Alpha Equations and Inequalities Test
2015 Mu Alpha Theta National Convention

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Answer Key

1	B
2	A
3	C
4	D
5	C
6	C
7	D
8	D
9	B
10	B
11	C
12	A
13	D
14	C
15	C
16	D
17	D
18	A
19	B
20	C
21	B
22	E
23	C
24	A
25	D
26	E
27	A
28	B
29	C
30	A

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Solutions:

1) B: First, solve for $3x - 3 = 3$ to find x=2. Plug x into the equation, yielding $2^3 + 6 * 2^2 + 12 * 2 + 7 = 63$.
2) A: Work from inside out, creating a tree of solutions: first tree is based on x < -2
   If x < -2, then $|x + 1 - 2x - 4| = 6$, which simplifies to $|-x - 3| = 6$. This has 2 solutions, x=-9, and x =3. x=3 is not a valid solutions because it is Greater than -2. The other branch of the tree is for x > -2, yielding $|x + 1 + 2x + 4| = 6$, which simplifies to $|3x + 5| = 6$. This has 2 solutions, -11/3, and 1/3. Of these 2 solutions, only 1/3 is valid. Therefore there are 2 valid solutions, x=-9, 1/3.
3) C: Identify that the equation is equal to (sin(x)+1)^3=0, which simplified to sinx = -1, yielding x=3Pi/2.
4) D: x^3-y^3 = (x-y)*(x^2+xy+y^2) = 8*(x^2 -2xy + y^2 + 3xy) = 8*(8^2+3*6)=656
5) C: The equation describes an ellipse centered at the origin. From the ellipse equation, it is clear that there are at least 4 solutions. Checking x=1 yields y=sqrt(27/4), combined with symmetry rules out any remaining integer pair solutions, so the correct answer is 4.
6) C: The ln is the limiting term, since the absolute value protects the square root from operating on a negative operand. 3x+2 > 0, so x > -2/3.
7) D: Find a common denominator: (x-4)/((x-6) (x-3/2))>=0. Then test for regions where it equals 0: x=4, x=6, x=3/2. This yields $x > 6, \frac{3}{2} < x \leq 4$.
8) D: Simplify the nested fraction, yielding 4-(x-4) = 10, which simplifies to x=-2.
9) B: Any negative value of y will produce 0 solutions, so the minimum number of solutions is 0. Sketch the graph to find the max values. The graph has a horizontal asymptote in both directions at y=3/5. There is a vertical asymptote at x=1, which will yield 2 solutions for y>3/5. As x approaches –inf or +inf there are 2 more solutions. Finally, there is a hump for -1/2<x<1/2, so there will be 4 solutions in this interval.
10) B: g(2*4) = -2, and f(-2) = 25.
11) C: Identify that $\sqrt{x + 6} = 6$. Square both sides, x+6 = 36. x =30
12) A: The word problem describes the inequality $100m < \sum_{i=1}^{m} 50 * .1 * i$
    This reduces to 20m<(m+1)*m/2 -> 40 < m+1, 39<m. The problem asks for when he will have more money, which is after 40 deposits.
13) D: The area is of interest, so we can shift the graph to the origin, simplifying the equation to $|x| + 2|y| < 8$. This is a diamond with diagonals of 8 and 16, resulting in an area of 8*16*1/2 = 64.
14) C: This is the same as finding the number of positive integral divisors of 60, except that we don't include 60 because there are only 30 questions on the test. There are 12 positive integral factors of 60, so there are 11 different numbers of problems James could work (1, 2, 3, 4, 5, 6, 10, 12, 15, 20, or 30 questions, averaging 60, 30, 20, 15, 10, 6, 5, 4, 3, 2, or 1 minutes per problem, respectively.
15) C: The ages are Alice = 16, Bob = 14, Chris = 15. There are several ways to solve the problem since there is extra information included. The sum of the ages is 45.
16) D: Identify that there is an even number of terms on top with alternating even/odd exponents, making -1 is the only real solution.
17) D: Start by substituting the line into the parabola yielding x=x^2+5x+17 which simplifies to x^2+4x+17 = 0. Calculate the discriminant to find the number of real solutions: sqrt(16-4*17) is imaginary, so there are no real solutions.

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18) A: One solution is x=0. Factor out this zero. Because the x^2 term is negative and the constant term is positive, there must be an even number of negative solutions. Additionally, because there are missing terms all the solutions cannot be negative, so there must be 2 even solutions and 2 odd solutions. This means 0 will be the median solution.
19) B: Calculate the determinant to be 7x^2-34x+35. Product of the roots is equal to c/a.
20) C: Combine the fractions and eliminate the denominator using conjugates. Simplifies to 148/85+44i/85.
21) B: The exponents of the two terms must be equal. Simplify the second term to 2x+x-6 which simplifies to 3x-6 which must equal 3, so x is equal to 9/3=3.
22) E: tan(x) equal sqrt(3) at pi/3, decreaing until the lower bound of 0, so the interval is $0 \leq x \leq \frac{\pi}{3}$.
23) C: The absolute value is irrelevant since the y is squared, so the answer is simply $-\sqrt{6} < y < \sqrt{6}$
24) A: Find the difference between consecutive terms, 24-17=7, 33-24=9, 44-33=11. Take the difference between those terms: 9-7 = 2, 11-9 = 2. Therefore the b coefficient of the quadratic is 2 and the a coefficient is 1. Plug in any of the points to find that c =9, so c/a = 9.
25) D: This is asking for the product of the roots taken 2 at a time divided by the sum of the roots, which is equal to 11/-6. Alternatively, simply factor the polynomial into (x+1)(x+2)(x+3).
26) E: There is no solution – the square root cannot equal -2.
27) A: If the form is ax^2 + bx + c, since a = 1, the sum of the roots is –b, and the product of the roots is c. –(-10) + 23 = 33.
28) B: There are several approaches to this problem. The only pair that works is x=6, y=5.
29) C: Partial fractions approach can be used here. A(x-5) + B(x+2) = 6x+7.
    Substitute x = 5 to find b=37/7
    Substitute x=-2 to bind a = 5/7
    A + B = 42/7
30) A: If the vectors are perpendicular, the dot product is equal to zero, producing the following equation: 4a+4 + 5a+ 10 + 6a + 18 = 0, so a = -32/15