For all of the following problems, the answer choice "NOTA" denotes "None of the Above", meaning the correct answer is not one of the choices provided. Good luck!

1. Luke is trapped in a garbage compactor which consists of a floor and two walls; the walls are spaced 20 meters apart. Not knowing what to do, as the walls begin to close in on him, Luke paces back and forth between the two walls, beginning at one of the walls. The walls are moving toward each other at such a rate that between each time Luke reaches one of the walls, the wall he just left is 75% of the distance from him that the wall he just reached was when he began walking toward it (the wall he just reached is also moving toward him at the same rate). Supposing Luke travels at a constant rate of 5 meters per second, how many seconds elapse between the beginning of Luke's pacing and his eventual untimely demise (you can assume this demise occurs precisely at the moment the walls touch each other)?

A) 20 B) 8 C) 14 D) 17 E) NOTA

2. Suppose you have a cone frustum (that is, a cone with the top removed), with base radius 8, perpendicular height 9, and top radius 4. You fill half the volume with water. Let a denote the perpendicular height of the water in the frustum. Now, suppose you turn the frustum over so that what was the top is now the base. Let b denote the new perpendicular height of the water in the frustum. Find $a + b$.

A) $513\pi/2$ B) $1539\pi/2$ C) $\frac{14\sqrt{3}-9\sqrt{2}}{4}$ D) $\frac{54\sqrt{3}-6\sqrt{2}}{4}$ E) NOTA

3. Suppose you have two urns. In the first, there are eight green marbles, three red marbles, and four blue marbles. The second contains five yellow marbles, six red marbles, and nine blue marbles. Suppose you select an urn at random (both urns are equally likely to be selected) and draw a marble from it. If the marble is red, what is the probability that the urn you selected was the first urn (the one that contained green marbles)?

A) $1/5$ B) $2/5$ C) $4/5$ D) $1/3$ E) NOTA

4. In a standard 52 card deck, how many possible straight flushes exist? (A straight flush is 5 cards such that they are all of the same suit and of five consecutive ranks. An Ace can count lower than 2 or higher than a King, but the order does not wrap around, So A-2-3-4-5 and 10-J-Q-K-A of the same suit are both considered straight flushes, while Q-K-A-2-3 is not.)

A) 52 B) 40 C) 36 D) 56 E) NOTA

5. Let $A=\begin{bmatrix} 7 & -4 & 3 \\ 1 & 0 & 2 \\ -3 & 5 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 5 & 3 & -4 \\ 12 & 0 & -2 \\ -3 & 5 & 3 \end{bmatrix}$. Find the sum of the complex eigenvalues of AB.

A) 8 B) $3+\sqrt{6}$ C) 22 D) -4 E) NOTA

6. Let $i=\sqrt{-1}$ and $x=\sqrt{-2i}$. If $y=|x|$, what is the value of $|xy|$?

A) 2 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) not enough information
E) NOTA

7. Steve the space dog has a busy owner. Thus, he is tethered using the SpaceRope300™, a device which ensures Steve stays exactly *the same* distance from wherever the device is anchored (that is, he can neither be further away nor closer to the device when it is powered on). However, the poorly designed machine has malfunctioned, preventing Steve from entering a triangular region on the surface of the sphere on which he could enter if the device was working properly. Suppose the SpaceRope300™ is set such that the distance which Steve must be from the device is exactly 7 parsecs away. If the three spherical angles that make up the triangle are all equal to $2\pi/3$ radians, what is the total area (in parsecs^2) in which Steve is permitted to roam?

A) $1372\pi/3$ B) 147π C) 196π D) 343π E) NOTA

8. Suppose you can purchase a device today for $1000 which will save you $50 on your monthly energy bill for the next two years, after which it will self-destruct. If the effective monthly interest rate can be expressed as $i=1/96$, and if $\left(\frac{1}{1+i}\right)^{24}=0.78$, assuming interest is compounded monthly, is this a worthwhile investment?

A) no B) the two payment streams are equivalent C) yes
D) not enough information E) NOTA

9. Let the distance a Frisbee can travel in meters be modeled by the function $f(t,u)=A\cdot(1+tu)$, where *A* represents the height of the person throwing the Frisbee (in inches), *t* represents the initial force with which the Frisbee is thrown, and *u* represents the wind factor at the time the Frisbee is thrown. If a 6'2" individual throws the Frisbee 518 meters given a certain initial force and wind factor, how far can a 5'6" individual throw the Frisbee if the initial force is $4/3$ that of the 6'2" individual, and the wind factor is $3/8$ that of the 6'2" individual?

A) 858 meters B) 462 meters C) 518 meters D) 354.75 meters E) NOTA

10. Bogdan deposits $10 into a bank account at the end of each year for ten years. If the nominal interest rate is 10%, compounded semiannually, which of the following is equivalent to how much Bogdan will have at the end of the ten-year period (including the final deposit of $10, immediately after the interest is compounded at the end of the tenth year)?

A) $10\left(\frac{1-1.1^{10}}{1-1.1}\right)$ B) $10\left(\frac{1}{1.05^2-1}\right)$ C) $10\left(\frac{1-1.05^{20}}{1-1.05^2}\right)$ D) $10\left(\frac{1}{1.1-1}\right)$ E) NOTA

11. The function $f(\theta)=a\theta$, in polar coordinates, where a is a non-zero real number, goes by what name?

A) Cassinian Oval B) Cayley's Sextic C) Archimedes's Spiral D) Cissoid of Diocles E) NOTA

12. What is the remainder when $612!$ is divided by 613?

A) 612 B) 1 C) 3 D) 17 E) NOTA

13. Dr. K is an experimental chef. He has four ingredient receptacles. The first receptacle contains 7 ingredients, the second receptacle contains 5 ingredients, the third receptacle contains 6 ingredients, and the last receptacle contains 2 ingredients. Dr. K combines 3 ingredients from the first receptacle with 2 ingredients from the second receptacle, 4 ingredients from the third receptacle, and 1 ingredient from the last receptacle. Further, the order in which the ingredients are added from the third receptacle affects the final product, while the order of addition from the other containers do not affect the outcome. Suppose that one-fourth of the concoctions created in this method are lethal. What is the total number of non-lethal recipes which Dr. K can create from these ingredients?

