

“For all questions, answer choice “E. NOTA” means none of the above answers is correct.”

1. Muzi graphed a rose curve with  $n$  petals. Lillian took a look at it and said, “Muzi, you graphed it wrong!” Muzi complained, “But you haven’t even seen the equation yet!” Lillian replied, “A rose curve with that many petals is impossible.” Unfortunately for Muzi, Lillian was correct. Find the remainder when  $n^2$  is divided by 8.  
A) 0  
B) 1  
C) 4  
D) Not enough information  
E) NOTA
2.  $z$  is a complex number such that  $z + 2i\bar{z} = 1 + 17i$ . Find  $|z|$ .  
A) 6  
B)  $\sqrt{58}$   
C) 10  
D)  $\sqrt{146}$   
E) NOTA
3. Let  $A, B$  be sets, and  $^c$  attached to a set denotes the complement of the set. Which of the following is equivalent to  $(A \cup B^c) \cap (A^c \cup B) \cap (A \cup B)^c$ ?  
A) The null set  
B)  $A \cup B$   
C)  $A^c \cup B^c$   
D)  $A^c \cap B^c$   
E) NOTA
4. For what value of  $k$  does the system 
$$\begin{cases} 2x - y + 3z = 4 \\ 7x - 2y + kz = 7 \\ 5x - 2y + z = 11 \end{cases}$$
 have no solution?  
A)  $-9$   
B)  $-1$   
C) 1  
D) 9  
E) NOTA
5. To the nearest thousandth,  $\log_6 2 = 0.387$  and  $\log_6 5 = 0.898$ . Using this information, evaluate  $\log_6 540$ .  
A) 3.172  
B) 3.511  
C) 3.672  
D) Not enough information.  
E) NOTA

6. Which of the following functions is different from the others?
- A)  $y = \cos\left(x + \frac{\pi}{6}\right)$
  - B)  $y = -\sin\left(x - \frac{\pi}{3}\right)$
  - C)  $y = \sin\left(\frac{\pi}{3} - x\right)$
  - D)  $y = -\cos\left(\frac{5\pi}{6} - x\right)$
  - E) NOTA
7. The Gateway Arch in St. Louis is 630 feet tall at its center and 630 feet wide at its base. Sanika is standing one third of the way from one foot of the arch to the other. Assuming the arch is perfectly parabolic, how far above the ground is the arch directly above her?
- A) 210 feet
  - B) 350 feet
  - C) 420 feet
  - D) 560 feet
  - E) NOTA
8.  $f(x) = \frac{2x^3 + 5x^2 - x - 6}{x^2 - x - 6}$  is graphed along with all of its asymptotes. Find the sum of the coordinates of all the x and y intercepts of  $f$  and its asymptotes.
- A) 3
  - B) 3.5
  - C) 5
  - D) 7
  - E) NOTA
9.  $A$  and  $B$  are  $n \times n$  matrices. How many of the statements below are true? You may assume  $A$ ,  $B$ , and  $A + B$  are all non-singular.
- |  |  |
|--|--|
| I. $\det(AB) = \det(A) \cdot \det(B)$                  | II. $\det(A + B) = \det(A) + \det(B)$                |
| III. $\text{tr}(AB) = \text{tr}(A) \cdot \text{tr}(B)$ | IV. $\text{tr}(A + B) = \text{tr}(A) + \text{tr}(B)$ |
| V. $(AB)^T = A^T B^T$                                  | VI. $(A + B)^T = A^T + B^T$                          |
| VII. $(AB)^{-1} = A^{-1} B^{-1}$                       | VIII. $(A + B)^{-1} = A^{-1} + B^{-1}$               |
- A) 3
  - B) 4
  - C) 5
  - D) 6
  - E) NOTA

10. Find the period of  $f(x) = \cos\left(\frac{x}{3}\right) + \sin\left(\frac{x}{4}\right)$
- A)  $2\pi$
  - B)  $10\pi$
  - C)  $24\pi$
  - D)  $48\pi$
  - E) NOTA
11. Jonathan and Henrik live in Perfectville, where the busses are always perfectly on time. To visit Henrik, Jonathan can take either Route 8 or Route 15. Route 8 stops in front of Jonathan's house every 10 minutes, starting at 8 minutes after the hour. That is, 7:08, 7:18, 7:28, etc. Route 15 stops in front of his house every 15 minutes, on the quarter hours. That is, 7:00, 7:15, 7:30, etc. If Jonathan walks outside at a random time to go visit Henrik, and takes the first suitable bus that stops, what is the probability he gets on a Route 15 bus?
- A) 0.3
  - B) 0.4
  - C) 0.6
  - D) 0.7
  - E) NOTA
12. The graph of  $y = |3x + 7| - |2x - 3| + |x - 5|$  on  $-100 \leq x \leq 100$  contains a number of line segments. Find the sum of the absolute values of the slopes of these segments.
- A) 4
  - B) 8
  - C) 12
  - D) 16
  - E) NOTA
13. Let  $r, s, t$  be the three solutions to the equation  $x^3 - x^2 + 3x - 4 = 0$ . Find  $r^2 + s^2 + t^2$ .
- A)  $-5$
  - B)  $-2$
  - C) 4
  - D) 7
  - E) NOTA
14. Find the sum of the solutions to  $3 \log_x 2 + \log_8 x = \log_4 32 - \log_{64} x$ .
- A) 4
  - B) 8
  - C) 12
  - D) 16
  - E) NOTA

