1. C
2. D
3. D
4. A
5. B
6. E
7. D
8. D
9. A
10. C
11. A
12. B
13. A
14. C
15. B
16. C
17. B
18. D
19. D
20. B
21. C
22. B
23. C
24. D
25. D
26. C
27. B
28. C
29. A
30. B

1. C When $r = \cos(a\theta)$ is graphed, it either results in a rose curve with a petals if a is odd, or $2a$ petals if a is even. So the number of petals is either odd, or a multiple of 4. Thus n must be even while not a multiple of 4, and n^2 leaves a remainder of 4 when divided by 8.
2. D Let $z = a + bi$, then the equation becomes $(a + bi) + 2i(a - bi) = 1 + 17i$. Separate out the real and imaginary parts to get $a + 2b = 1$ and $2a + b = 17$. Solving the system of equation to get $z = 11 - 5i$. So $|z| = \sqrt{11^2 + 5^2} = \sqrt{146}$
3. D $A \cup B^C$ excludes the region only in *B*. $A^C \cup B$ excludes the region only in A. $(A \cup B)^C$ is the region entirely outside of *A* and *B*, which is contained within the first 2 sets. Thus the intersection of the three sets is simply the third set, which can be equivalently expressed as $A^C \cap B^C$.
4. A For the system to have no solution, the determinant of the coefficient matrix must be 0. So $\begin{vmatrix} 2 & -1 & 3 \\ 7 & -2 & -k \\ 5 & -2 & 1 \end{vmatrix} = -k - 9 = 0$. Solving to get $k = -9$.
5. B 540 factors into $2^2 \cdot 3^3 \cdot 5$. However, since $\log_6 3$ is not provided, we can rewrite 540 as $\frac{6^3 \cdot 5}{2}$. So $\log_6 540 = 3 + \log_6 5 - \log_6 2 = 3 + 0.898 - 0.387 = 3.511$.
6. E All four functions are equivalent. It can be seen A and C are equal using cofunction identity $\cos x = \sin\left(\frac{\pi}{2} - x\right)$, similarly for B and D. B and C are equal because sine is an odd function.
7. D Placing the Gateway Arch on a coordinate plane, with the origin on the ground in center of the arch, with x-axis extending along the base, and y-axis represent the height. Then the parabolic arch can be expressed as $y = -\frac{2x^2}{315} + 630$. If Sanika is one third of the way from one foot to the other, she is 105 feet from the center. So substituting in 105 for *x* to get $y = 560$.
8. D First factor the numerator and denominator to get $f(x) = \frac{(x-1)(x+2)(2x+3)}{(x-3)(x+2)}$. So there is a removable discontinuity at $x = -2$, making it neither a vertical asymptote nor an x-intercept. There is one vertical asymptote at $x = 3$, which as an x-intercept at $(3, 0)$. $f(x)$ has two x-intercepts at $(1, 0)$ and $\left(-\frac{3}{2}, 0\right)$. The y-intercept of $f(x)$ can be found by substituting in 0 for *x*, at $(0, 1)$. Finally, the slant asymptote can be found by dividing the numerator by the denominator, which is $y = 2x + 7$. It has intercepts at $\left(-\frac{7}{2}, 0\right)$ and $(0, 7)$. Adding up all the coordinates of these intercepts to get 7.
9. A Statements I, IV, and VI are true.
10. C The period of $\cos\left(\frac{x}{3}\right)$ is 6π, and the period of $\sin\left(\frac{x}{4}\right)$ is 8π, so the period of $f(x)$ is the least common multiple of the two, which is 24π.
11. A The bus schedule cycles every 30 minutes, so looking at any half-hour window is sufficient, say 7:00 to 7:30. The only time Jonathan would get on a Route 15 bus is if he walked outside between 7:08 and 7:15, or between 7:28 and 7:30, which combine to span 9 of the 30 minutes. So the desired probability is $\frac{9}{30} = 0.3$.

12. B The graph contains 4 line segments. On the interval $\left[-100, -\frac{7}{3}\right]$, the slope is $(-3) - (-2) + (-1) = -2$. On $\left[-\frac{7}{3}, \frac{3}{2}\right]$, the slope is $3 - (-2) + (-1) = 4$. On $\left[\frac{3}{2}, 5\right]$, the slope is $3 - 2 + (-1) = 0$. On $[5, 100]$, the slope is $3 - 2 + 1 = 2$. Adding up the absolute values of these slopes to get 8.

13. A Using Vieta's formulas, $r + s + t = 1$ and $rs + rt + st = 3$.
So $r^2 + s^2 + t^2 = (r + s + t)^2 - 2(rs + rt + st) = -5$.