A) 252,000 B) 63,000 C) 504,000 D) 189,000 E) NOTA

14. Cyrus is stationed at the origin. Suppose each time Cyrus moves, there is a $1/3$ chance he will move exactly one unit positively in the y-direction (and no movement in the x-direction), a $1/24$ chance he will move exactly one unit negatively in the y-direction (and no movement in the x-direction), a $1/4$ chance he will move exactly one unit positively in the x-direction (and no movement in the y-direction), and a $3/8$ chance he will move exactly one unit negatively in the x-direction (and no movement in the y-direction). What is Cyrus's expected position after his sixth movement?

A) $\left(7/4,3/4\right)$ B) $\left(-3/4,7/4\right)$ C) $(0,0)$ D) $\left(-3/4,-7/4\right)$ E) NOTA

15. The graph of $4x^2-4xy+7y^2-24=0$ is an example of which of the following conic sections?

A) hyperbola B) circle C) ellipse D) parabola E) NOTA

16. Find the least positive multiple of 10 which leaves remainder 2 when divided by 3 and remainder 3 when divided by 7.

A) 80 B) 60 C) 110 D) 90 E) NOTA

17. If the domain of $3f(x)$ is $[-1,8]$, what is the domain of $2f(4x-3)$?

A) $\left[-13/3, 23/3\right]$ B) $\left[1/2, 11/4\right]$ C) $[-7,29]$ D) $[-14,58]$ E) NOTA

18. Let $\frac{4x^2+13x-7}{x^3+6x^2-x-30}$ be expressed in the form $\frac{A}{x+j}+\frac{B}{x+k}+\frac{C}{x+m}$, where *A*, *B*, *C*, *j*, *k*, and *m* are real numbers, find the value of $A\cdot B\cdot C-j\cdot k\cdot m$.

A) -28 B) -32 C) 32 D) 28 E) NOTA

19. Let *x* denote the number of distinct permutations of the word "XXXYYZ", and let *y* denote the number of distinct permutations of the word "XXXXXXYZA". Which of the following words has exactly $x\cdot y$ distinct permutations?

A) XXXYYZABC B) XXXYYZZZZ C) XXXYYZAB D) XYZZZZZ E) NOTA

20. How many values *x* on the interval $(0,2\pi)$ satisfy $1+\sec x=\tan x+\sin x$?

A) 2 B) 1 C) 4 D) 3 E) NOTA

21. There are two circles of equal length radius, circle A and circle B. The curve which defines circle A contains the origin, but lies entirely in quadrants II and III. The curve which defines circle B contains the origin, but lies entirely in quadrants I and IV. Now suppose the point P is the point on circle B precisely where the curve that defines circle B intersects with the origin (that is, the point P is initially defined as (0,0)). Circle B is now rotated counterclockwise around circle A, such that the distance between their centers remains constant, and there is only one point of tangency between circle A and circle B at any time, and the circle moves continuously without slipping. What is the name of the curve formed by the path of point P during this rotation?

A) circle B) kampyle C) cardioid D) conchoids E) NOTA

22. Given that $x^2+y^2=8xy$ and $0<y<x$, find the value of $\frac{x+y}{x-y}$.

A) 1 B) $\sqrt{15}/5$ C) $3\sqrt{7}/7$ D) $\sqrt{15}/3$ E) NOTA

23. What is the coefficient of x^6 in the binomial series expansion of $\sqrt{1-x^2}$?

A) $1/8$ B) $-1/16$ C) $2/3$ D) $-1/4$ E) NOTA

24. Evaluate the following infinite sum: $\frac{1}{2}+\frac{1}{2}+\frac{3}{8}+\frac{1}{4}+\ldots+\frac{n}{2^n}+\ldots$

A) 1 B) $4/3$ C) 2 D) $8/3$ E) NOTA

25. Solve for x: $-3e^{3-x}+2e^{-2x+6}=20$

A) no solution B) $3-\ln 0.4$ C) $3-\ln 2.5$ D) $3-\ln 4$ E) NOTA

26. Which of the following is NOT the equation of a degenerate conic section?

A) $3x^2-6x+2y^2+12y+21=0$ B) $2x^2-y^2+28x+2y+95=0$
C) $9x^2+8x+2y^2-24y+80=0$ D) $2x^2+8x+2y^2-24y+80=0$ E) NOTA

27. Tommy has eight brown socks and seven blue socks. If he draws four socks at random from his drawer, what is the probability that they are all blue?

A) $1/39$ B) $4/7$ C) $1/1365$ D) $2/7$ E) NOTA

28. How many four-digit numbers greater than 6000 have exactly one digit that is a 4?

A) 452 B) 712 C) 501 D) 607 E) NOTA

29. Find the distance traveled by an ant walking on the polar curve $r=\cos\theta$ for $0\le\theta\le 2\pi$.

A) π B) 2π C) 4π D) 8π E) NOTA

30. Let $(1,4)$, $(3,2)$, $(2,-3)$, $(-1,-2)$, and $(-3,1)$ be the coordinates of the vertices of a convex polygon. Find the area enclosed by this polygon.

A) 25 B) 20 C) 16 D) 12 E) NOTA