

Where applicable, “E) NOTA” indicates that none of the above answers is correct.

1. Answer C: $1 + 3 + 5 + \dots + (1 + (n - 1)(2))$

$$a_{2015} = 1 + 2014(2) = 4029$$

$$S_{2015} = \left(\frac{1 + 4029}{2}\right)(2015) = 2015^2$$

The square root of the sum is 2015.

2. Answer C: $[\log_{abc}(a^c)][1 + \log_a b + \log_a c] =$
 $[\log_{abc}(a^c)][\log_a a + \log_a b + \log_a c] = [\log_{abc}(a^c)][\log_a abc]$
 $\left[\frac{\log_a a^c}{\log_a abc}\right][\log_a abc] = \log_a a^c = c = 340$

3. Answer A: $a_5 = 5!, a_6 = 6!, r = \frac{6!}{5!} = 6$

Since $a_5 = a_4(r)$ then $a_4 = \frac{a_5}{r} = \frac{5!}{6} = \frac{120}{6} = 20$

4. Answer C: $2e^x + e^{-x} = 3$
 $(2e^x + e^{-x})(e^x) = 3e^x$
 $2e^{2x} + e^0 = 3e^x$
 $2e^{2x} - 3e^x + 1 = 0$
 $(2e^x - 1)(e^x - 1) = 0$
 $e^x = \frac{1}{2} \quad e^x = 1$

The sum of the solutions for e^x is $\frac{1}{2} + 1 = \frac{3}{2}$

5. Answer C:

$$\frac{1}{\log_2 100!} + \frac{1}{\log_3 100!} + \frac{1}{\log_4 100!} + \dots + \frac{1}{\log_{100} 100!} =$$

$$\frac{\log 2}{\log 100!} + \frac{\log 3}{\log 100!} + \frac{\log 4}{\log 100!} + \dots + \frac{\log 100}{\log 100!} =$$

$$\frac{1}{\log 100!} (\log 2 + \log 3 + \log 4 + \dots + \log 100) =$$

$$\frac{1}{\log(2 \cdot 3 \cdot 4 \cdot \dots \cdot 100)} (\log(2 \cdot 3 \cdot 4 \cdot \dots \cdot 100)) = 1$$

6. Answer B: $\sqrt[3]{\frac{17\sqrt{7}+45}{4}} = \sqrt[3]{\frac{34\sqrt{7}+90}{8}} = \frac{\sqrt[3]{(3+\sqrt{7})^3}}{2} = \frac{3+\sqrt{7}}{2} = \frac{a+\sqrt{b}}{c}$

$$a + b + c = 3 + 7 + 2 = 12$$

7. Answer D: $27^x - 9^{x-1} - 3^{x+1} + \frac{1}{3} = 0$
 $3^{3x} - 3^{2x-2} - 3^{x+1} + 3^{-1} = 0$
 $3^2(3^{3x} - 3^{2x-2} - 3^{x+1} + 3^{-1}) = 3^2(0)$
 $9(3^{3x}) - 3^{2x} - 27(3^x) + 3 = 0$
 Let $y = 3^x$ $9y^3 - y^2 - 27y + 3 = 0$
 $y^2(9y - 1) - 3(9y - 1) = 0$
 $(y^2 - 3)(9y - 1) = 0$
 $y = \pm\sqrt{3} \quad y = \frac{1}{9}$

Since $y = 3^x$ then $3^x = \sqrt{3}$ and $3^x = \frac{1}{9}$. ($3^x = -\sqrt{x}$ is undefined)

Therefore the solutions are $x = \frac{1}{2}$ and $x = -2$ and the sum of solutions is $\frac{1}{2} + (-2) = -\frac{3}{2}$.

