1. B. Menaechmus is highly regarded as the discoverer of conic sections and solving the Delian problem.

2. C. 5 points.

3. B. If two conic sections share 5 points, then they must be the same conic section, so 4 is the maximum number of intersections possible.

4. E. All of these are possible degenerate conic sections.

5. D. $B^2 - 4AC = 4^2 - 4(3)(1) = 4 > 0 \Rightarrow$ hyperbola (and the graph is non-degenerate)

6. B. The angle of rotation can be calculated by the formula $\tan 2\theta = \frac{B}{A-C}$. This gives us using the tangent double-angle formula:

$$\tan 2\theta = 2$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = 2$$

$$\tan^2\theta + \tan\theta - 1 = 0$$

$$\tan\theta = \frac{\sqrt{5}-1}{2}$$

Solving for the radius gives us $r = \sqrt{x^2+y^2} = \sqrt{10-2\sqrt{5}}$ and $\csc\theta = \frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}$. When squared and reduced, we have $\csc^2\theta = \frac{5+\sqrt{5}}{2}$.

7. B. By transformation of axes.

8. A. We need to convert the given equations from complex to Cartesian format. For the first equation, we have $iz^2 - i\bar{z}^2 = 4 \Leftrightarrow i(z+\bar{z})(z-\bar{z}) = 4 \Leftrightarrow i(x+yi+x-yi)(x+yi-x+yi) = 4$ $\Leftrightarrow i(2x)(2yi) = 4 \Leftrightarrow xy = -1$ which is a rectangular hyperbola. The second equation becomes $4|z|^2 + z^2 + \bar{z}^2 = 3 \Leftrightarrow 4x^2 + 4y^2 + x^2 - y^2 + 2xyi + x^2 - y^2 - 2xyi = 3 \Leftrightarrow 6x^2 + 2y^2 = 3$, which is an ellipse. Finally, $z + \bar{z} - (z-\bar{z})^2 = 4 \Leftrightarrow x+yi+x-yi-(-2iy)^2 = 4 \Leftrightarrow 2x + 4y^2 = 4$, which is a parabola, leaving a circle as not included.

9. C. The hyperbola will collapse to a single line or two lines in the infinite case.

10. D. The generated 3x3 matrix determinant is $\begin{vmatrix} 3 & 2 & -3 \\ 2 & 1 & -3 \\ -3 & -3 & 7 \end{vmatrix} = -7$

The trace is the sum of the elements on the upper left-lower right diagonal, giving 11. 11^2-4(-7)=149.

11. A. Standard form: $(x+3)^2 + (y-0.5)^2 = 8$, center is (-3, 0.5) and sum is -2.5

12. E. The width of the rectangle is determined by twice the focal length 2*c*. The height can be established through the equation of the ellipse. *x* must be *c*, and solving for *y* gives

$$\frac{c^2}{a^2}+\frac{y^2}{b^2}=1$$
$$y=\pm b\sqrt{1-\frac{c^2}{a^2}}$$
$$y=\pm b\sqrt{\frac{a^2-\left(a^2-b^2\right)}{a^2}}$$
$$y=\pm\frac{b^2}{a}$$

with a length of $\frac{2b^2}{a}$. Therefore, the area must be $A=\frac{4b^2c}{a}$.

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13. C. The x-coordinate of the minimum value is obtained by $-\frac{b}{2a}=\frac{\sqrt{2}}{2e\pi}$. Substituting this in gives $y=ep\left(\frac{\sqrt{2}}{2ep}\right)^2-\sqrt{2}\left(\frac{\sqrt{2}}{2ep}\right)+\frac{3}{ep}=\frac{5}{2ep}$.

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14. E. The eccentricity of a hyperbola is related to the secant of the angle between the asymptotes. Thus, $\sec\theta=e=\sqrt{3}$, which does not relate to any of the angles provided.

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15. A. A right triangle is formed with vertices at (-3,0), (0,4), and (0,0). Thus, the hypotenuse is 5 (3-4-5 triangle). The radius can be calculated through congruent triangles as $r=\frac{a+b-c}{2}=1$. This puts the center of the inscribed circle at (-1,1). The standard form becomes $(x+1)^2+(y-1)^2=1\Leftrightarrow x^2+y^2+2x-2y+1=0$.

