Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Given $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, find $|A|$.

   (A) 5 (B) -2 (C) -6 (D) -10 (E) NOTA

2. Let B be the sum of all values of b such that the matrix $\begin{bmatrix} 1 & 4 & 0 \\ 3 & b & -2 \\ 5 & -3 & b \end{bmatrix}$ is singular. What is the value of $4B$?

   (A) 40 (B) 44 (C) 48 (D) 52 (E) NOTA

3. Evaluate $\begin{bmatrix} 4 & -8 & 3 \\ 6 & 5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -4 \\ -3 & 9 & 4 \end{bmatrix}$.

   (A) $\begin{bmatrix} 5 & -10 & -1 \\ 3 & 15 & 6 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & -6 & -1 \\ 3 & 14 & 6 \end{bmatrix}$

   (C) $\begin{bmatrix} 5 & -6 & 1 \\ 3 & 14 & 6 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & -6 & -7 \\ 3 & 15 & 6 \end{bmatrix}$ (E) NOTA

4. Given the system of linear equations

$$\begin{aligned} 7x + 5y - 3z &= 23 \\ 3x - 5y + 2z &= -5 \\ 5x + 3y - 7z &= 5 \end{aligned}$$

   what is the value of $2x - y + 7z$?

   (A) 12 (B) 13 (C) 14 (D) 15 (E) NOTA

5. For what value(s) of t will the points $(-1, t), (3,8), (t, 2)$ be collinear?

   (A) 0 (B) 10 (C) 1, 10 (D) 0, 11 (E) NOTA

6. An $n \times n$ matrix with n distinct nonzero eigenvalues has how many square roots? A square root of a matrix A is a matrix B such that $B^2 = A$.

   (A) $2n$ (B) 2^n (C) n^2 (D) n - 1 (E) NOTA

7. Evaluate $\begin{bmatrix}3 & 3\\5 & 1\end{bmatrix}\begin{bmatrix}5 & 6\\10 & 4\end{bmatrix}$.

(A) $\begin{bmatrix}45 & 30\\35 & 34\end{bmatrix}$ (B) $\begin{bmatrix}35 & 34\\45 & 30\end{bmatrix}$ (C) $\begin{bmatrix}35 & 30\\35 & 34\end{bmatrix}$ (D) $\begin{bmatrix}45 & 34\\35 & 30\end{bmatrix}$ (E) NOTA

8. Let A be a 3×3 matrix where $A_{ij} = 3i - 5j$. What is the sum of the elements in A?

(A) -36 (B) -33 (C) -30 (D) -27 (E) NOTA

9. Evaluate $\langle \sqrt{3}\cos\frac{\pi}{8}, \log_3 343, \begin{vmatrix}5 & 8\\-3 & 5\end{vmatrix}\rangle \cdot \langle sin\frac{\pi}{8}, \log_7 9, 3\rangle$

(A) $\frac{\sqrt{6}+153}{4}$ (B) $\frac{\sqrt{6}}{4} + 153$ (C) $\frac{\sqrt{6}+153}{2}$ (D) $\frac{\sqrt{6}}{2} + 153$ (E) NOTA

10. Which of the following describes the matrix $\begin{bmatrix}3 & 3 & 3\\3 & 3 & 3\\3 & 3 & 3\end{bmatrix}$?

I. symmetric
II. nonsingular
III. square
IV. skew-symmetric

(A) II and III only (B) I and III only (C) I, II, and III only (D) III and IV only
(E) NOTA

11. Which of the following lines contains the point (4, 7, -3) and lies in the direction $(-1, 9, 2)$?

(A) $(-t + 5)i + (9t - 2)j + (2t - 5)k$ (B) $(4t - 5)i + (7t + 2)j + (2t + 5)k$
(C) $(-t - 5)i + (9t + 2)j + (2t + 5)k$ (D) $(4t + 5)i + (7t + 2)j + (2t - 5)k$
(E) NOTA

12. What is the equation of the plane that contains the points (3, 6, 1), (1, 4, 1), (5, 0, -2)?

(A) $3x - 3y + z + 8 = 0$ (B) $10x - 16y + 6z + 60 = 0$
(C) $-6x + 6y - 16z - 2 = 0$ (D) $-2x - 6y + 3z + 3 = 0$ (E) NOTA

13. What is the adjoint of the matrix $\begin{bmatrix} 1 & 4 & 3 \\ -8 & 9 & 0 \\ 2 & 7 & 6 \end{bmatrix}$?

(A) $\begin{bmatrix} 54 & -10 & -27 \\ 48 & 2 & -24 \\ -74 & 1 & 41 \end{bmatrix}$ (B) $\begin{bmatrix} 54 & -3 & -27 \\ 48 & 0 & -24 \\ -74 & 1 & 41 \end{bmatrix}$

(C) $\begin{bmatrix} 54 & -3 & -27 \\ 48 & -12 & -24 \\ -74 & 15 & 23 \end{bmatrix}$ (D) $\begin{bmatrix} 54 & -10 & -27 \\ 48 & 0 & -24 \\ -74 & 15 & 23 \end{bmatrix}$ (E) NOTA

14. Which of the following is not an eigenvector of $\begin{bmatrix} 8 & 8 & 16 \\ 4 & 4 & 8 \\ -4 & -4 & -8 \end{bmatrix}$?