8. Answer C: $0 = \ln t$ when $t = e^0 = 1$
 @ $t = 1$ $y = e^2$
 Hence the y intercept must be at $(0, e^2)$.

9. Answer E: If $x = \frac{2015}{2015+x}$ then $x^2 + 2015x - 2015 = 0$
 Using the quadratic formula and discarding the extraneous solution,
 $x = \frac{-2015 + \sqrt{(2015)^2 - 4(1)(-2015)}}{2} = \frac{-2015 + \sqrt{2015(2015+4)}}{2} = \frac{\sqrt{2015(2019)} - 2015}{2} = \frac{\sqrt{ab} - a}{2}$
 $a - b = 2015 - 2019 = -4$

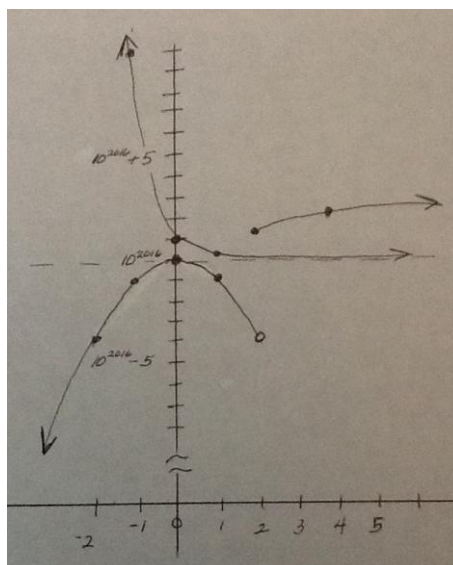
10. Answer D: To find $g(3)$, x must be found by $\sqrt{\frac{x-1}{x+1}} = 3$.
 $\frac{x-1}{x+1} = 9$ and therefore $x - 1 = 9x + 9$. Hence $x = \frac{-5}{4}$.

Since $g\left(\sqrt{\frac{x-1}{x+1}}\right) = 3x$ then $g(3) = g\left(\sqrt{\frac{\frac{-5}{4}-1}{\frac{-5}{4}+1}}\right) = 3\left(\frac{-5}{4}\right) = -\frac{15}{4}$

11. Answer A: $\log_2(\log_3(\log_5(\log_7 M))) = 11$.
 $2^{11} = \log_3(\log_5(\log_7 M))$
 $3^{2^{11}} = \log_5(\log_7 M)$
 $5^{3^{2^{11}}} = \log_7 M$
 $M = 7^{5^{3^{2^{11}}}} = 7^{3^{30}}$ Therefore 7 is the only prime number.

12. Answer A:

There are no points in common.



13. Answer C: $|\sqrt{n-5}| < 1$
 $-1 < \sqrt{n} - 5 < 1$
 $-4 < \sqrt{n} < 6$
 $16 < n < 36$ There are 19 integers between 16 and 36.

14. Answer D: $3 = k(2)^r$ and $15 = k(4)^r$
 $\frac{15}{3} = \left(\frac{4}{2}\right)^r$
 $5 = 2^r$
 $r = \log_2 5$

15. Answer C: $y^x = \left(\left(1 + \frac{1}{n}\right)^{n+1}\right)^{\left(1 + \frac{1}{n}\right)^n} = \left(\left(1 + \frac{1}{n}\right)^{n+1}\right)^{\frac{(n+1)^n}{n^n}} = \left(1 + \frac{1}{n}\right)^{\frac{(n+1)(n+1)^n}{n^n}} =$
 $\left(1 + \frac{1}{n}\right)^{\frac{(n)(n+1)^{n+1}}{(n)n^n}} = \left(1 + \frac{1}{n}\right)^{\frac{(n)(n+1)^{n+1}}{n^{n+1}}} = \left(1 + \frac{1}{n}\right)^{n\left(\frac{n+1}{n}\right)^{n+1}} = \left(1 + \frac{1}{n}\right)^{ny} =$
 $\left[\left(1 + \frac{1}{n}\right)^n\right]^y = x^y$