15. When the complex number  $a + bi$  is plotted on the Argand plane, it is equidistance to  $7 + 2i$ ,  $9 + 6i$ , and  $3 - 2i$ . Find  $a + b$ .
- A)  $-3$   
B)  $5$   
C)  $\frac{25}{3}$   
D)  $11$   
E) NOTA
16. During a practice, Mr. Lu randomly handed out 12 cards to 12 students to determine the teams for that practice. There were four cards each of hearts, clubs, and diamonds, and the students who received cards with matching suits are placed on a team. Michelle was the third to receive a card, and Stephanie was the fourth. What is the probability that they are on the same team?
- A)  $\frac{1}{11}$   
B)  $\frac{1}{4}$   
C)  $\frac{3}{11}$   
D)  $\frac{1}{3}$   
E) NOTA
17. If  $\sin \alpha = \frac{5}{13}$ ,  $\tan \beta = -\frac{15}{8}$ , and  $\frac{\pi}{2} < \alpha < \beta < 2\pi$ . Find  $\cos(\alpha + \beta)$
- A)  $-\frac{171}{221}$   
B)  $-\frac{21}{221}$   
C)  $\frac{21}{221}$   
D)  $\frac{171}{221}$   
E) NOTA
18. Given points  $B$  at  $(4, 17)$ ,  $E$  at  $(7, 8)$ , and  $V$  at  $(10, 9)$ . Ray  $EL$  bisects  $\angle BEV$ . Find the slope of ray  $EL$ .
- A)  $-\frac{4}{3}$   
B)  $1$   
C)  $\frac{3}{2}$   
D)  $2$   
E) NOTA

19. One of the first things you will learn in calculus is the special limit  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ . Use this fact to find  $\lim_{x \rightarrow 0} \frac{\sin^2 5x}{x^2}$ .
- A)  $\frac{1}{5}$   
B) 1  
C) 5  
D) 25  
E) NOTA
20. The function  $L(n)$  returns the units' digit of positive integer  $n$ . Evaluate  $\sum_{k=0}^{2016} L(2^k + 3^k)$ .
- A) 10080  
B) 10082  
C) 10085  
D) 10087  
E) NOTA
21. For  $0 \leq x < 2\pi$ , find the sum of the solutions of  $\cos^4 x - 4 \cos^2 x + 1 = 0$
- A) No solutions  
B)  $2\pi$   
C)  $4\pi$   
D)  $8\pi$   
E) NOTA
22. Four fair standard 6-sided dice are rolled. The probability of getting a sum of exactly 10 can be expressed as  $\frac{k}{6^4}$ . Find the value of  $k$ .
- A) 76  
B) 80  
C) 84  
D) 88  
E) NOTA
23. In  $\triangle ABC$ ,  $AB = AC = 6$ , and  $BC = 4$ . Find the product of the lengths of the three medians in  $\triangle ABC$ .
- A)  $\frac{512\sqrt{2}}{9}$   
B)  $64\sqrt{2}$   
C)  $68\sqrt{2}$   
D)  $100\sqrt{2}$   
E) NOTA

24. Line  $l$  passes through points  $(3, -5, -2)$  and  $(-1, 3, -6)$ . Plane  $P$  can be described by the equation  $7x + 2y - z = 12$ . Let  $\theta$  be the smallest angle formed between line  $l$  and plane  $P$ . Find  $\cos \theta$ .
- A)  $\frac{1}{9}$   
B)  $\frac{\sqrt{17}}{9}$   
C)  $\frac{8}{9}$   
D)  $\frac{4\sqrt{5}}{9}$   
E) NOTA
25.  $\{a_1, a_2, a_3, \dots\}$  is an infinite geometric sequence. If  $\sum_{n=0}^{\infty} a_{2n+1} = 19$  and  $\sum_{n=0}^{\infty} a_{3n+1} = 15$ , find the sum of the absolute values of the possible common ratios.
- A)  $\frac{2}{15}$   
B)  $\frac{4}{15}$   
C)  $\frac{8}{15}$   
D)  $\frac{16}{15}$   
E) NOTA
26. Let  $f(x) = 6x^4 + ax^3 + bx^2 + ax + 6$ , where  $a$  and  $b$  are positive integers. If  $f(x)$  has four distinct rational roots, find  $f(1)$ .
- A) 0  
B) 100  
C) 144  
D) 196  
E) NOTA
27. Two parallel chords in a circle have lengths 18 and 30. If the distance between these chords is 18, find the area of the circle.
- A)  $225\pi$   
B)  $250\pi$   
C)  $324\pi$   
D)  $394\pi$   
E) NOTA

28. The 7 solutions of  $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$  are plotted on the Argand plane. These 7 points are then connected in order to form a convex heptagon. Find the area contained in this heptagon.
- A)  $\frac{3\sqrt{2}}{2}$   
B)  $\frac{4\sqrt{2}-1}{2}$   
C)  $\frac{3\sqrt{2}+1}{2}$   
D)  $2\sqrt{2}$   
E) NOTA
29. Evaluate  $\sum_{n=1}^{500} 2 \sin 1 \cos(2n - 1)$
- A)  $\sin 1000$   
B)  $-\sin 1000$   
C)  $\cos 1000 - 1$   
D)  $1 - \cos 1000$   
E) NOTA
30. The typical security protocol for a WiFi connection, WiFi Protected Access (WPA), uses a 256-bit key. In other words, a 256-digit binary number is used as password. A (poor) program performs brute force attack on a WiFi connection by trying all the possible numbers as passwords. If the program is capable of trying a billion numbers each second, the number of years to actually try all the possible numbers can be expressed as  $10^k$ , for some real number  $k$ . Which of the following is the closest to the value of  $k$ ? (This is called Fermi estimation, which approximates quantities only by their orders of magnitude. For example, there are 10 questions on this test. The dispute room reads 100 disputes following each round of topic tests. There are 1000 attendees at this national convention, taking in a total of 10000 meals during the convention.)
- A) 55  
B) 60  
C) 65  
D) 70  
E) 75