(A) $\begin{bmatrix} -e \\ e \\ 0 \end{bmatrix}$ (B) $\begin{bmatrix} -6 \\ 2 \\ 2 \end{bmatrix}$ (C) $\begin{bmatrix} -4 \\ 2 \\ 2 \end{bmatrix}$ (D) $\begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix}$ (E) NOTA

15. Let $C = \begin{bmatrix} \frac{3}{2} & \frac{-3\sqrt{3}}{2} \\ \frac{3\sqrt{3}}{2} & \frac{3}{2} \end{bmatrix}^5$. What is the value of C_{12}?

(A) $-\frac{243}{2}$ (B) $\frac{243}{2}$ (C) $-\frac{243\sqrt{3}}{2}$ (D) $\frac{243\sqrt{3}}{2}$ (E) NOTA

16. Let $T = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$. Monalice uses elementary row operations to obtain matrix H, the reduced row-echelon form of T. What is the value of H_{23}?

(A) 3 (B) 1 (C) 0 (D) 5 (E) NOTA

17. Let θ equal the angle between the vectors $\langle 1, 3, 1\rangle$ and $\langle 3,4, -2\rangle$. What is the value of $csc\theta$?

(A) $\frac{\sqrt{1914}}{30}$ (B) $\frac{5\sqrt{1914}}{319}$ (C) $\frac{\sqrt{1740}}{30}$ (D) $\frac{5\sqrt{1740}}{290}$ (E) NOTA

18. Consider the graph H with incidence matrix $G = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ and vertices v_1, v_2, v_3, v_4. How many walks of length 2 are there from v_1 to v_4?

(A) 1 (B) 3 (C) 4 (D) 6 (E) NOTA

19. Evaluate: $\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}^{2015}$

(A) $3^{2013}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ (B) $3^{2014}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

(C) $3^{2015}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ (D) $3^{2016}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$ (E) NOTA

20. What is the rank of matrix $= \begin{bmatrix}2 & 1 & 3 & 2\\ 3 & 2 & 5 & 1\\ -1 & 1 & 0 & -7\\ 3 & -2 & 1 & 17\end{bmatrix}$?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

21. What is the volume of the parallelepiped with adjacent vertices at the points (-3, 4, 2), (1, -5, 0), (1, 4, -4), and (2, 0, 4)?

(A) 275 (B) 276 (C) 277 (D) 278 (E) NOTA

22. A matrix M is reduced by a series of elementary row operations to the matrix $3I$, where I is the identity matrix. What matrix will be the result of the same sequence of row operations applied to $4I$?

(A) $81M^{-1}$ (B) $64M^{-1}$ (C) $7M^{-1}$ (D) $12M^{-1}$ (E) NOTA

23. What is the shortest distance between the skew lines $q_1\colon\ x+2=\frac{y-3}{2}=\frac{z-5}{3}$ and

$q_2\colon\ -x=\frac{y-1}{3}=\frac{z-4}{7}$?

(A) $\frac{341\sqrt{1985}}{1985}$ (B) $\frac{\sqrt{1985}}{341}$ (C) $\frac{\sqrt{1985}}{6}$ (D) $\frac{5\sqrt{6}}{1985}$ (E) NOTA

24. Let A be a 7 X 7 matrix such that $|A| = 4$. If $\det(8A) = 2^n$, what is the value of n?

(A) 20 (B) 21 (C) 22 (D) 23 (E) NOTA

25. What is the area of a triangle with vertices at $(1,-5),(-2,4)$, and $(-3,-7)$?

(A) 10 (B) 16 (C) 21 (D) 24 (E) NOTA

26. Let B be the inverse of the matrix $\begin{bmatrix} 1 & -1 & 8 \\ 1 & -2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$. What is the value of B_{32}?

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $-\frac{2}{9}$ (E) NOTA

27. What is the shortest distance between the planes $4x - 2y + 4z = 7$ and $y = 2x + 2z - 1$?

(A) $\frac{1}{6}$ (B) 6 (C) 1 (D) $\frac{5}{6}$ (E) NOTA

For questions 28-30, let $\vec{u} = \langle 8,-2,7\rangle, \vec{v} = \langle 1,3,-4\rangle$.

28. What is the area of the parallelogram formed between the two vectors $\vec{u}$ and $\vec{v}$?

(A) $2\sqrt{221}$ (B) $13\sqrt{14}$ (C) 30 (D) $\sqrt{91}$ (E) NOTA

29. What is $\vec{u} \times \vec{v}$?

(A) $\langle -13,39,26\rangle$ (B) -30 (C) $\langle 8,-6,-28\rangle$ (D) $\langle 9,1,3\rangle$ (E) NOTA

30. What is the projection of $\vec{u}$ onto the vector $\vec{v}$?

(A) $\langle 1,3,-4\rangle$ (B) $\langle -1,-3,4\rangle$ (C) $\langle 8,-2,7\rangle$ (D) $\langle -\frac{16}{9},\frac{4}{9},\frac{-14}{9}\rangle$ (E) NOTA