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16. B. a=3 and b=2, so $c=\sqrt{a^2-b^2}=\sqrt{5}$. $e=\frac{c}{a}=\frac{\sqrt{5}}{3}$.

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17. C. The area enclosed by a chord perpendicular to the axis of symmetry and the parabola is given by the formula $A=\frac{2}{3}bh$, essentially 2/3 of the area formed by the parallelogram. The height is the focal length and the base is the length of the latus rectum. Converting the equation of the parabola to standard form gives $(y-3)^2=12(x+1)$. The focal length is 12/4=3, the height must be 3 and width 12, 2/3(36)=24.

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18. A. Converting to standard form gives $\frac{\left(y+4\right)^2}{16}-\frac{\left(x+5\right)^2}{8}=1$. The negative slope asymptote is given by the equation $y+4=-\frac{4}{2\sqrt{2}}\left(x+5\right)\Leftrightarrow y=-\sqrt{2}x-\left(5\sqrt{2}+4\right)$. The product of the slope $-\sqrt{2}$ and x-intercept $\frac{5\sqrt{2}+4}{-\sqrt{2}}$ is $5\sqrt{2}+4$.

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19. B. The slope to the line tangent to the circle is given by $-\frac{x}{y}$, or consequently the negative reciprocal of the tangent at that point. Starting at $\left(-2, 2\sqrt{3}\right)$, the equation of the tangent line is given as $y-2\sqrt{3}=\frac{\sqrt{3}}{3}(x+2) \Leftrightarrow y=\frac{\sqrt{3}}{3}x+\frac{8\sqrt{3}}{3}$. The equation of the tangent line to $\left(2\sqrt{2}, -2\sqrt{2}\right)$ is $y+2\sqrt{2}=x-2\sqrt{2} \Leftrightarrow y=x-4\sqrt{2}$. Setting the two lines equal gives

$$\frac{\sqrt{3}}{3}x+\frac{8\sqrt{3}}{3}=x-4\sqrt{2}$$

$$\left(\frac{\sqrt{3}}{3}-1\right)x=-\frac{8\sqrt{3}}{3}-4\sqrt{2}$$

$$x=\frac{8\sqrt{3}+12\sqrt{2}}{3-\sqrt{3}}\left(\frac{3+\sqrt{3}}{3+\sqrt{3}}\right)=\frac{24+36\sqrt{2}+24\sqrt{3}+12\sqrt{6}}{6}=4+6\sqrt{2}+4\sqrt{3}+2\sqrt{6}$$

$$\therefore y=x-4\sqrt{2}=4+2\sqrt{2}+4\sqrt{3}+2\sqrt{6}$$

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20. A. In order for the ellipse to hold, we need to make *a* fit the definition of an ellipse, such that $d(F_1,P)+d(P,F_2)=2a$ and $e=c/a$ is between 0 and 0.5. The focal length is half the distance between the foci, so $c=\frac{1}{2}\sqrt{(1-5)^2+(-2-1)^2}=2.5$. Thus,

$$0<e<0.5$$

$$0<\frac{2.5}{0.5\left(\sqrt{(1-x)^2+(-2-y)^2}+\sqrt{(x-5)^2+(y-1)^2}\right)}<0.5$$

$$10<\sqrt{(1-x)^2+(-2-y)^2}+\sqrt{(x-5)^2+(y-1)^2}$$

When (1,2) is substituted in, you get that $10<4+\sqrt{17}$, close to 8 and does not satisfy the inequality.

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21. D. In standard form, the parabola has equation $(y-5)^2=-8(x+1)$. The vertex is at (-1,5), and *a*=2, so the focus is at (-3,5) and the directrix is the line x=1. The endpoints of the latus rectum are (-3,1) and (-3,9) since the total length is 4a. At x=-3, the derivative is 1. Point-slope form and solving for y gives $y-1=(1)+3 \Leftrightarrow y=5$.

