 5 D) 6 E) NOTA

4. Which of the following cases is not a possible degenerate conic section?

A) Null set B) Point C) One line D) Two lines E) NOTA

5. Which conic section is described by the equation $3x^2+4xy+y^2-6x-6y+7=0$?

A) Circle B) Ellipse C) Parabola D) Hyperbola E) NOTA

6. Let q be the acute angle of rotation to standard axes for the conic section given by the equation $3x^2+4xy+y^2-6x-6y+7=0$. Evaluate $\csc^2 q$.

A) $\frac{5}{4}$ B) $\frac{1}{2}\left(5+\sqrt{5}\right)$ C) $\frac{1}{2}\left(5-\sqrt{5}\right)$ D) $10+4\sqrt{5}$ E) NOTA

7. In order to rotate the conic section to a standard axes $(x,y)\mapsto(x',y')$ through an angle q, which of the following expressions is substituted for *x*?

A) $x'\sin q+y'\cos q$ B) $x'\cos q-y'\sin q$ C) $x'\cos q+y'\sin q$ D) $x'\sin q+y'\cos q$ E) NOTA

8. In the complex plane where $z=x+yi, i=\sqrt{-1}$, and $\overline{z}$ is the complex conjugate, which conic section is NOT described by the equations $iz^2-i\overline{z}^2=4$, $4|z|^2+z^2+\overline{z}^2=3$, or $z+\overline{z}-\left(z-\overline{z}\right)^2=4$?

A) Circle B) Ellipse C) Parabola D) Hyperbola E) NOTA

9. As the eccentricity of a non-degenerate conic section approaches infinity, what does the conic degenerate into?

A) Null set B) Point C) 1 or 2 lines D) 2 points E) NOTA

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

10. A conic section can be written as the matrix equation $\mathbf{x}^T A_Q \mathbf{x} = 0$, where $\mathbf{x} = [x, y, 1]^T$ and

$$A_Q = \begin{bmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{bmatrix}$$

using the coefficients in general form. For the conic section described by the equation $2x^2 + 4xy + y^2 - 6x - 6y + 7 = 0$, let τ be the trace of A_Q and let Δ be the determinant. Evaluate $\tau^2 - 4\Delta$.

A) -11 B) 37 C) 101 D) 149 E) NOTA

The next 20 questions are evenly split four ways for each of the four conic sections, denoted as either C (circle), E (ellipse), P (parabola), or H (hyperbola).

11. (C1) A circle has the equation $4x^2 + 4y^2 + 24x - 4y + 5 = 0$ with center (h,k). What is $h+k$?

A) -2.5 B) 2.5 C) -10 D) 10 E) NOTA

12. (E1) What is the area of the inscribed rectangle in an ellipse with equation $b^2x^2 + a^2y^2 = a^2b^2$ and $a>b>0$ such that two of the sides of the rectangle are the latus recti, where c is the focal length?

A) $\frac{2ab^2}{c}$ B) $\frac{4ab^2}{c}$ C) $\frac{2a^2c}{b}$ D) $\frac{4a^2c}{b}$ E) NOTA

13. (P1) What is the minimum value of the parabola with equation $y = epx^2 - \sqrt{2}x + \frac{3}{ep}$?

A) $\frac{\sqrt{2}}{2e\pi}$ B) $\frac{1}{ep}$ C) $\frac{5}{2ep}$ D) $\frac{3}{4ep}$ E) NOTA

14. (H1) If the eccentricity of a hyperbola is $\sqrt{3}$, what is the measure of the smaller angle between the two asymptotes, in degrees?

A) 30 B) 45 C) 60 D) 90 E) NOTA

15. (C2) What is the equation of the circle inscribed in the triangle formed by the lines $x=0$, $y=0$, and $-4x+3y=12$?

A) $x^2 + y^2 + 2x - 2y + 1 = 0$ C) $x^2 + y^2 + 2\sqrt{3}x - 2\sqrt{3}y + 3 = 0$ E) NOTA

B) $x^2 + y^2 + 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0$ D) $4x^2 + 4y^2 + 12x - 12y + 9 = 0$

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

16. (E2) What is the eccentricity of an ellipse in quadrant I of the Cartesian plane, tangent to (2,0) and (0,3)?

A) $\frac{\sqrt{5}}{2}$ B) $\frac{\sqrt{5}}{3}$ C) $\frac{1}{2}$ D) $\frac{1}{3}$ E) NOTA

17. (P2) What is the area of the figure enclosed by the parabola $y^2 - 12x - 6y - 3 = 0$ and its latus rectum?

A) $4.5p$ B) 18 C) 24 D) 27 E) NOTA

18. (H2) A hyperbola is given by the equation $y^2 - 2x^2 - 20x + 8y - 50 = 0$. What is the product of the slope and the x-intercept of the asymptote with negative slope?