16. Answer D: $\sqrt[3]{L^3 \sqrt[3]{L^3 \sqrt[3]{L}}} = \left(L \left(L \left(L \frac{1}{3}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \left(L \left(L^{\frac{4}{3}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}} = \left(L \left(L^{\frac{4}{9}}\right)\right)^{\frac{1}{3}} = \left(L^{\frac{13}{9}}\right)^{\frac{1}{3}} = L^{\frac{13}{27}}$

$$\begin{aligned}
 17. \text{ Answer B: } & x^{1+\log_{\frac{1}{2}} x} > \frac{x}{4} \\
 & \log_x x^{1+\log_{\frac{1}{2}} x} > \log_x \frac{x}{4} \\
 & 1 + \log_{\frac{1}{2}} x > \log_x x - \log_x 4 \\
 & 1 + \log_{\frac{1}{2}} x > 1 - \log_x 4 \\
 & \log_{\frac{1}{2}} x > -\log_x 4 \\
 & \frac{\log_2 x}{\log_2 \frac{1}{2}} > -\frac{\log_2 4}{\log_2 x} \\
 & \frac{\log_2 x}{-1} > -\frac{-2}{\log_2 x} \\
 & (\log_2 x)^2 < 2
 \end{aligned}$$

If $y = \log_2 x$ then $y^2 - 2 < 0$ and $(y - \sqrt{2})(y + \sqrt{2}) < 0$. So $-\sqrt{2} < y < \sqrt{2}$.

Converting back $-\sqrt{2} < \log_2 x < \sqrt{2}$ and $2^{-\sqrt{2}} < x < 2^{\sqrt{2}}$.

$$\begin{aligned}
 18. \text{ Answer B: } & 9 - x^2 \geq 0 & \text{and} & & 9 - |2x + 5| > 0 \\
 & x^2 - 9 \leq 0 & & & |2x + 5| < 9 \\
 & (x - 3)(x + 3) \leq 0 & & & -9 < 2x + 5 < 9 \\
 & -3 < x < 3 & & & -7 < x < 2
 \end{aligned}$$

So the domain must be the interval $[-3, 2)$. Since the positive portion of this interval is $(0, 2)$ is $\frac{2}{5}$ of the domain, the answer must be 40%.

$$\begin{aligned}
 19. \text{ Answer A: } & 7.4 = -\log[H]^+ & \text{and} & & 7.2 = -\log[H]^+ \\
 & -7.4 = \log[H]^+ & & & -7.2 = \log[H]^+ \\
 & [H]^+ = 10^{-7.4} & & & [H]^+ = 10^{-7.2}
 \end{aligned}$$

$$\text{\% increase is } \frac{10^{-7.2} - 10^{-7.4}}{10^{-7.4}} (10^2) = \frac{10^{-7.2} - 10^{-7.4}}{10^{-9.4}} = \frac{10^{-7.2}}{10^{-9.4}} - \frac{10^{-7.4}}{10^{-9.4}} = 10^{2.2} - 10^2$$

20. Answer D: Here are some facts that will simplify the expression

$$x^3 = 1 = x^{2016} \text{ and } x^2 = x^{2015}$$

$$x^3 - 1 = 0 \text{ means } (x - 1)(x^2 + x + 1) = 0.$$

$$\log\left(x^2 + \frac{1}{x^2} + x^3 + \frac{1}{x^3}\right) = \log\left(x^2 + \frac{1}{x^2} + 1 + 1\right) = \log\left(x^2 + \frac{x^3}{x^2} + 1 + 1\right) =$$

$$\log(x^2 + x + 1 + 1) = \log(x^2 + x + 1 + 1) = \log(0 + 1) = \log(1) = 0$$

$$\begin{aligned}
 21. \text{ Answer D: } & 2^{x^2} = 16^{2x-3} \\
 & 2^{x^2} = (2^4)^{2x-3} \\
 & 2^{x^2} = 2^{8x-12} \\
 & x^2 = 8x - 12 \\
 & x^2 - 8x + 12 = 0 \\
 & (x - 2)(x - 6) = 0 \\
 & x = 2 \text{ or } x = 6
 \end{aligned}$$

The sum of the solutions is $2 + 6 = 8$.