14. C Manipulate the terms of the equation a bit, the equation can be rewritten as $\frac{1}{\log_8 x} + \log_8 x = \frac{5}{2} - \frac{1}{2}\log_8 x$. Make the substitution $a = \log_8 x$, the equation becomes $\frac{1}{a} + \frac{3}{2}a - \frac{5}{2} = 0$, or $3a^2 - 5a + 2 = 0$ when multiplying through by $2a$. This can easily be solved to get $a = 1, \frac{2}{3}$. So $x = 8^1, 8^{\frac{2}{3}}$, or 8 and 4. So the sum of the solutions is 12.

15. B Let points A, B, and C be $(7, 2), (9, 6)$, and $(3, -2)$, respectively. The point that is equidistance to A, B, and C is the circumcenter of ΔABC, or the point of intersection of the perpendicular bisectors. Segment *AB* has a slope of 2 and midpoint at $(8, 4)$, so its perpendicular bisector is $y = -\frac{1}{2}x + 8$. Segment *AC* has slope of 1 and midpoint at $(5, 0)$, so its perpendicular bisector is $y = -x + 5$. Solving the system of equations to get their point of intersection to be $(-6, 11)$, so $a + b = -6 + 11 = 5$.

16. C The order the cards are received does not matter. There are a total of $\binom{12}{2} = 66$ ways for Michelle and Stephanie to receive their cards, and a total of $3\binom{4}{2} = 18$ ways for them to receive the same suit. So the probability is $\frac{18}{66} = \frac{3}{11}$.

17. B Of the 3 quadrants α is restricted to, the only quadrant where value of sine is positive is Quadrant II. Both quadrants II and IV have negative tangent values, but since β has a larger reference angle than α, it cannot be in Quadrant II based on the restrictions. Thus $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(-\frac{12}{13}\right)\left(\frac{8}{17}\right) - \left(\frac{5}{13}\right)\left(-\frac{15}{17}\right) = -\frac{21}{221}$.

18. D Since the question only asks about the slope, the exact position of the points is not relevant. For convenience, shift point *E* to the origin to have *B* at $(-3, 9)$ and *V* at $(3, 1)$. Note that $BE = 3EV$, so scale BE by a factor of $\frac{1}{3}$ to have point B' at $(-1, 3)$. Then $\Delta B'EV$ is an isosceles triangle with vertex at *E*, and the angle bisector of $\angle BEV$ would pass through the midpoint of $B'V$, which is at $(1, 2)$, so the slope of ray EL is 2.

19. D The idea is to match the denominator with the argument of sine in the numerator, so $\lim_{x\to 0}\frac{\sin^2 5x}{x^2} = \lim_{x\to 0}\frac{5\sin 5x}{5x}\cdot\frac{5\sin 5x}{5x}$. Based on the information given, $\lim_{x\to 0}\frac{\sin 5x}{5x} = 1$, so the limit is simply 25.

20. B With exception of $k = 0$, the last digits of powers of 2 and 3 cycle for every 4th power. Specifically, $L(2^1 + 3^1) = 5, L(2^2 + 3^2) = 3, L(2^3 + 3^3) = 5, L(2^4 + 3^4) = 7$. For $1 \le k \le 2016$, there are 504 cycles of 4, so the sum is $2 + 504(20) = 10082$.

21. C The equation is a quadratic in terms of $\cos^2 x$, solving by either quadratic formula or completing the square to get $\cos^2 x = 2 \pm \sqrt{3}$. $2+\sqrt{3} > 1$, which is impossible for $\cos^2 x$, so $\cos x = \pm\sqrt{2-\sqrt{3}}$. These correspond to 4 solutions, one in each quadrant, all with the same reference angle. So the sum of these solutions is 4π.

22. B Let the results of the 4 dice be $a, b, c,$ and d. Then $a+b+c+d=10$. Since each roll is at least 1, 1 can be subtracted from each variable to get $a'+b'+c'+d'=6$. The problem becomes the number or ordered non-negative quadruples that satisfy the equation, without any of the variables having a value exceeding 5. This is then equivalent to distributing 6 indistinguishable balls into 4 distinguishable buckets, which can be done in $\binom{9}{3}=84$ ways. There are 4 ways where all the balls end up in the same bucket, so the number of ways to roll a sum of 10 with 4 dice is $84-4=80$.

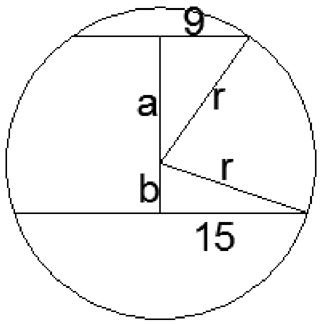
23. C The length of the median from point A is simply $\sqrt{6^2-2^2}=4\sqrt{2}$, as A is the vertex of the isosceles triangle. The medians from points B and C are also clearly equal. Let D be the midpoint of AC, and let $BD = x, m\angle BDC = \theta$, so $m\angle BDA = \pi - \theta, DC = DA = 3$. Using law of cosines on ΔBDC and ΔBDA, both at the vertex D to get $3^2 + x^2 - 6x\cos\theta = 4^2$ and $3^2 + x^2 + 6x\cos\theta = 6^2$, since $\cos(\pi-\theta) = -\cos\theta$. Adding the two equations together to get $18 + 2x^2 = 52$, or $x^2 = 17$. So the product of the lengths of the medians is $17(4\sqrt{2}) = 68\sqrt{2}$.