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22. D. When rewritten in standard form, the Pell-Fermat equation becomes $\frac{x^2}{1}-\frac{y^2}{1/n}=1$. To find c and consequently the eccentricity,

$$c^2 = a^2 + b^2 = 1 + \frac{1}{n}$$
$$\therefore\ c = \sqrt{1 + \frac{1}{n}} = \sqrt{\frac{1+n}{n}}$$
$$e = \frac{c}{a} = c = \sqrt{\frac{1+n}{n}}$$

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23. B. The equation of the circle in standard form is $(x-4)^2 + (y+3)^2 = \frac{9}{4}$. Using Pappus' Centroid Theorem, we know that $S = 2\pi rC = 2\pi(3\pi)r = 6\pi^2 r$. Given the surface area of $24\pi^2$, the radial distance from the centroid (4,-3) to the axis of revolution is 4. Thus the set is a circle centered at (4,-3) with radius 4. The annulus then has area $\pi\left(4^2 - \left(\frac{3}{2}\right)^2\right) = \frac{55\pi}{4}$.

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24. D. If we center the ellipse of the whispering ceiling at (0,5) because of the 5 ft walls, and the foci are 30 ft apart as the "whispering points" with a 20 ft ceiling, then c=15 ft and b=20 ft, so $a = \sqrt{c^2 + b^2} = \sqrt{400 + 225} = 25$, giving the ellipse equation as $\frac{x^2}{25^2} + \frac{(y-5)^2}{20^2} = 1$. At the foci at x=15, we need to solve for y.

$$\frac{15^2}{25^2} + \frac{(y-5)^2}{20^2} = 1$$
$$(y-5)^2 = 256$$
$$y = 5 \pm 16$$
$$y = 21$$

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25. B. Statements 2 and 5 are always true.

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26. A.

$$9x^2-16y^2+18x+64y-199=0\to\frac{(x+1)^2}{16}-\frac{(y-2)^2}{9}=1$$

$$c=\sqrt{16+9}=5 \qquad F(4,2)$$

Asymptotes: $y-2=\frac{3}{4}(x+1)\to 3x-4y+11=0$

$$d=\frac{|3(4)-4(2)+11|}{\sqrt{3^2+4^2}}=\frac{15}{5}=3$$

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27. B. I prefer finding the intersection of the perpendicular bisectors of the chords to get the center. For the chord (5,5) to (6,-2), the midpoint is (5.5, 1.5). The slope between the two points is -7, so the slope of the perpendicular line is 1/7. The perpendicular bisector has equation $y-1.5=\frac{1}{7}(x-5.5)$. For the chord (6,-2) to (2,-4), the midpoint is (4,-3) and the slope is ½, so the perpendicular line is -2, so the line is $y+3=-2(x-4)$. Setting the two equations equal to find the intersection gives (2,1) as the center and the radius of 5. The triangle is inscribed in the circle, so a side length of the triangle is $5\sqrt{3}$. Thus, the triangle has area $A=\frac{s^2\sqrt{3}}{4}=\frac{(5\sqrt{3})^2\sqrt{3}}{4}=\frac{75\sqrt{3}}{4}$.

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28. A. In order to generate the general form equation, we need to pick an arbitrary point (x,y) on the ellipse. The eccentricity is defined as the distance from the focus to an arbitrary point divided by the distance from that point to the directrix. The distance from the focus to a point is $f=\sqrt{(x-3)^2+y^2}$ and the distance from that point to the directrix is given as $d=\frac{x+y-1}{\sqrt{1^2+1^2}}$.

Knowing that $e=\frac{f}{d}=\frac{1}{2}$, we solve and convert to the general form:

$$2\sqrt{(x-3)^2+y^2}=\frac{1}{\sqrt{2}}(x+y-1)$$

$$4\left(x^2-6x+9+y^2\right)=\frac{1}{2}\left(x^2+2xy+y^2-2x-2y+1\right)$$

$$7x^2-2xy+7y^2-46x+2y+71=0$$

And the constant term is 71.

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29. C. The equation in function form is $y = 1.5x^2 + 15x + \frac{147}{4}$. The sum of the roots is given as ⍰ b/a, or -10, and they are real.

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30. E. This equation places a focus on the origin with form $r = \frac{a(e^2 - 1)}{1 - e\cos\theta}$. The eccentricity is 3 and thus *a*=16/8=2. The focal length, *c* is equal to *ea*, or 6.