22. Answer C: $(\sqrt[3]{x+9} - \sqrt[3]{x-9})^3 = 3^3$

$$x + 9 - 3\sqrt[3]{(x+9)^2}\sqrt[3]{x-9} + 3\sqrt[3]{x+9}\sqrt[3]{(x-9)^2} - (x-9) = 27$$

$$-3\sqrt[3]{(x+9)^2}\sqrt[3]{x-9} + 3\sqrt[3]{x+9}\sqrt[3]{(x-9)^2} = 9$$

$$\sqrt[3]{(x+9)^2}\sqrt[3]{x-9} - \sqrt[3]{x+9}\sqrt[3]{(x-9)^2} = -3$$

$$\sqrt[3]{(x^2-81)(x+9)} - \sqrt[3]{(x^2-81)(x-9)} = -3$$

$$\sqrt[3]{x^2-81}(\sqrt[3]{x+9} - \sqrt[3]{x-9}) = -3$$

$$\sqrt[3]{x^2-81}(3) = -3$$

$$\sqrt[3]{x^2-81} = -1$$

$$x^2 - 81 = -1$$

$$x^2 = 80$$

23. Answer C:

$$\sum_{n=1}^{\infty} \left(\frac{1}{\log_2 4^{2^n}} \right) = \sum_{n=1}^{\infty} \left(\frac{1}{\log_2 (2^2)^{2^n}} \right) = \sum_{n=1}^{\infty} \left(\frac{1}{\log_2 (2)^{2^{n+1}}} \right) = \sum_{n=1}^{\infty} \left(\frac{1}{2^{n+1}} \right) = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{\frac{1}{4}}{1 - \frac{1}{2}} = \frac{1}{2}$$

24. Answer A: Let $x = \sqrt[3]{26 + 15\sqrt{3}}$ and $y = \sqrt[3]{26 - 15\sqrt{3}}$ then $z = x + y$.

$$z^3 = (x + y)^3 = 26 + 15\sqrt{3} + 3x^2y + 3xy^2 + 26 - 15\sqrt{3}$$

$$z^3 = 52 + 3xy(x + y)$$

$$z^3 = 52 + 3\sqrt[3]{(26^2) - (15^2)(3)}(z)$$

$$z^3 = 52 + 3\sqrt[3]{676 - 675}(z)$$

$$z^3 = 52 + 3(z)$$

$$z^3 - 3z - 52 = 0$$

$$(z - 4)(z^2 + 4z + 13) = 0$$

Since $z^2 + 4z + 13$ has no real roots, $z = 4$.

25. Answer C: $x = -\sqrt{5 - \sqrt{5 - \sqrt{5 - \sqrt{5 - \dots}}}}$

$$x = -\sqrt{5 + x}$$

$$x^2 = 5 + x$$

$$x^2 - x - 5 = 0$$

Using the quadratic formula $x = \frac{1 \pm \sqrt{21}}{2}$. However, since x must be negative $x = \frac{1 - \sqrt{21}}{2}$

26. Answer D: Let $x = \log_6 16$ and use $a = \log_{12} 27$ to combine expressions using $\frac{\log 3}{\log 2}$.

$$a = \log_{12} 27 = \log_{12} 3^3 = 3 \log_{12} 3 = \frac{3 \log 3}{\log 12} = \frac{3 \log 3}{\log 3 + \log 4} = \frac{3 \log 3}{\log 3 + 2 \log 2}$$

$$\frac{3}{a} = \frac{\log 3 + 2 \log 2}{\log 3} = 1 + 2 \frac{\log 2}{\log 3}$$

$$\frac{\log 2}{\log 3} = \frac{1}{2} \left(\frac{3}{a} - 1 \right) = \frac{3 - a}{2a}$$

$$\frac{\log 3}{\log 2} = \frac{2a}{3 - a}$$

$$x = \log_6 16 = \log_6 2^4 = 4 \log_6 2 = \frac{4 \log 2}{\log 6} = \frac{4 \log 2}{\log 3 + \log 2}$$

$$\frac{4}{x} = \frac{\log 3 + \log 2}{\log 2} = \frac{\log 3}{\log 2} + 1$$

$$\frac{\log 3}{\log 2} = \frac{4}{x} - 1 = \frac{2a}{3 - a} \text{ from above.}$$

$$\frac{4}{x} = \frac{2a}{3 - a} + 1 = \frac{2a + 3 - a}{3 - a} = \frac{a + 3}{3 - a}$$

$$\frac{x}{4} = \frac{3 - a}{a + 3} \text{ so } x = \frac{4(3 - a)}{a + 3}$$

27. Answer E: The vertical asymptote of $y = \log_2 x$ is $x = 0$ and its domain is $(0, \infty)$. The graph of $y = -1 - 5 \log_2(-2x + 3)$ can be rewritten in the form

$$y = -1 - 5 \log_2 \left[-2 \left(x - \frac{3}{2} \right) \right].$$

Therefore $x = \frac{3}{2}$ is its vertical asymptote.