24. D The vector describing the direction line *l* is in is $< -4, 8, -4 >$, which can be reduced to $< 1, -2, 1 >$. The normal vector to plane *P* is $< 7, 2, -1 >$. The angle between these two vectors is the complement of θ, so $\cos\left(\frac{\pi}{2}-\theta\right) = \frac{7-4-1}{\sqrt{6}\cdot\sqrt{54}} = \frac{1}{9}$, and $\cos\theta = \sqrt{1-\left(\frac{1}{9}\right)^2} = \frac{4\sqrt{5}}{9}$.

25. D Let *r* be the common ratio of the series a_k. The summations given are infinite geometric series of common ratio r^2 and r^3. So $\frac{a_1}{1-r^2} = 19$, and $\frac{a_1}{1-r^3} = 15$. Combining the two equations to get $\frac{1-r^3}{1-r^2} = \frac{19}{15}$. Reducing the fraction on the left hand side by $(1-r)$ to get $\frac{1+r+r^2}{1+r} = \frac{19}{15}$, cross multiply and rearrange to get $15r^2 - 4r - 4 = 0$, or $(5r+2)(3r-2) = 0$. So the possible common ratios are $-\frac{2}{5}$ and $\frac{2}{3}$, and $\frac{2}{5}+\frac{2}{3} = \frac{16}{15}$.

26. C By Descartes' Rule of Signs, all the roots of $f(x)$ must be negative. Since the coefficients of $f(x)$ are symmetric, all the roots of $f(x)$ must either be ± 1 or come in reciprocal pairs. Since all roots of $f(x)$ are distinct, -1 is not a possible root. Therefore, the four distinct negative roots of $f(x)$ must be $-3, -2, -\frac{1}{2}, -\frac{1}{3}$. In other words, $f(x) = (x+3)(x+2)(2x+1)(3x+1)$, and $f(1) = 4\cdot 3\cdot 3\cdot 4 = 144$.

27. B



In the figure on the left, a and b are the distances away from the center for chords of lengths 18 and 30 respectively, and r is the radius of the circle. Using the two right triangles, $a^2 + 9^2 = r^2$ and $b^2 + 15^2 = r^2$. So $a^2 + 9^2 = b^2 + 15^2$, in other words, $a^2 - b^2 = 15^2 - 9^2$, which factors to $(a + b)(a - b) = 144$. The distance between the chords is 18, which can either be $(a + b)$ or $(a - b)$ depending on whether the chords are on the same side of the center, with the other being $\frac{144}{18} = 8$. So clearly, $a + b = 18, a - b = 8$, or $a = 13, b = 5$. So $\pi r^2 = 250\pi$.

28. C The polynomial can be viewed as a finite geometric series, so it can be re-expressed as $\frac{x^8-1}{x-1} = 0$. Therefore, the solutions to the polynomials are the 8th roots of unity, with the exception of 1. The heptagon is composed of 6 isosceles triangles with legs of length 1 and vertex angle of $45°$ along with an isosceles right triangle with legs of length 1. So the overall area is $6\left(\frac{1}{2} \cdot 1^2 \sin 45°\right) + \frac{1}{2} \cdot 1^2 = \frac{3\sqrt{2}+1}{2}$.

29. A Using the product-to-sum formula, $2 \sin 1 \cos(2n - 1) = \sin 2n - \sin(2n - 2)$. Thus the sum becomes $\Sigma_{n=1}^{500}(\sin 2n - \sin(2n - 2)) = \sin 2 - \sin 0 + \sin 4 - \sin 2 + \cdots + \sin 1000 - \sin 998$ which is telescopes to $\sin 1000 - \sin 0 = \sin 1000$.

30. B The number of years to go through all the numbers is $\frac{2^{256}}{10^9 \cdot 3600 \cdot 24 \cdot 365.25}$. Of course, this is an enormous number to actually compute, so estimates must be made. $2^{10} = 1024$, which can be approximated as 10^3, so $2^{256} = 2^6(2^{10})^{25} \approx 64(10^{75}) \approx 10^{77}$. A rough computation of $3600 \cdot 24 \cdot 365.25$ yields the number of seconds in a year to be on the order of 10^7. So the overall expression can be approximated as $\frac{10^{77}}{10^9 \cdot 10^7} = 10^{61}$, making B the closest choice. In reality, the denominator was underestimated, making 61 a little large. The actual answer to the computation is about 3.7×10^{60} years $(k = 60.56)$, or 2.7×10^{50} times the age of the universe.