A) $4 + 5\sqrt{2}$ B) $4\sqrt{2} - 10$ C) $2\sqrt{2} - \frac{5}{2}$ D) $5 - \frac{5\sqrt{2}}{2}$ E) NOTA

19. (C3) Lines are drawn tangent to the circle $x^2 + y^2 = 16$ at the points $\left(-2, 2\sqrt{3}\right)$ and $\left(2\sqrt{2}, -2\sqrt{2}\right)$. What is the y-coordinate of the intersection of the two tangent lines?

A) $4 + 6\sqrt{2} + 4\sqrt{3} + 2\sqrt{6}$ C) $6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$ E) NOTA

B) $4 + 2\sqrt{2} + 4\sqrt{3} + 2\sqrt{6}$ D) $6 - 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$

20. (E3) An ellipse has foci located at (1,-2) and (5,1) with eccentricity less than 0.5. Which of the following points cannot exist on the ellipse?

A) (1,2) B) (-2,1) C) (5,5) D) (-3,2) E) NOTA

21. (P3) At what value of *y* does the line tangent to a parabola with equation $y^2 + 8x - 10y + 33 = 0$ at the lower endpoint of its latus rectum intersect its directrix if said tangent has slope 1?

A) -3 B) 1 C) 4 D) 5 E) NOTA

22. (H3) The Pell-Fermat equation is a Diophantine equation of the form $x^2 - ny^2 = 1$ for *n* any nonnegative integer used to solve for integer values of x and y to approximate the square root of *n* as x/y, so long that *n* is not a perfect square. It also happens to be a hyperbola in the Cartesian plane! What is the eccentricity of the hyperbola?

A) $\sqrt{n - 1}$ B) $\frac{\sqrt{n^2 - 1}}{n}$ C) $\frac{\sqrt{n^2 + 1}}{n}$ D) $\sqrt{\frac{n+1}{n}}$ E) NOTA

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

23. (C4) The circle with equation $4x^2+4y^2-32x+24y+91=0$ is revolved about a line in the Cartesian plane to generate a torus with surface area $24\pi^2$. If we consider the solution set of all lines that can be the axis of rotation to generate such a torus, we generate infinitely many lines that are tangent to a circle that is concentric with the revolved circle. Thus, an annulus is formed from the two circles. What is the area enclosed by this annulus?

A) $\frac{25\pi}{4}$ B) $\frac{55\pi}{4}$ C) $\frac{943\pi}{36}$ D) 28π E) NOTA

24. (E4) Consider a semi-elliptically-arched ceiling in a whispering gallery. The vertical walls are of height 5 feet, the ceiling reaches 20 feet above the vertical walls at its highest point, and the whispering points are located 30 feet across from each other at a height of 5 feet. What is the height of the ceiling above the whispering points?

A) 11 B) 15 C) 16 D) 21 E) NOTA

25. (P4) Consider two distinct points on an arbitrary parabola P_1 and P_2 with corresponding points on the directrix Q_1 and Q_2 such that P_1Q_1 and P_2Q_2 are perpendicular to the directrix, and the focus of the parabola is F. How many of the following statements are always true?

- The distance from P_1 to Q_1 is the same as the distance from P_2 to Q_2.
- The distance from P_1 to Q_1 is the same as the distance from P_1 to F.
- The distance from P_2 to Q_2 is the same as the distance from Q_2 to F.
- The distance from P_1 to P_2 is the same as the distance from Q_1 to Q_2.
- The line through P_1 and Q_1 is parallel to the line through P_2 and Q_2.

A) 1 B) 2 C) 3 D) 4 E) NOTA

26. (H4) A hyperbola has the equation $9x^2-16y^2+18x+64y-199=0$. What is the shortest distance from a focus of the hyperbola to either of its asymptotes?

A) 3 B) 5 C) $\frac{15}{4}$ D) $\frac{23}{4}$ E) NOTA

27. (C5) On the coordinate plane, a circle is formed from the three points (5,5), (6,-2) and (2,-4). An equilateral triangle is then inscribed in the circle, not necessarily including any of the above points. What is the area enclosed by the equilateral triangle?

A) $\frac{25\sqrt{3}}{4}$ B) $\frac{75\sqrt{3}}{4}$ C) $\frac{25\sqrt{3}}{18}$ D) $\frac{25\sqrt{3}}{6}$ E) NOTA

28. (E5) An ellipse has a focus (3,0), a directrix with equation $x+y-1=0$, and an eccentricity of 0.5. Given the equation of the ellipse in general quadratic form (positive x^2 coefficient, all coefficients are relatively prime integers), what is the constant term?

A) 71 B) -71 C) 73 D) -73 E) NOTA

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

29. (P5) A parabola has equation $4y+3=6(x+5)^2$. What is the sum of the x-intercepts of this parabola?

A) -2.5 B) 2.5 C) -10 D) 10 E) NOTA

30. (H5) A hyperbola has polar equation $r=\frac{16}{1-3\cos\theta}$. What is the distance from a focus of this hyperbola to the vertex of the parabola closer to this focus?

A) 1 B) 3 C) 8 D) 16 E) NOTA