$$28. \text{ Answer A: } \begin{vmatrix} x & e^{\ln 6} & 2e^{\ln 1} \\ e^{2 \ln 3 - 3 \ln 2} & 0 & e^{-\ln 5} \\ e^{\ln 5} & 5^{\ln e^2} & e^{\pi i} \end{vmatrix} = \begin{vmatrix} x & 6 & 2 \\ \frac{9}{8} & 0 & \frac{1}{5} \\ 5 & 25 & -1 \end{vmatrix} = -\frac{9}{8} \begin{vmatrix} 6 & 2 \\ 25 & -1 \end{vmatrix} - \frac{1}{5} \begin{vmatrix} x & 6 \\ 5 & 25 \end{vmatrix} = 0$$

$$-\frac{9}{8}(-6 - 50) - \frac{1}{5}(25x - 30) = 0. \text{ Therefore } 63 - 5x + 6 = 0 \text{ and } x = \frac{69}{5}$$

29. Answer B: Let $y = \ln x$ and k be an integer.

$\sin(y) + 2 \cos(3y) \sin(2y) = 0$ can be rewritten using a product to sum identity.

$$\sin(y) + 2 \left(\frac{\sin(3y + 2y) - \sin(3y - 2y)}{2} \right) = 0$$

$$\sin(y) + \sin(5y) - \sin(y) = 0$$

$$\sin(5y) = 0; 5y = \pi k; y = \frac{\pi k}{5}; y = \ln x = \frac{\pi k}{5}; x = e^{\frac{\pi k}{5}}.$$

Since x must be the smallest value greater than 1,

$$x = e^{\frac{\pi}{5}} \text{ which makes } a = 1, b = 5 \text{ and } ab = 5.$$

30. Answer D: Let $x = \sqrt{a + \sqrt{a + \dots}}$ and $y = \frac{1}{a + \frac{1}{a + \dots}}$.

$$x = \sqrt{a + x}$$

$$x^2 = a + x$$

$$x^2 - x = a$$

$$x^2 - x + \frac{1}{4} = a + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = a + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right) = \sqrt{a + \frac{1}{4}}$$

$$x = \sqrt{a + \frac{1}{4}} + \frac{1}{2}$$

$$y = \frac{1}{a + y}$$

$$ay + y^2 = 1$$

$$y^2 + ay + \left(\frac{a}{2}\right)^2 = 1 + \left(\frac{a}{2}\right)^2$$

$$\left(y + \frac{a}{2}\right)^2 = 1 + \frac{a^2}{4}$$

$$y + \frac{a}{2} = \sqrt{1 + \frac{a^2}{4}}$$

$$y = \sqrt{1 + \frac{a^2}{4}} - \frac{a}{2}$$

$$x - y = \sqrt{a + \frac{1}{4}} + \frac{1}{2} - \left(\sqrt{1 + \frac{a^2}{4}} - \frac{a}{2}\right)$$

$$2(x - y) = 2\sqrt{a + \frac{1}{4}} + 1 - 2\sqrt{1 + \frac{a^2}{4}} + a$$

$$2(x - y) = \sqrt{4a + 1} + 1 - \sqrt{4 + a^2} + a = a + 1 + \sqrt{4a + 1} - \sqrt{4 + a^2}$$

So, we are looking for the solution to:

$$a + 1 + \sqrt{4a + 1} - \sqrt{4 + a^2} > 14 \text{ or}$$

$$a + \sqrt{4a + 1} - \sqrt{4 + a^2} > 13$$

By guess and check, $a = 43$.